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THE VOLUTE IN ARCHITECTURE AND ARCHITECTURAL DECORATION

BY

REXFORD NEWCOMB

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ENGINEERING EXPERIMENT STATION

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REXFORD NEWCOMB

ASSISTANT PROFESSOR OF ARCHITECTURAL HISTORY

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THE VOLUTE IN ARCHITECTURE AND ARCHITECTURAL DECORATION

I. INTRODUCTION

1. *Purpose of the Study.*—Concerning the use of the volute in connection with the Ionic order much has been written. Varied and fanciful theories have been advanced to explain or prove its origin. But scarcely anything has been recorded as to the wide use of the volute in other structural and decorative architectural forms, or the lessons to be drawn from this use. As a matter of fact the volute is one of the most universally occurring of all art motifs, its early use as decoration antedating by many centuries its incorporation into the Ionic capital. It was with a view to pointing out the universality of the use of the volute and the bearing that this fact has upon the development of the Ionic order, that the present study was undertaken.

It is hoped, furthermore, that it will be possible to prove that the volute, as an art-form, is the natural outcome and the logical result of man's environment; that the volute is inseparably linked with nature, being one of the few configurations convenient to living cellular forms; that with the appreciation, by man, of this principle in nature, it becomes a part of his natural experience, a thing to be felt, as its rhythm or color; and that it would seem childish, therefore, to explain the introduction of the volute into the Ionic order as the result of man's imitation of the curve of any particular natural form, such as that of the sea shell, or the ram's horn, or to attempt to account for that introduction by the fact that certain spiral forms were extant in this country or that at a given time. It is intended to show that spiral forms are to be found in all countries at all times under one guise or another; that the volute has a much deeper and more fundamental meaning than any of those that have been claimed for it in the past; that it is a principle in nature, a very potent and fundamental law of growth, appreciable in all forms from the lowest to the highest; that what man experiences so universally in nature and feels so fundamentally, he is pretty likely to incorporate into his artistic expression; or, in other words, that the volute had become.

in very early times, an abstract, though dynamic, principle in thought and life, and, in art, "an element of beauty," appreciated by the mind of man,—that appreciation predicated, to be sure, upon biological and geological pre-conditions—"an element," akin to unity, rhythm, balance, and the various geometrical progressions in nature that have found their way into art.

It is the purpose of this study to develop this method of approach in order to explain the extremely wide and varied use of the volute in architecture, and ultimately to present what is apparently the natural and logical reason, as well as the valid argument, for the existence of the volute in an order of architecture.

H. THE VOLUTE

2. *Definition.*—The volute or spiral is defined in mathematics as any plane curve formed by a point that moves around a fixed center continually increasing its distance from it, or again, as the locus of the extremity of a line (or radius vector) which varies in length as it revolves about a fixed point or origin. These are general definitions and do not adequately describe the regular spiral, which is known as the "Spiral of Archimedes," and defined as a curve formed by a point moving with uniform angular velocity and receding from the center at a uniform rate.*

This last definition more nearly describes the spiral that is most frequently encountered in art. It is not, however, the spiral of nature. In other words, the spiral of art and architecture is not the spiral of nature, any more than any other art-motif is identical with the thing in nature that inspired it,† but is the spiral of nature, idealized and conventionalized into something more nearly approaching the abstract or ideal. That is, after all, the artistic process: art, if it is anything, is nature plus man's idealism. So then the volute in architecture occupies a place analogous to that of the Venus de Milo in sculpture. The Venus de Milo represents not what any Greek woman ever was, but what every Greek woman would like to have been—that is, the Greek woman idealized. Similarly the volute is not the spiral of the ram's horn, of the nautilus, of the snail-shell, nor of wave motion; it is all of these, or the spirit common to all, idealized and conventionalized from the natural originals.

It must be borne in mind that the great majority of art-motifs owe their birth to attempted realism. Forms are meant first to be lifelike, or at least, to suggest real objects. Conventionalization follows realism.‡ But even primitive man was not content merely to represent

* T. L. Heath, in the "Works of Archimedes," defines the spiral thus: "If a straight line drawn in a plane revolve at a uniform rate about one extremity which remains fixed and return to the position from which it started, and if, at the same time as the line revolves, a point move at a uniform rate along the straight line beginning from the extremity that remains fixed, the point will describe a spiral in the plane."

† Ruskin, in "Modern Painters," Vol. IV, Chap. XVII, discusses the mathematical development of the spiral of nature. Its development is illustrated by Fig. 1.

‡ Haddon, "Evolution in Art," p. 7.

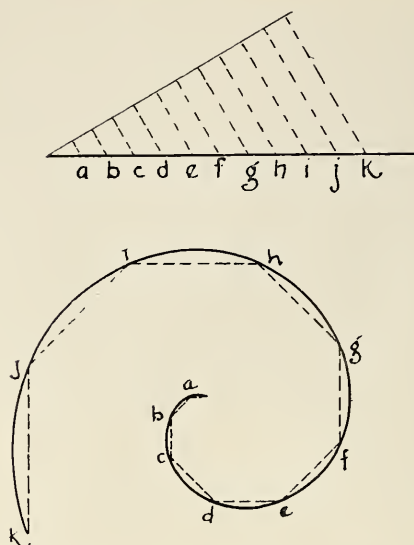


FIG. 1. THE SPIRAL OF NATURE, AFTER RUSKIN

realistically the various spiral forms. The earliest spirals found, although they are used in connection with animal or plant-forms, are conventional in treatment rather than naturalistic.

3. *Attributes of the Volute.*—In this connection it will be interesting and instructive to attempt to interpret something of the spirit or feeling of the volute. The characteristics of the volute are, perhaps, two fold. It is particularly unified in that it is related to a definite point or center, yet it is universal and all-embracing in that it is ever-widening as it recedes from that center in its generation, and infinite in its conception. Moreover, the volute is capable of two interpretations depending upon the way the mind follows the curve. It may mean from a very particular definite point out into the everywhere, or it may mean just the reverse—from infinity into the finite particular center. Thus it can be employed as a terminating motif in art, serving equally well as a beginning or as an ending.

This terminating characteristic is appreciated in nature. In certain of the animal-forms the center is the beginning, the process of growth taking place from this center outward; this is true of the spiral shell-forms such as the chambered nautilus, snails, and other

forms. In others, the process is reversed, as in the ram's horn, which grows out and curls up as it grows. This latter method of growth is also found in the case of most plant tendrils. On the other hand an unfolding fern-frond obeys neither progression; it comes through the surface of the ground fully voluted and then unrolls. Abstractly, however, we generally conceive the volute as progressing from the center outward in an ever-widening sweep.

In addition, the volute always inspires one with its dynamic character. It is one of the most engrossing and interesting curves in nature or art, and though not so difficult to trace as some decorative interlacings, it is, nevertheless, a rather involved curve for the human eye. The involved form, and the suggestion of the infinite, are enough to generate a mysterious interest and, perhaps for this reason among others, the curve has been widely used in art. Ruskin* has remarked "the inherent power of all representations of infinity over the human heart." The volute, however is scarcely as beautiful as some of the subtle "S" curves found in nature and perpetuated in art.

4. *Origin of the Volute*.—The introduction of the spiral or volute into art and its travel from the place of its early conception as an art-motif to the various early cultural centers have been the subjects of many discussions. So wide is the geographical distribution of the spiral in art, savage and civilized, that it would seem that it originally must have been introduced at many widely separated and geographically unrelated points.† May not its use have sprung up spontaneously in many cultural centers?‡

Just what was the source of the spiral motif is not so plain. Two theories concerning this have a place in any discussion; these are the theories that (a) derive the spiral from nature: or, (b) attribute to it a technomorphic origin.

Concerning the first theory much may be said. Nature has been very prodigal in her use of spirals and spirally inclined curves. The spiral is found throughout the earth in one form or another,

* Ruskin, "Modern Painters," Vol. II, p. 99.

† See Breasted, "A History of the Ancient Egyptians," p. 253. Bell, "Architecture of Ancient Egypt," p. 76. Hamlin, "History of Ornament," p. 46. Evans, "Scripta Minoa," p. 126. Hall, H. R., "Decorative Art of Crete in the Bronze Age," p. 9. Baikie, "Sea-Kings of Crete," pp. 48, 193, etc.

‡ Taylor, "The Origins of the Aryans," pp. 52, 132.

and of the universality of this curve we have ample proof, not only in these spirals of terrestrial origin, but also in the spiral nebulae found in the heavens about us.* In plant life, the spiral is a frequently recurring form, appearing in fern-fronds, tendrils, and

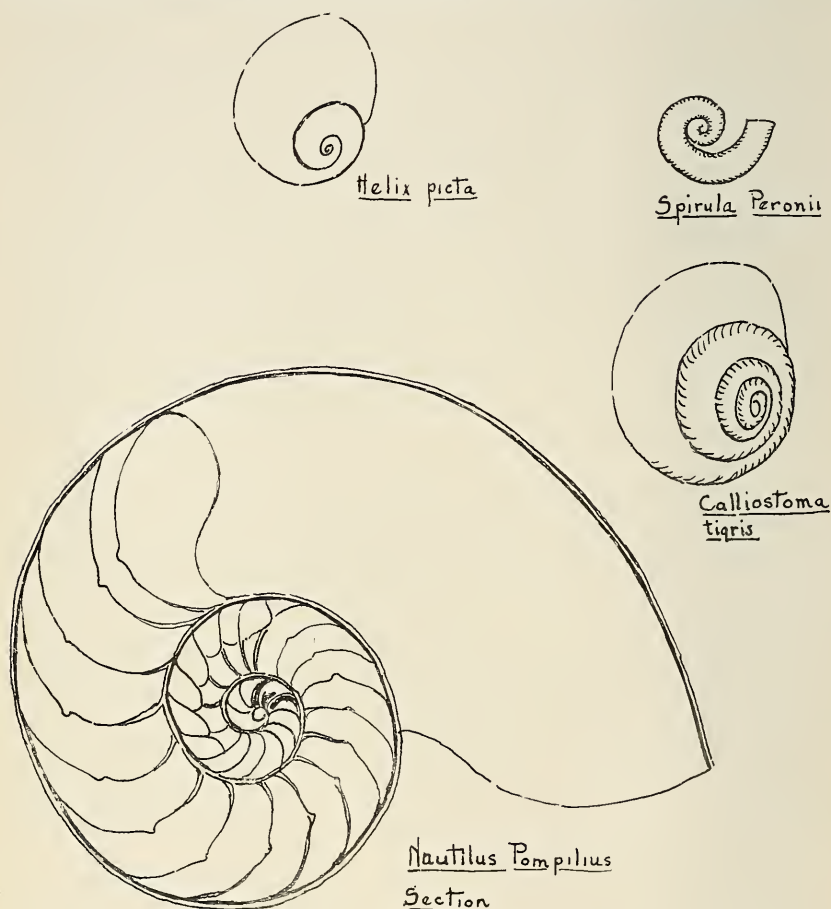


FIG. 2. SHELL SPIRALS, FROM NATURE

leaf curvatures. The vegetable origin of many volutes in art is betrayed by their use with vegetable or floral forms, and indeed this affinity is so marked that it has been the basis for an argument deriving all spiral or voluted forms from the Egyptian lotus.† In animal

* Herbert, "First Principles of Evolution," p. 15.

† Goodyear, "Grammar of the Lotus."

life, the spiral is, if anything, of more frequent occurrence than in plant life, being found in such forms as the chambered nautilus, various spirulæ, volutidæ, and numberless others (Fig. 2). It is characteristic of a whole family of land shell-bearing creatures, typified by our common snails. Moreover, it is a habitual bodily configuration of many animal forms such as worms and snakes, and is the curve found in the horns of certain sheep and goats, as well as being a characteristic termination of the antennæ of many insects. The origin of the spiral in art has also been attributed to a conventionalization of wind and water* motion, and for this reason the curve is often called the "wave-motif."†

The idea of the technomorphic origin of the spiral has been well developed by Dr. Holmes‡ in connection with his attempt to account for this motif on much of the pottery of the American Indian. Basket building with the Indian consisted, in many cases, of "raising" the basket by a coiling of grass ropes previously prepared, and of stitching or securing each succeeding coil to the one below it. This same method, in the absence of a pottery wheel, was used in "raising" pottery forms,¶ such as bowls or vases, and has, no doubt, been practised in all lands. When the basket or bowl thus formed was ready for decoration, what motif would be more appropriate and natural than the spiral, the symbol typifying the spirit or principle of the construction?§

From the earliest times, as everyone knows, women have wound their hair into coils of more or less spiral form. This habit has been utilized by the Latin authority Vitruvius** in his explanation of the Ionic volute. Moreover, the spiral or volute is arrived at, technomorphically, by the rolling up of skins, metal or paper.

Now it appears probable that in certain quarters of the globe the spiral may have had a technomorphic origin; but with the wide

* Holmes and Cushing, *Annual Report of the Bureau of Ethnology*, Vol. IV., p. 460, 1886. "This same figure, in use by the Indians of the interior of the continent, is regarded as symbolic of the whirlwind, and it is probable that any symbolizing people will find in the features and phenomena of their environment, whatever it may be, sufficient resemblance to any of their decorative devices to lead to a symbolic association." See also p. 515.

† Hamlin, *Op. Cit.*, pp. 46, 97.

‡ Holmes, "Origin and Development of Form and Ornament in Ceramic Art," *Annual Report of the Bureau of Ethnology*, Vol. IV, 1886.

¶ Mason, "The Origins of Inventions," Chap. V.

§ Elliott, "Prehistoric Man and His Story," pp. 296-7.

** Vitruvius, Book IV, Chap. I, ". . . in the capital they placed the volutes, hanging down at the right and left like curly ringlets."

distribution of spirals and spirally-inclined forms in all nature it seems likely that man must have appreciated the curve of the sea-shell, the spiral motion of the breaker upon the beach, or the spiral tendency of floral forms, long before he actually discovered the spiral in connection with his manipulation of materials. Indeed this very principle, appreciated in nature as a natural and logical method of construction since it is so prevalent in plant and animal structure, may have led to this method of basketry or vase-making.

The spiral then is a natural form, a form predicated upon natural structure. Curvature is a law of nature remarked by Ruskin,* and will be found in one form or another in all and through all phases of nature. These curves in nature have in a measure determined man's mental habits. Thus the human mind finds curves more pleasant than right lines, and the curves that approach infinity the most interesting and pleasing of all. The spiral, because it is among the curves more nearly infinite in conception, has from very early times greatly appealed to the mind of man, and has for that reason entered very deeply into his artistic expression.

Whether or not it can be established that the spiral is conventionalized wave-motion, the representation of a sea-shell, or the curve of a ram's horn, is immaterial. It is possible that all of these concrete phenomena and many more have inspired early artists with a love of the spiral at one time or another, but it was the beautifully rhythmic curve—the law and spirit behind all these specific forms—that man loved, and still loves, not the particular natural form. Whether or not the spiral was technically given form originally through a technomorphic process is likewise immaterial. Nature with her wealth of beautiful spirals was all about primitive man as she is about man today, and nature shaped his mental processes long before he began to manipulate materials. It would seem reasonable therefore to conclude that this curve entered art like most other forms from nature-inspired beginnings.

So then, in the last analysis, it must appear that the spiral is typical or representative of a law of growth in nature, and stands for a more fundamental principle in life and art than the mere representation imitatively by man of any one particular form would indicate. Once into art, from whatever source, it became a popular and engrossing motif for reasons already pointed out. Its popularity

* Ruskin, "Modern Painters," Vol. II, p. 200; Vol. IV, p. 187.

has prevailed and will continue to prevail as long as the mind of man bears the same relation to nature as in the past and the present. In other words the volute continues in art because it is in nature.

Now, while the volute as a motif has continued perennially interesting and beloved, the application of the volute in art is a thing that has changed and will continue to change. It is with this changing use of the volute that we are now concerned.

III. EARLY USE OF THE VOLUTE

5. *In the Arts.*—Early man by virtue of his mental limitations demanded that he see the actual line of the spiral curve. This demand led to the execution of the primitive spirals found in many parts of the world and dating back to the Bronze and even to the Stone Age.* In this connection it is interesting to note that the earliest spirals found in art are entirely abstract and completely divorced from their inspiration in nature. In later times, however, there are found cases where the single spiral and even running spirals form the motif of much decoration but in which the line of the spiral is largely hidden and obliterated by other detail. Even paintings have been conceived upon the volute as a motif, the volute itself being implied in the finished work. A good example of this type is Burne-Jones' "Golden Stair" (Fig. 3).

The earliest spirals of historic ornament appear as pure geometricalizations of nature, although some personal ornaments from the Bronze Age seem to be of technomorphic origin, and to have originated in the form of small coils of wire. Many examples of the two classes have been found in excavations, principally in Scandinavia, Ireland, Hungary, North Italy, Greece, and Egypt.

Most of these early spirals with the exception of some in Ireland, Greece, and Egypt, were used as decoration upon useful objects—arms, utensils, amulets, and fibulae. The spirals of Ireland, good examples of which we have surviving to this day, were cut upon stone monuments of one sort or another. Mr. Rolleston, in speaking of the spiral stone decorations at New Grange, Ireland, says: "Except for the large stone with spiral carvings and one other at the entrance of the mound, the intention of these sculptures does not appear to have been decorative except in a very crude and primitive sense, ... The designs are, as it were, scribbled upon the walls anyhow and anywhere. Among them everywhere the spiral is prominent. The triple and double spirals are also found, as well as lozenges and zig-zags."†

* Avebury, "Prehistoric Times," pp. 19, 44, 46. etc. Ward, "Historic Ornament," Figs. 39, 40, 41, 47, 50, 55 and 57. Munro, "Paleolithic Man," Plates LII, XXXIX, LXVIII and LXX. Reinach, "Apollo," p. 13, Fig. 12.

† Rolleston, *Op. Cit.*, p. 70.

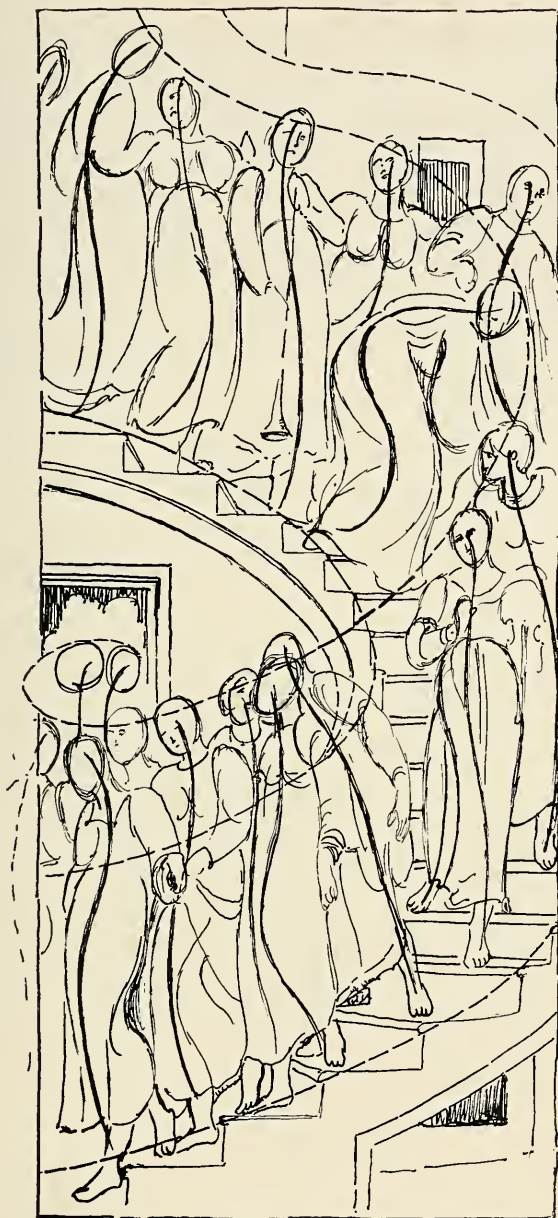


FIG. 3. "THE GOLDEN STAIR" OF BURNE-JONES, AN ANALYSIS



FIG. 4. GOLDEN DISKS FROM MYCENÆ AFTER SCHLIEMANN

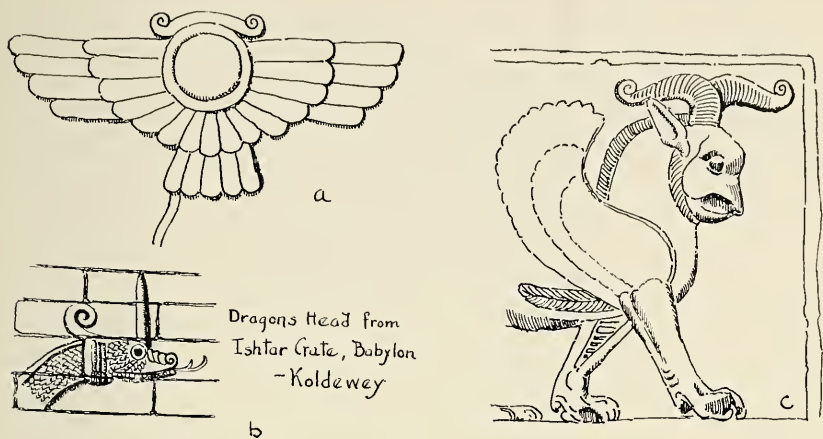


FIG. 5. ASSYRIAN ANIMAL SPIRALS, FROM VARIOUS SOURCES

Lord Avebury* in remarking on the differences in ornament in the Stone and Bronze Ages of northern Europe shows that circles and spirals were more prevalent in the Bronze Age than in the Stone Age. The Egyptian spirals of earliest occurrence were carved upon amulets and were usually geometrically executed, thus showing that the abstract curve was appreciated and loved aside from its association with any particular natural form. In the early Egyptian amulets the spiral was used as a space-filling motif around the scarabæus.†

This is not true, however, of the spirals executed in a period just a little later in the history of the race. The introduction of metals, bronze, gold, or silver, made possible a much wider artistic expression, and especially was this true in the case of Mycenæ, where have been found such golden disks as are shown in Fig. 4. Here spirals appear, although geometrical in spirit, as the antennæ of butterflies and the arms of octopi. These disks, together with certain vases found at Mycenæ and in Crete, show spiral forms midway between the very abstract spirals of early peoples and the more naturalistic spirals as found in Egypt and especially in Assyria, where antennæ, animal horns, and even hair have been given a spiral turn, but always in a naturalistic spirit‡ and never in a geometric as at Mycenæ (Fig. 5).

* Avebury, *Op. Cit.*, pp. 19, 44, 46.

† Hamlin, *Op. Cit.*, pp. 51, 53.

‡ See Jastrow, "Civilization in Assyria and Babylonia," Plates 31, 53, etc.

Spirals of floral origin make their first appearance principally in Egypt and Assyria. Perhaps the earliest of this type is the Egyptian lotus spiral. The priority, even here, is a matter of great debate among archæologists, some maintaining that Egypt originated the spiral motif and passed it on to the Tigris-Euphrates civilization and the Cretans, while others maintain the reverse was true and that the spiral came from Crete into Egypt.*

6. *In Architecture*.—Bell,† in speaking of the palace of Amen-hotep IV, situated upon the right bank of the Nile two hundred miles below Thebes, says: “It is said that the spiral was first used as an architectural ornament in this building.” However this may be, spirals of distinctly floral origin are to be found as wall and ceiling decorations in the Theban tombs of the eighteenth to the twentieth dynasties,‡ the use of spiral ornament becoming well authenticated and much used from that time on. Nor was the spiral ornament of the lotus type confined to Egypt. Many examples have been found



FIG. 6. ASSYRIAN IVORY CARVING, FROM THE BRITISH MUSEUM

* Baikie, "Sea Kings of Crete," p. 142.

† Bell, "Architecture in Ancient Egypt," p. 82.

‡ 1600 to 1100 B.C. (Berlin dating), 1580 to 952 B.C. (Petrie).



FIG. 7. ASSYRIAN IVORY CARVING, FROM THE BRITISH MUSEUM

in early Greek cultural centers, especially at Mycenæ, in the Palace of Tiryns,* and upon the ceiling of the flat-roofed chamber of the "beehive" tomb of Orchomenos† in Boetia, which is a well known case. Comparing the last mentioned Greek lotus spiral with the well known Egyptian examples it certainly appears, on account of its less skilful handling of the flower, and its cruder execution, the copy rather than the original.

Other examples of floral spirals are to be found in Babylonia and Assyria. Fig. 6 shows an Egyptian king holding a lotus stalk.‡ This small ivory carving was found in Assyria and goes to prove, among other things, the intimate connection between Egypt and Assyria at a very early period. The interesting thing, however, from the present view point is the voluted leaf at the base of the stalk. Here the spirally inclined forms are leaves and not the flowers themselves.

Another floral spiral form is to be found upon another small carved ivory block from the same case in the British Museum¶

* See Dürm, "Baukunst der Griechen," plate op., p. 31.

† See Guide to the Babylonian and Assyrian Antiquities, British Museum, 1908, p. 22.

‡ See Guide to the Babylonian and Assyrian Antiquities, British Museum, 1908, p. 22.

¶ Table-case F. No. 87, Nimroud Gallery, British Museum.

(Fig. 7). It represents the Assyrian sacred tree, a highly conventionalized form. The curious thing about these volutes is that they more nearly resemble animal horns than floral forms at all. Near the top of this fragment are shown two pairs of volutes set vertically against the central trunk and bound with thongs. These volutes resemble very closely in form and position the volutes used later, in Persian times, upon the columns of the Propylæa at Persepolis (Fig. 8). In Assyria this use of coupled voluted forms appears to have been widespread, examples in most cases having the curious little point at the division of the volutes noted in Fig. 9. In this connection it will be noticed that in all the Assyrian volutes the spirals spring vertically from the stalk or stem, and always appear as added decoration, and not as in any way related to structure. Mention has already been made of the wave-motion spirals, and spirals of this origin are often encountered in Assyrian ornament. They occur usually in conventional representations of water, the general body of water being represented by parallel chevrons or parallel undulating lines, with here and there a spiral wave. In these Assyrian examples the spirals are isolated* and not "running" as was true of the "Greek Wave" of a later period (Fig. 10).

These varied occurrences of the spiral in early art around the Mediterranean are introduced to prove the wide distribution of this universally loved curve. The examples present methods of handling so varied in the different countries that the conclusion must be drawn that the volute was developed, even if it did not originate, independently in many widely separated cultural centers. In the cases so far considered in this study the spiral has appeared as a motif of pure decoration used either as a beginning or as an ending motif in connection with other forms, animal or floral, as linear ornament, or as all-over decoration upon plane surfaces; and so far the spiral or spirally-inclined curves have been encountered in Egypt, Assyria, and the Ægean Area, the three early centers of culture as far as classical civilization is concerned. These facts indicate that the spiral form was appreciated in all these countries from very early times. It should also be noticed that the spiral was little used in connection with architecture except in Egypt and the Ægean centers of the Greek mainland, and even there it appeared as applied decoration, sculp-

* See Jastrow, *Op. Cit.*, Plates VII and LXIX. Maspero, "Ancient Egypt and Assyria," pp. 256, 257 and 334.

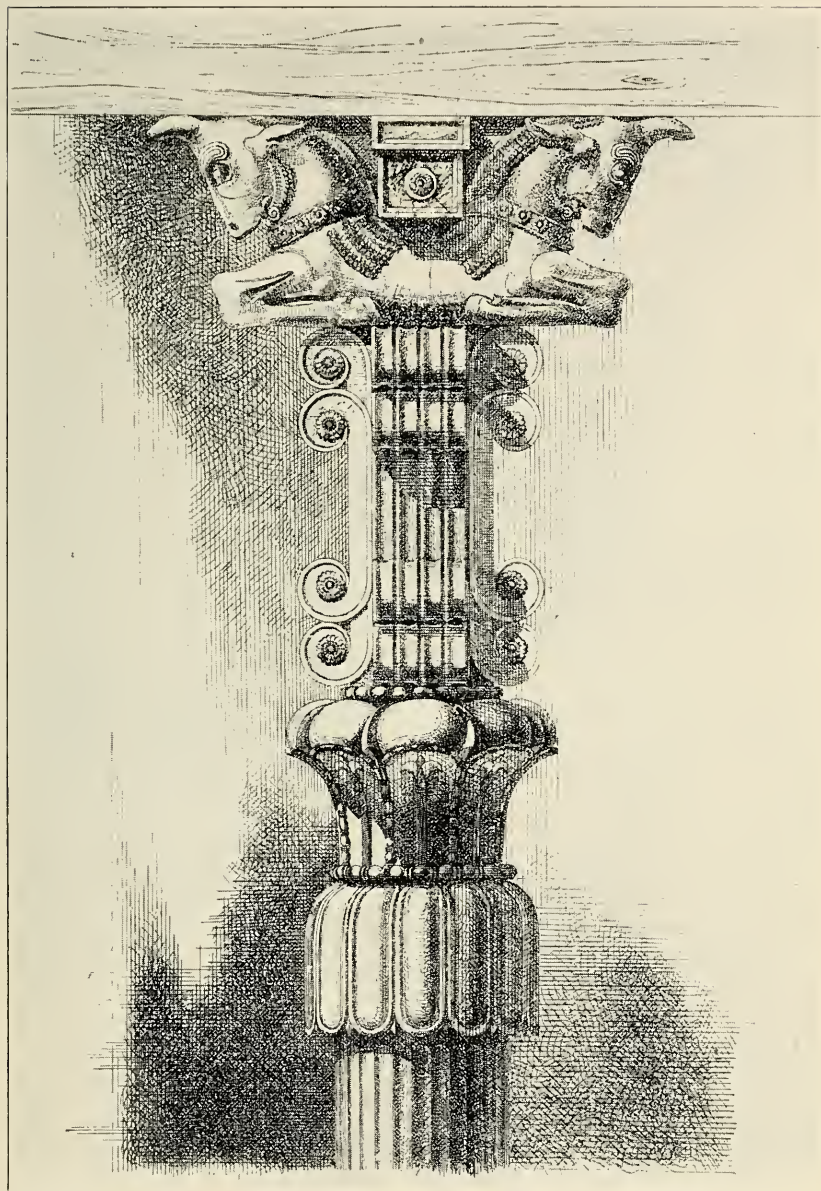
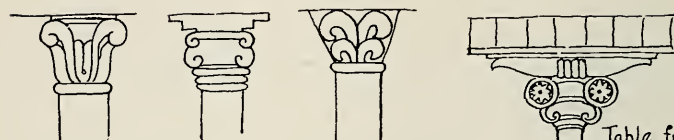


FIG. 8. PERSIAN CAPITAL WITH VOLUTES, FROM DIEULAFOY



c Volute Capitals from Assyrian Reliefs
From Reber

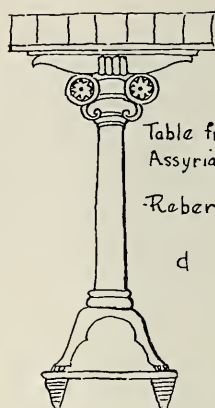
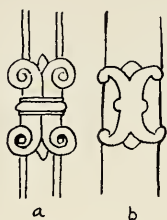


Table from an
Assyrian Relief
-Reber

d



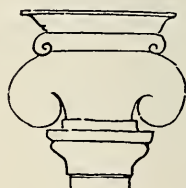
a

b

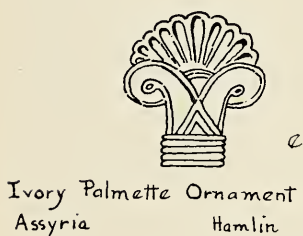
Spiral Ornament on Chairs

(a) Assyrian Relief

(b) From Miletus -Reber



f



Ivory Palmette Ornament
Assyria Hamlin

Assyrian Volute
-Hamlin-

FIG. 9. VARIOUS ASSYRIAN SPIRALS

tural or painted, rather than as pure architectural* ornament. Far less use was made of the spiral in the Tigris-Euphrates Basin than in Egypt or Greece, and there almost entirely in connection with

* Pure architectural ornament is ornament suggested or dictated by structure, such as bases, capitals, cornices, modillions, etc.

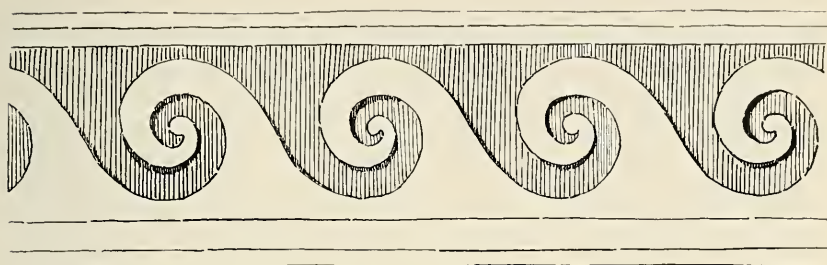


FIG. 10. WAVE MOTIF, TEMPLE OF MARS ULTOR, ROME, AFTER D'ESPOUY

pictorial representation (Figs. 5, 6, 7, and 9), and never as geometric decoration or architectural ornament.

Now the question as to the place of origin and subsequent route of the spiral naturally arises in the student's mind. Did the spiral originate in Egypt, pass into Assyria and then on to Asia Minor and the Ionian lands and Greece, as has often been claimed, or did it pass with the Ionians in the migration from Boetia and Attica to Ionia on the west coast of Asia Minor?

At first the settlement of this question would seem to have little bearing upon the question of the origin of the Ionic capital. A spiral would seem to be a spiral by whatever route it arrived. But in reality, the settlement of this question will determine whether it will be necessary to consider the Ionic capital as having been developed from the decorative, spirally-inclined, floral motive of Assyria into the structural motif of Ionia, or whether it will be possible to see in this form the indication of a lost structural prototype. If it came from Assyria, decoration preceded structure,* and the perpetuation of a decorative form occasioned the introduction of a structural entity; if it came from Greece, it may be possible to account for the structural origin of the capital first, and then its decoration as a logical elaboration of that structure.

The question as to whether the spiral came from Egypt into early Greece, i.e., Crete, Boetia, and Mycenæ, need not be considered here. It is not vital to the discussion whether this or the reverse† was true. In early Greek art is to be found, in both the hand-crafts

* This is the explanation of the origin held by all adherents of the decoration theory.

† As Hall (H. R.) and Evans contend.

and architecture, the geometrical spiral as contrasted with the spirally-inclined decorative forms of Assyria. The question stands: Did the spiral go to Ionia by way of Greece or by way of Assyria?*

The Ionians, in the time of Herodotus, claimed to have inhabited twelve cities in the northern Peloponnese and to have been driven from these cities into Attica by the Achæans, who were in turn driven out by the on-coming Dorians.† From Attica they were led by the son of King Codrus of Athens to the Asiatic coast, where they settled at Miletus. Lang,‡ in summing up the movements of these people, says: "It thus appears that the people later called, in Asia, 'Ionians,' had been dwellers on the coasts of Boetia and Attica as well as in the northern Peloponnese." The composition of the people known as Ionians of Asia Minor is not plain. Herodotus describes them as a "mixed multitude" and includes among them Phocians, Arcadians, Cretans, and others.§ The name,§ no doubt, was applied after the arrival of the emigrants in Asia.

The time of the Ionic migration is as yet not definitely settled, but late research places it after the Dorian invasion, and thus after the Ægean period of Greek history. "Without assuming any definite date, we may say," says Sir Edward H. Bunbury, "that recent research has tended to support the popular Greek idea that Ionia received its main Greek element rather late, and therefore after any part of the Ægean period."*** These conclusions are based upon archæological finds, and archæology is now depended upon to solve the Ionic question.†† In the absence of architectural remains of the earliest Ionic period that would prove, beyond a doubt, the origin of the Ionic order, other archæological evidence must obviously be considered.

Due to the excavations of Schliemann, Evans, and others, some idea of the culture that flourished before the departure of the Ionians for Asia Minor is possible. The resemblance between the structures

* A mass of evidence, not to be overlooked, is encountered in the finds upon the Danube and Po Rivers, in Scandinavia and in Ireland. Why is Assyrian influence not found in these spirals?

† Lang, "World of Homer," p. 140.

‡ Ibid, p. 142.

§ Leaf, "Homer and History," pp. 66, 296-297.

§ After their fabled leader, Ion.

** Bunbury, "Ionia," in *Encyclopædia Britannica*, 11th Edition.

†† Hogarth, "Ionia and the East," p. 12.

of Sparta,* in the Peloponnese, and one found at Neandria,† in the Troad, has been remarked by other writers. The votive objects found at Sparta include pottery in the "geometric post-Ægean style," together with ivory-covered fibulæ bearing many Ægean motifs, among which is the "double-coil," also characteristic of the Po and Danube Valleys.‡ The objects found upon the site of the Ionic Temple of Artemis at Ephesus (700 B.C. ?), as a result of the excavations of Mr. Hogarth, serve to show the intimate relationship of early Ionic art of Asia to that of the Greek mainland. Here Mr. Hogarth found thousands of votive offerings in gold, bone, ivory, paste, and crystal, ornamented with well accepted Ægean motifs, such as the "spectacles," a kind of double spiral, the "bouble axe" spirals, two animals opposed in heraldic fashion.§ These finds|| which he dates from about 700 B.C., or about two hundred years after the traditional migration, bore a closer resemblance to pure Ægean work than did the contemporaneous work in Greece. This fact will not seem strange, perhaps, when the mixture of blood taking place upon the mainland due to the immigration of northern elements is considered.

Attention is called to this same resemblance between the Ionic minor arts and those of the Mycenæan centers by Fowler and Wheeler,** who say: "In general it may be said that in the development of early Ionic ceramic art there are found more reminiscences of Mycenæan art than appear in the early styles of Greece proper, and this fact is rightly deemed of high importance, since the direct relation of Mycenæan art with that of Greece is as yet insufficiently established. . . . Motives taken from plant life are characteristic and point very likely to a Mycenæan survival." Mr. J. H. Hopkinson†† recognizes the "Sub-Mykenean" character of the ware of the Ionic Islands.

Professor John L. Myres‡‡ of Oxford, in speaking of Ionic culture, says: "Ionic culture and art, though little known in their earlier

* The Spartan finds were made within the precinct of Artemis Orthia at Sparta by the members of the British School of Athens. It is described as "the remnants of a temple in crude brick with wooden framework."

† Lang, *Op. Cit.*, p. 145.

‡ Munro, *Op. Cit.*, pp. 385, 415, 467.

§ As upon the Gate of the Lions at Mycenæ.

|| There is no doubt now that these were executed in the temple, as goldsmith's refuse has also been discovered there.

** Fowler and Wheeler, "Greek Archaeology," p. 455.

†† Hopkinson, *Journal of Hellenic Studies*, XXII, p. 47.

‡‡ Myres, "Ionians," in *Encyclopædia Britannica*, 11th Edition.

phases, derive their inspiration on one side from those of the old Ægean (Minoan) civilization, on the other from the oriental (mainly Assyrian) models which penetrated through the Hittite civilization of Asia Minor.* Egyptian influence is almost absent until the time of Psammetichus, but then becomes predominant for a time." Thus it will be seen that the culture carried into Asia Minor was Ægean and that there is a question as to the extent of the so-called Hittite influence upon the Ionian Greeks, who occupied a situation very near to Greece proper but considerably removed from the center of Hittite power. Here again must be recalled the wide difference between the spirals of Greece proper and those of Assyria.

The great number of true geometric spiral ornaments† occurring upon the Greek mainland during the Ægean period, as well as the general use of spirals of single and double roll in the valleys of the Danube and Po, must indicate something of the derivation, or at least the early distribution of this form, so far as the north side of the Mediterranean is concerned. The wide prevalence of this form is one of the striking archæological facts in connection with the arts of primitive peoples from Greece to Ireland. Upon the other hand the spiral ornaments of Assyria, Persia, and the Tigris-Euphrates Basin in general, are not pure geometrical spirals, such as we have in Ireland, the Danube Valley, or at Mycenæ and Knossos, but are spirally-inclined curves used in connection with other forms principally animal and vegetable. Had the use of the spiral among the Assyrians been anything like as widespread as among the Greeks, the excavations would surely have yielded some examples before now. Moreover, the absence of any spiral architectural ornament such as has been found in Greece from the earliest of times, certainly argues against the possibility of the volute having come from the East. The origin of the Ionic spirally-decorated capital must be sought not in Assyria but in Greece, or at least among Grecian peoples; and furthermore, too much emphasis should not be placed upon spirals which, from all appearances, are after all only decoration for a form that called for such decoration, and not the inspiration of that form in the first place. One must look further and beyond this decorative feature for

* Hogarth, under "Hittites," *Encyclopædia Britannica*, says: "The Greeks came too late in Asia Minor to have any contact with Hittite power, obscured from their view by the intermediate and secondary state of Phrygia."

See Spiral Fibula, "The Argive Heræum," Vol. II, Plate 85.

the origin of the form. The shape, the configuration of parts, had to exist structurally before it could be decorated at all.

An intensive study of architectural evolution has led to the conviction that what is at one time structure becomes, in a succeeding age, decoration. There are many examples to prove this law in architectural evolution and in other evolutionary processes as well. In biology there is the case of organs that persist long after their function has ceased. In art the mind demands, after long association, the perpetuation of forms that have ceased to have a significance, and these elements persist always as ornament upon succeeding structural forms. If the old* theory of the origin of the Ionic order be true, and the structural volute of the Ionic capital be derived from the decorative Assyrian floral volutes, it is the only example of a reversal of the foregoing law with which the writer is acquainted. The old theory stands at variance with all the ages of architectural experience, and this fact in itself serves to make the theory questionable.

Semper,† a German architect-critic, appears to have been among the earlier devotees of the Assyrian palmette theory, in which he derives the Ionic capital from the Assyrian palmette by a "process of gradual suppression of the leafy part and an increase of the scroll." Dr. Clarke‡ further elaborates and supports the theory of Semper. Goodyear sees no reason for seeking an Assyrian origin when evidence of direct lineage from the Egyptian lotus is so apparent. Marquand§ seems to agree as to this origin of the motif, when he says: "This type of decoration seems to have been derived from a floral prototype, possibly that of the Egyptian lotus." It will be noted that in each of these cases the decoration and not the form itself seems to have received the major consideration.

This purely decorative origin of the voluted capital has, however, been questioned by others. The contentions of Hittorff and Viollet-le-Duc, French authorities, have been well summed up by Professor Dürm in his "Baukunst der Griechen," when he says: "Diese

* See Goodyear, *Op. Cit.*, pp. 115-137.

Hittorff and Zanth, "Architecture antique de la Sicile," p. 335, Note 1 (Atlas, Plates 80-82).

Fletcher and Fletcher, "History of Architecture," p. 77.

Perrot and Chipiez, VII, Plates 52 and 53.

† Semper, "Der Stil in den technischen und tektonischen Künsten."

‡ A Proto-Ionic Capital from the Site of Neandria, *American Journal of Archaeology*, Vol. II, p. 1, 1886.

§ Marquand, "Greek Architecture," p. 195.

Umstände haben wohl Hittorff und Viollet-le-Duc schon vor langer Zeit bei ihren Erklärungen des jonischen Kapiells veranlasst, das geschnittene oder geschnitzte und bemalte Sattelholz als Ursprungsmotiv anzunehmen. Seine Form in Stein übertragen—unter Berücksichtigung der Materialeigen thümlichkeiten, also unter Vermeidung der zu grossen seitlichen Ausladungen—wird zum jonischen Säulen-Kapitell, besonders wenn dem Stammende noch eine entsprechende Bekrönung zugefügt wird.”*

Here there is at least an attempt, and a very plausible attempt, to explain upon structural grounds the origin of the capital. One would scarcely question the opinion of such distinguished authorities upon a problem of this sort. They were convinced, evidently, of some sort of structural origin for the capital. A similar conviction has led to a search for a more conclusive theory than that set forth by either Hittorff or Viollet-le-Duc.†

The wooden bolster-cap theory has appeared logical and in accordance with the early examples, but, upon the other hand, not quite the whole story. Why should the spiral, even though it was, as has already been shown, very much loved and universally used in decoration, have suggested itself as a capital decoration, especially when its carving upon the wooden bolster-cap meant the carving of the spiral across the grain? The longitudinal dimension of the volute cylinder would naturally be expected to run with the grain. These considerations have led to a search for a further explanation; and in this connection some preliminary assumptions have been made. In the first place, the fact that the Ionic capital presents two very different aspects depending upon whether one views the capital front-on or from the side, leads to the assumption that originally the form was placed so as to be seen from front or back and not from the side; this assumption accords with its use in many places in Asia Minor, especially upon such rock-cut tombs as those of Amyntas, and others in Lycia;‡ secondly, the volute cylinders standing normal to the plane of the façade, as they do, must be, since they indicate structure, the descendants of a former structural form now obsolete by virtue of a change of materials; thirdly, the attenuated proportions of the Ionic shaft point to a wooden origin, which has been assumed; and lastly, the omission of the frieze upon many of the early buildings

* Dürm, "Baukunst der Griechen," p. 246.

† Viollet-le-Duc, "Entretiens sur l'architecture," Lecture II.

‡ See Sturgis and Frothingham, "History of Architecture," Figs. 147, 148, 153, etc.

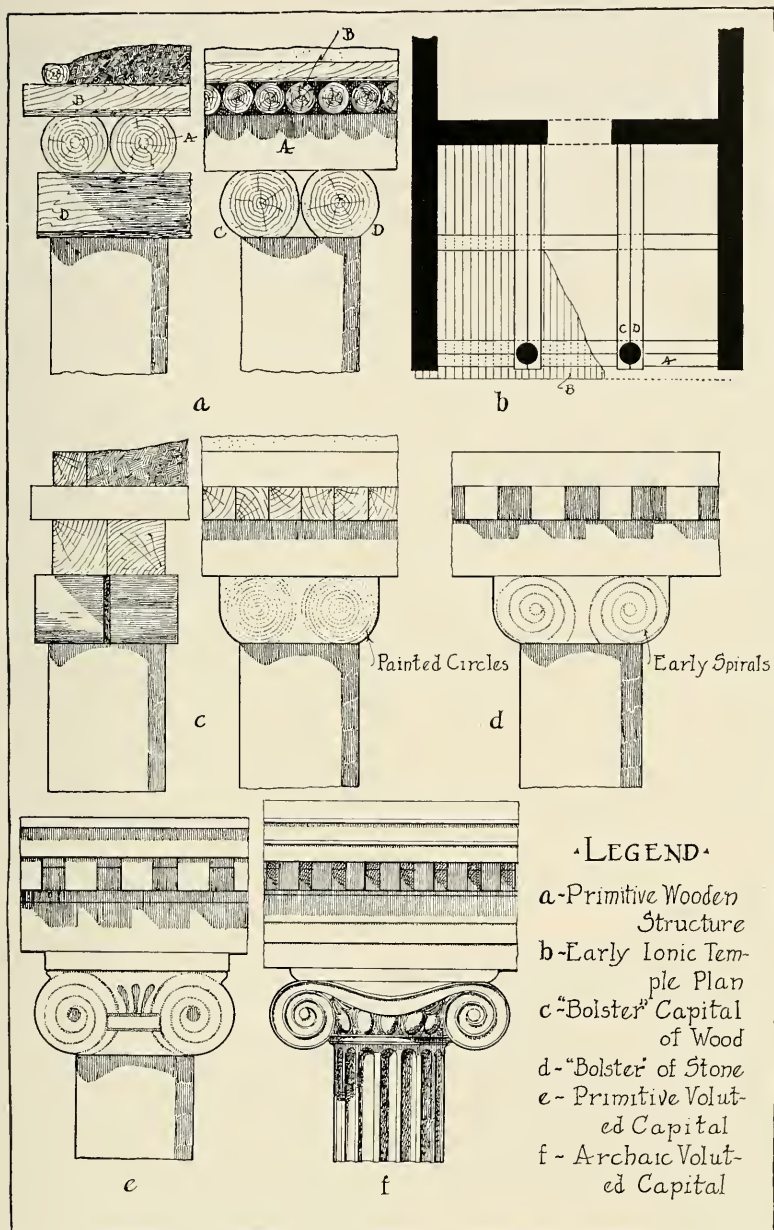


FIG. 11. EARLY DEVELOPMENT OF THE IONIC ORDER, A CONJECTURAL STUDY

in Asia Minor, and the enormous size of the dentils, are circumstances which necessitate a further investigation for a possible wooden origin, and an explanation that takes into account the structural significance of the capital, making it an integral part of the structure.

In any wooden restoration the architrave, obviously, becomes a row of beams spanning the distances between columns. These beams are indicated by A in Fig. 11a. A study of the Asia Minor tombs seems to indicate that the dentils were originally beams, or at least wooden members, running normal to the direction of the architrave. This structural significance of the dentils has long been held by many authorities and universally appreciated. These are restored as wooden beams at B, Fig. 11a. A study of the plan of the typical peristyle ceiling, seen from below, shows beams running back from the columns normal to the architrave (Fig. 12). This would indicate that, in primitive times, there must have been a member at least as important as the architrave spanning the distance between the columns and the wall. In Fig. 12 it will be noted that this beam is much heavier than similar beams running perpendicular to it due to the fact that these smaller beams are borne upon it. In making the restoration shown in Fig. 11b, following the structural logic of primitive peoples, who often double or triple structural units in order to produce greater strength, two beams have been substituted for each of the heavier ones shown in Fig. 12 and one beam for each of the smaller ones. Since, unquestionably, the earlier Ionic temples were *distyle in antis* on plan, there was no peristyle, and the row of columns upon the flank, shown in Fig. 12, became a wall. (See Fig. 11b.) The double beams C and D suggest, at their ends, the cylinders of the Ionic capital.

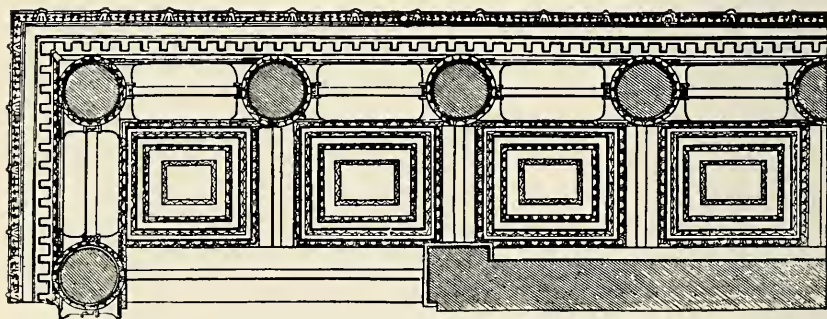


FIG. 12. CEILING PLAN, PERIPTEROS, MAUSOLEUM OF HALICARNASSOS, FROM REBER

But whence the volutes? It has long been noticed that, in primitive art, spirals and concentric circles have been used concurrently. This was true at Mycenæ, in Crete, and no doubt in Asia Minor. This tendency has been remarked by Haddon* and Dr. Montelius, both of whom noticed that series of circles, when joined with recurved lines, as is often the case upon the gold disks of Mycenæ and upon similar objects found in Central Europe, Scandinavia, and Ireland, have the appearance of spirals unless very closely observed. Indeed, this illusion is so prevalent that Dr. Montelius has called the form a "bastard spiral." Both these writers consider the concentric circles as degenerate spirals, but their arguments are applied to the use of spirals among primitive peoples. Now among very primitive peoples the truth of the argument may perhaps be granted, but it is scarcely permissible in the case of the Ionic Greek artist. The tendency of the artist in the ascendant period of Greek art was to seek refinement, rather than to reduce any of his art to rule of thumb or easy mechanical equivalent, a procedure which cheapened Roman work. With the log ends in use, the next natural consideration would be their decoration, and the concentric annular rings of the log suggested a set of concentric circles as ornament. This ornament was at first no doubt painted on and, indeed, the practice of painting these concentric circles upon the capital may have been continued long after the form suggested by the log ends had become the "bolster capital" of Hittorff and Viollet-le-Duc (Fig. 11c). Since it would have been difficult for these early people to carve wood across grain, especially if the pattern were that of concentric circles or spirals, it is reasonable to believe that the sculptural delineation of the ornament upon the capital may not have been practiced until after the translation of the capital into stone (Fig. 11d).

How long it took the concentric circles to become many-whorled tangent elementary spirals would be difficult to say, but probably the concentric circles persisted until after the log ends had given way to the "bolster capital" and, indeed, may have persisted down to the adoption of stone, but so far no capitals, however early, have been turned up that carry the concentric circles. On the other hand the earliest known capitals present primitive tangent many-whorled spirals, the sculpturing of which is so flat that the spirals seem scarcely

* Haddon, *Op. Cit.*, p. 93.

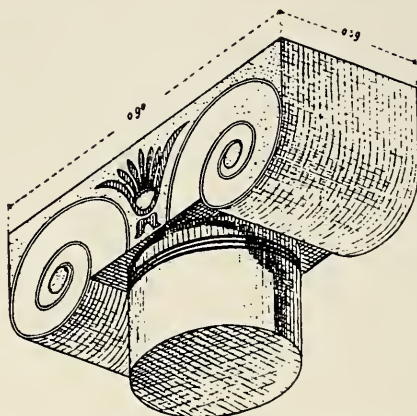


FIG. 13. IONIC CAPITAL FROM DELOS, FROM PERROT AND CHIEPIEZ

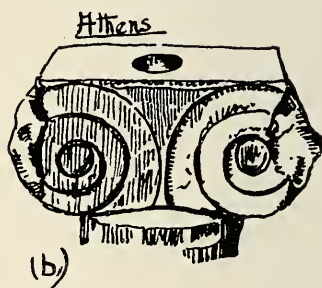
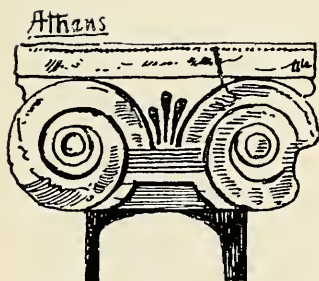


FIG. 14. ARCHAIC CAPITALS FROM ATHENS, AFTER DÜRM

more than shallow scratches upon the surface of the block. Indeed, so primitive are the Athens capitals (Fig. 14) that they appear almost as stone translations of the log ends, which persist now only as ornament, the beam itself having been raised to the level of the architrave and not indicated upon the elevation at all. These archaic

capitals from Athens are the most primitive Ionic capitals yet brought to light; and this fact has led some authorities to hold that the Ionic capital was developed in Greece proper. In these examples there has been no attempt to connect the volutes. There is on the more primitive of the two examples shown (Fig. 14*b*) no intermediate "palmette" as upon that shown in Fig. 14*a* and upon later capitals, and the projection is not great; Fig. 14*a* shows a tendency toward the linear type which has been called, for the sake of clearness, the "bolster" Ionic.

Obviously, the translation of this wooden structure into stone occasioned considerable modification, and it would seem in the light of some of these early Ionic capitals that some modification had already taken place before this change from wood to stone. The intermediate form between the conjectural wooden type shown in Fig. 11*a* and such capitals as those from Delos (Fig. 13) must have been a type very similar to the "bolster" capital of Hittorff or Viollet-le-Duc. In the Delos capitals the wide separation of the volutes and the general linear tendency points to a "bolster" cap prototype. Marquand* has pointed out this tendency in early Ionic capitals when he says: "The essential rectangularity of the Ionic capital is most evident in an archaic example from Delos, in which a single rectangular block has been but slightly modified in form." This rectangularity of some of the early capitals accounts, no doubt, for the bolster-block theory of the French authorities.

In these primitive types the echinus is omitted and thus it would seem to have been a later refinement. Its introduction dates, doubtless, from some time after the change in materials from wood to stone, and it may be only the echinus of the Doric capital used as a support for the voluted "bolster-block," the volutes of which naturally had to be separated in order to make a nice union with the echinus.

Such capitals as those found at Neandria in the Troad (Fig. 15), and Messa in the Island of Lesbos (Fig. 16), appear as elaborate attempts to make the early spiral capital born of structure a decorative form. Even here the spirals appear distinctly Ægean in character rather than Assyrian, being spirals of several turns, while most of the Assyrian volutes are only spirally-inclined forms. In these cases some oriental influence may be granted, but this would not nullify the arguments in favor of a structural origin. Oriental forms are

* Marquand, *Op. Cit.*, p. 92.

chiefly decorative and rarely simply structural, and Ionic artists may have welcomed some suggestion to make the new structural form decoratively agreeable. On the whole, however, it seems that the Assyrian influence upon Ionic peoples has been tremendously exaggerated. No Assyrian voluted capitals are known, unless the very crude spirally-inclined capitals copied from Assyrian wall paintings (Fig. 9) should be called voluted capitals.



FIG. 15. IONIC CAPITAL FROM NEANDRIA, AFTER DÜRM

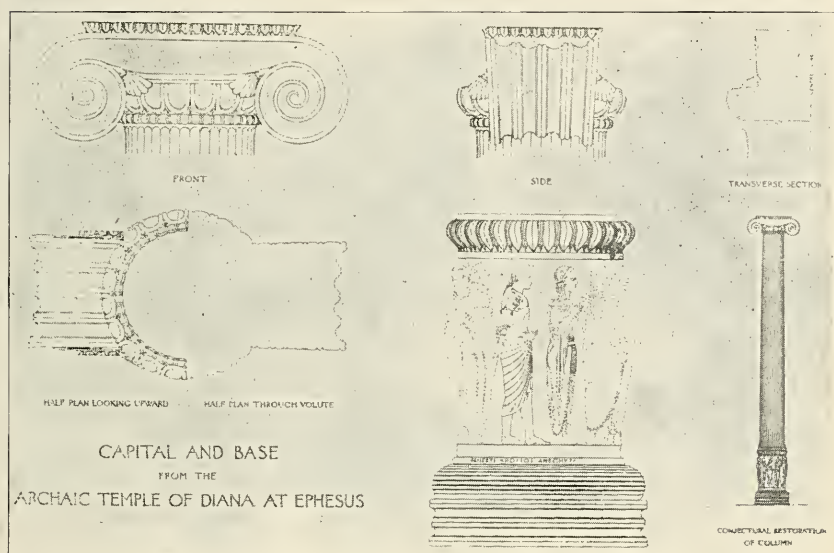


FIG. 16. IONIC CAPITAL FROM MESSA, AFTER DÜRM

It is strange that these forms should appear in wall paintings and yet not be realized in excavation, and thus appear in the guise of fanciful conventions of the artists rather than as living forms of architecture. It is probable also that had the voluted capital been known and used by the Assyrians it would have been perpetuated by their successors the Persians. The only Persian capital with volute decoration is that found upon the Propylæa at Persepolis (Fig. 8).

Here the volutes are used in the truly oriental decorative manner, a spiral decoration against a vertical member, a method noted in Fig. 7. Its late date, 480 B.C., would exclude this example from any consideration in studying the origin of the Ionic capital.

The earliest well authenticated example of the Ionic voluted capital is that from the Archaic Temple of Diana (Artemis) at Ephesus, which dates from 550 B.C., and is now preserved in the British Museum (Fig. 17). It is certain that the archaic capitals from Athens



(Fig. 14), Neandria* (Fig. 15), the Naxian votive column capital (Fig. 18), and the capital from Naukratis† (Fig. 19), appear older

* Koldewey holds that the Neandrian capitals are not proto-Ionic but a new type which he calls "Æolie." This capital with its vertically springing spirals is certainly more reminiscent of the Assyrian wall-painting spirals than any so far discovered. Yet even here may be seen, after all, an attempt actually to carve in stone the spiral ornament painted upon the block capitals of Athens.

† Naukratis was a center of cultural activity during the Ptolemaic Period, and just previous to the founding of Alexandria was the chief trade center of lower Egypt. It was given as a trading place to the Greek traders by Psammetichus I and due to the supremacy of the Greeks temples were built there to Greek gods. The capital referred to, the volutes of which are missing, is said to have come from a temple of Apollo. See King and Hall, "History of Egypt," p. 436.

than the Ephesian capital, principally upon account of their naïve archaic composition, but just how much older is a question. Anderson and Spiers* consider the Naukratis capital, an example discovered by Professor Flinders Petrie, and dated by him at about 650 B.C., the oldest capital of this variety known; but it must be admitted that the votive column of the Naxians at Delphi has claims to a greater age† than the Naukratis capital. Especially will this be noted in the insufficient attempt in the Naxian column to stop the flutes under the

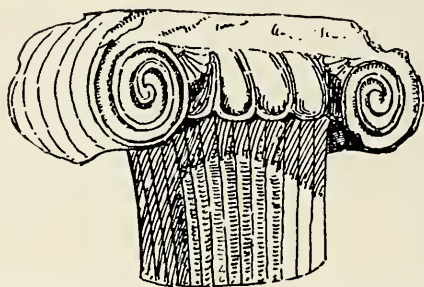


FIG. 18. NAXIAN VOTIVE COLUMN CAPITAL, DELPHI, AFTER ANDERSON AND SPIERS

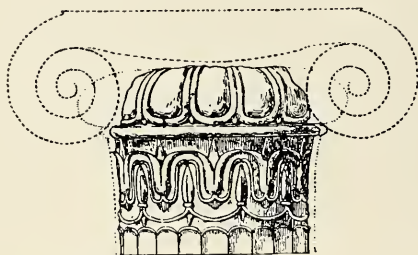


FIG. 19. CAPITAL FROM NAUKRATIS, EGYPT, FROM ANDERSON AND SPIERS

embryo echinus, a thing which is adequately accomplished in the Naukratis capital.

It is a failing of historians and archæologists to place anything found upon Egyptian soil in the list of Egyptian antiquities and to attribute to all Egyptian finds a great age. The Naukratis capital is clearly an Egyptian‡ interpretation of a Greek form entirely out of keeping with Egyptian thought, even of the Ptolemaic Period, but no doubt done upon Egyptian soil by Greek artists.

* Anderson and Spiers, "Architecture of Greece and Rome," p. 56.

† Upon the basis of internal evidence.

‡ Egyptian, in the sense that it was discovered upon Egyptian soil.

The difference between the Archaic examples from Athens and the Naxian capital is even more pronounced than the difference between the Naxian and the Naukratis types, and, from internal evidence, these capitals represent the most archaic Ionic type. In view of these finds at Athens it may be necessary to admit eventually that the earliest voluted capitals were developed in Greece, and carried into Ionia, where they were elaborated and modified decoratively by virtue of oriental association. The internal evidence* of course is not conclusive and the matter must be left thus until further finds of archaic types are reported. All the evidence so far presented, however, indicates that the Ionic voluted capital is not the product of Assyrian decoration but the development of a Grecian, or perhaps an Ægean, structure. Its birthplace was in the west of that day, not in the east (Tigris-Euphrates Basin).

* The internal evidence may indicate either of two things:

- (a) a more primitive type of development, or
- (b) a retrogression from an advanced type, due to a barbaric handling.

IV. THE VOLUTE AS A FEATURE OF THE CLASSIC ORDERS

7. *The Ionic Development.*—In the appendix is given a chronological arrangement of the various historic specimens of the Ionic voluted capital, but before discussing the voluted forms subsequent to the capital from the Archaic Temple of Diana at Ephesus, some summary should perhaps be made of the handling of the volute as a feature of the Ionic capital up to that time.

Upon the archaic examples from Athens (Fig. 14*b*), no attempt is made to connect the spiral ornaments of the block. In Fig. 14*a* there is apparent an attempt, naïve as it is, to join the two spirals in some kind of a composition. Here perhaps is found a reference to a floral inspiration, a thing very natural when the general use of spirals that had been made in Greece up to this time is remembered. In each of these examples the spirals spring vertically from the shaft, perhaps in obedience to the principle that transition from a vertical to a horizontal should be emphasized. In the case of Fig. 14*b*, however, the ornament does not suggest a floral derivation, and the spirals are so close together as to indicate that the ornament was meant to be abstract and not suggestive of any particular form.

In the Neandrian capital, which has been heralded as a distinct type by Koldewey, the spirals still spring vertically, and there is some attempt to fill the triangular space with floral ornament. This capital betrays most clearly its inspiration, and appears as an ultra-decorative version of the voluted form, rather than as following the line of development that culminates in the Ephesian capital.

In the case of the capital from Delos (Fig. 13) there is more of an attempt to unite the volutes, which here spring horizontally; but even here they still are separated by a small floral form. The echinus which later forms an important part of the Ionic capital, here becomes prominent, and, although the projection of the volute is still very great, and the unfluted shaft makes a poor junction with the echinus, unmistakable signs of development are apparent.

With the Ephesian capital there comes a complete union of the volutes, although the great projection sidewise is still a feature. In

this example also is found for the first time the cushion used upon the horizontal portion of the volutes, a feature which gives the Greek Ionic examples much of their grace and lightness, and a feature which was discarded later, due most likely to the increasing importance of the echinus, much to the detriment of the form.

From the Archaic Temple of Diana at Ephesus it is possible to trace the development of the Ionic capital to its perfection upon the Erechtheum. This can be done by a study of Figs. 20, 21, 22, and 23. The subsequent story of the Ionic order, its introduction and use at Rome, and its revival during the Renaissance, as well as its general employment during recent times, are matters of most common knowledge. It has not seemed necessary to review this mass of material so familiar now to the layman as well as to the student and architect, except perhaps in a pictorial way, and to this end a set of illustrations has been included (Figs. 24, 25, 26, 27, 28, and 29).

8. *The Corinthian Capital*.—Something of the use of voluted forms upon bell or basket-shaped capitals should be recalled here. This use of the volute is found not only upon the Corinthian capitals of Greece (Fig. 32), and Rome (Figs. 33, 34, 35, 36, and 37), but also upon such bell-shaped capitals as that from Philæ (Fig. 31), an example dating from the Ptolemaic Period and due most likely to Grecian influence. Fig. 30 will serve to recall the revival of this type during the Renaissance. But this use of volutes is now very generally appreciated, and these illustrations should serve to delineate the general trend of that use.

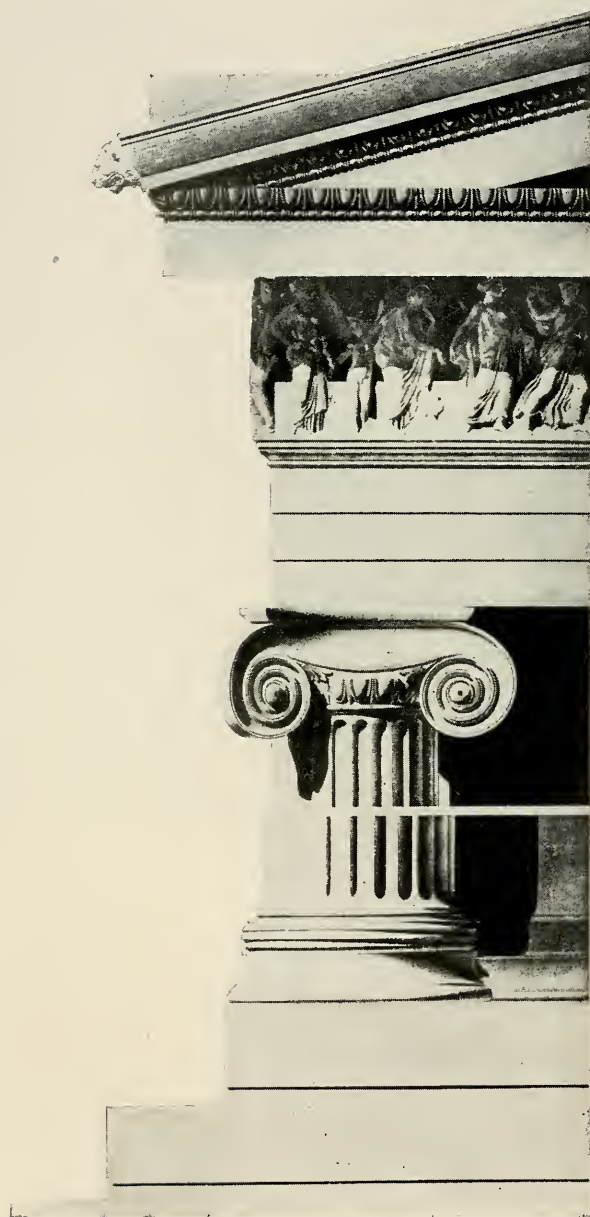


FIG. 20. CAPITAL FROM THE TEMPLE OF NIKÉ APTEPOS, FROM D'ESPOUY

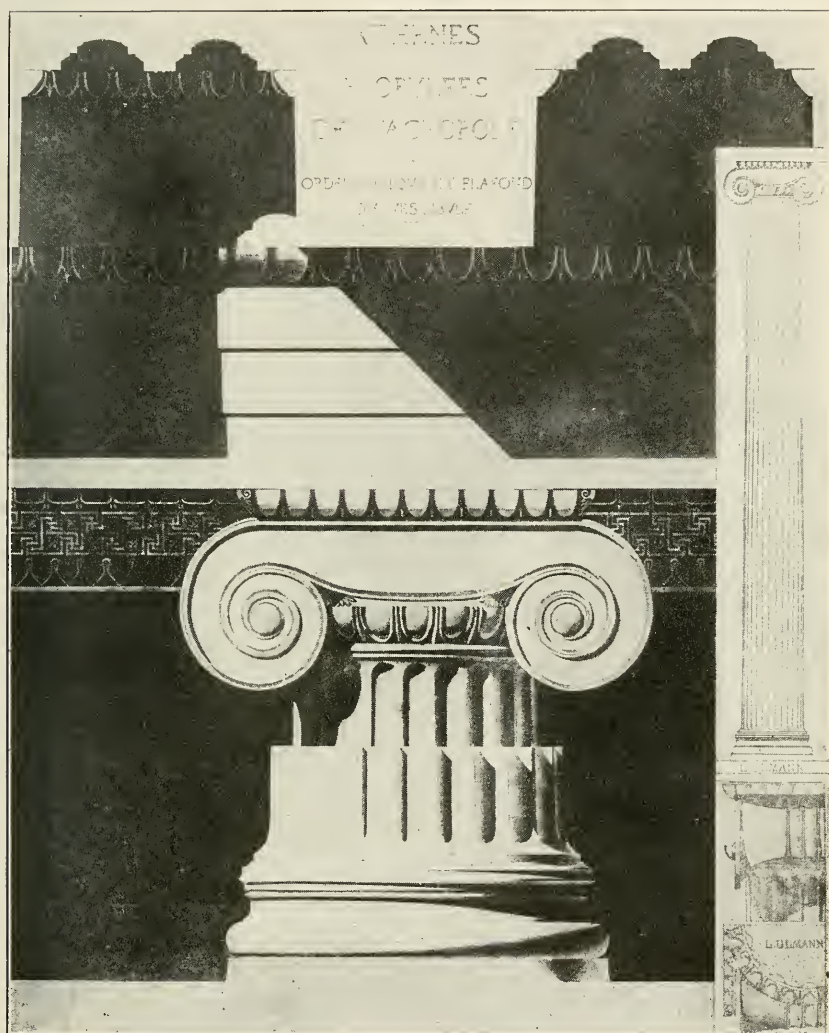


FIG. 21. CAPITAL FROM THE PROPYLÆA, ATHENS,
FROM D'ESPOUY

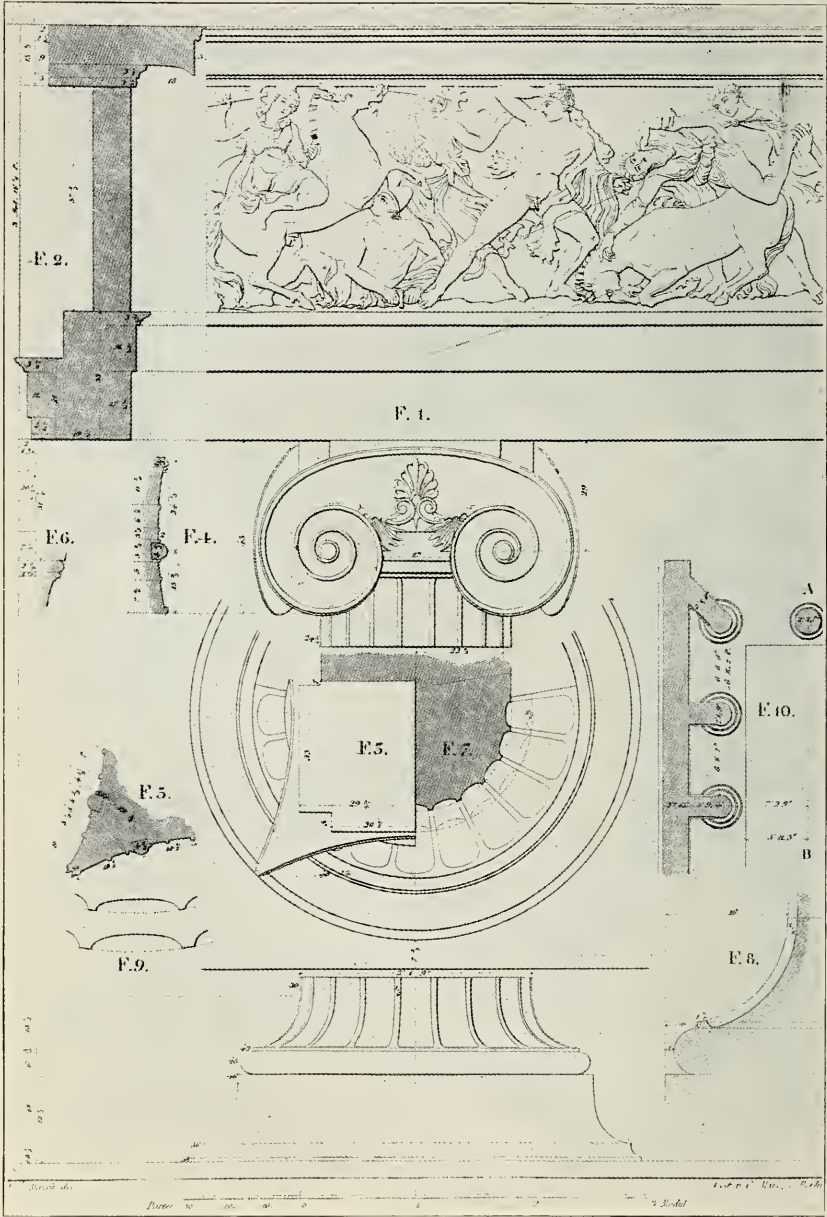


FIG. 22. CAPITAL FROM THE TEMPLE OF BASSÆ, FROM MAUCH

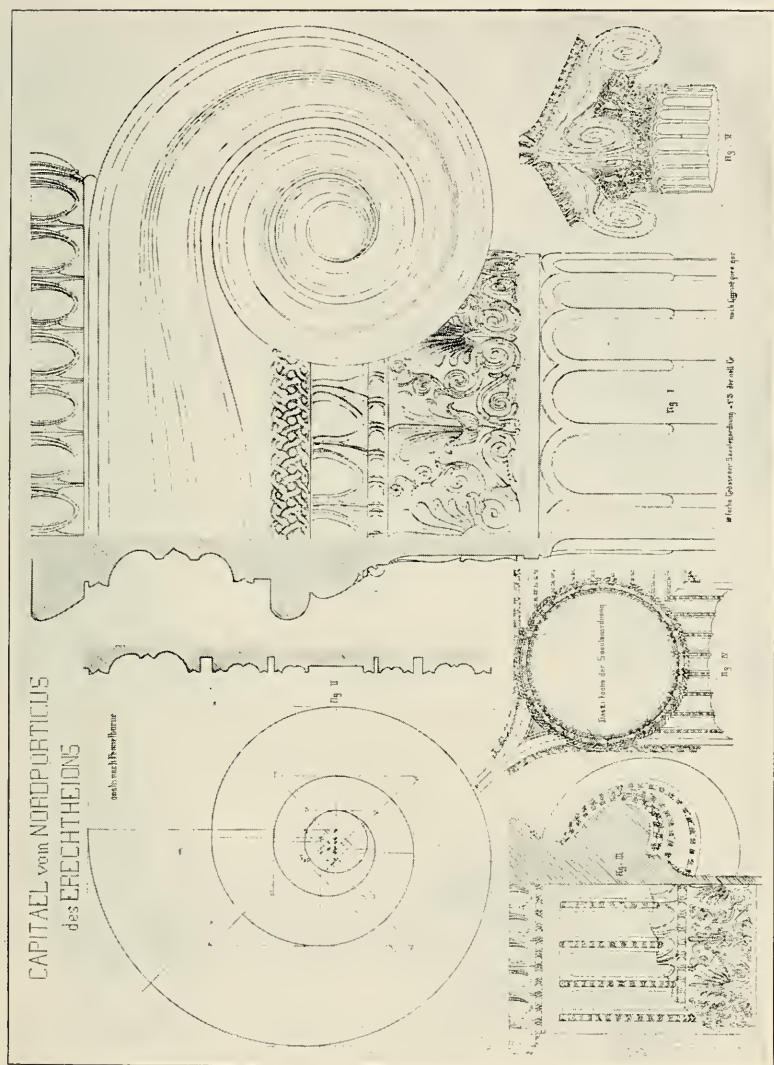


FIG. 23. CAPITAL FROM THE ERECHTHEUM, ATHENS, FROM UHDE.

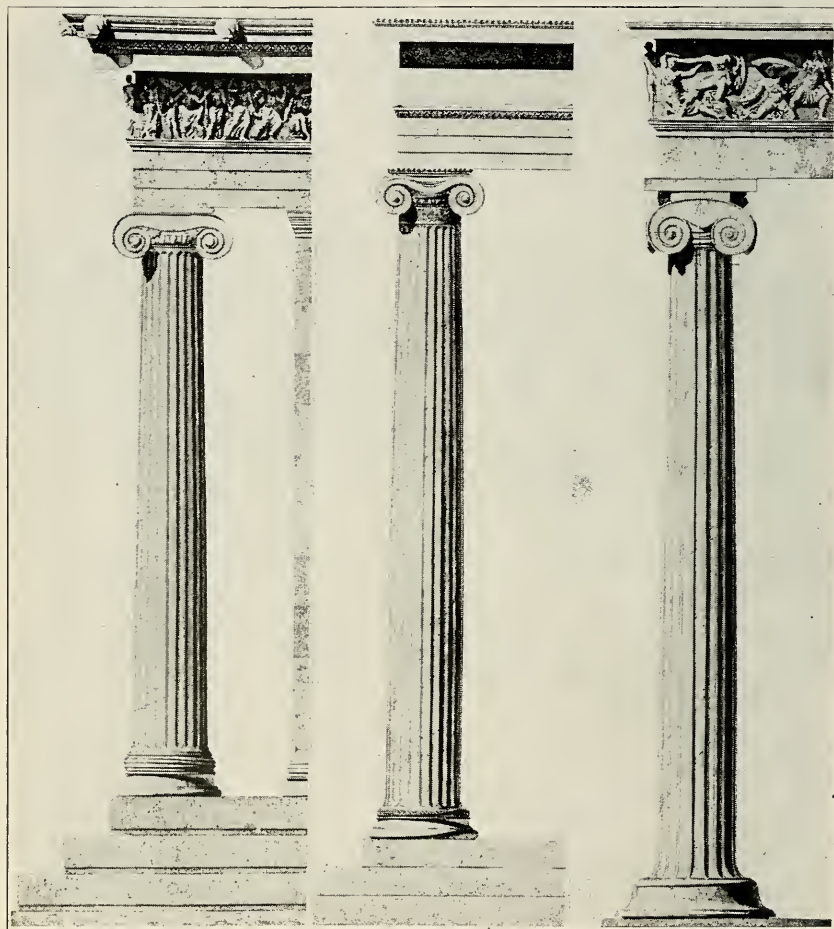


FIG. 24. COMPARATIVE STUDY OF IONIC CAPITALS, FROM UHDE

From left to right:

Column and Capital from Temple of Nike Apteros, Athens;
Column and Capital from North Portico of Erechtheum, Athens;
Column and Capital from Temple of Apollo Epikurios, Bassæ.

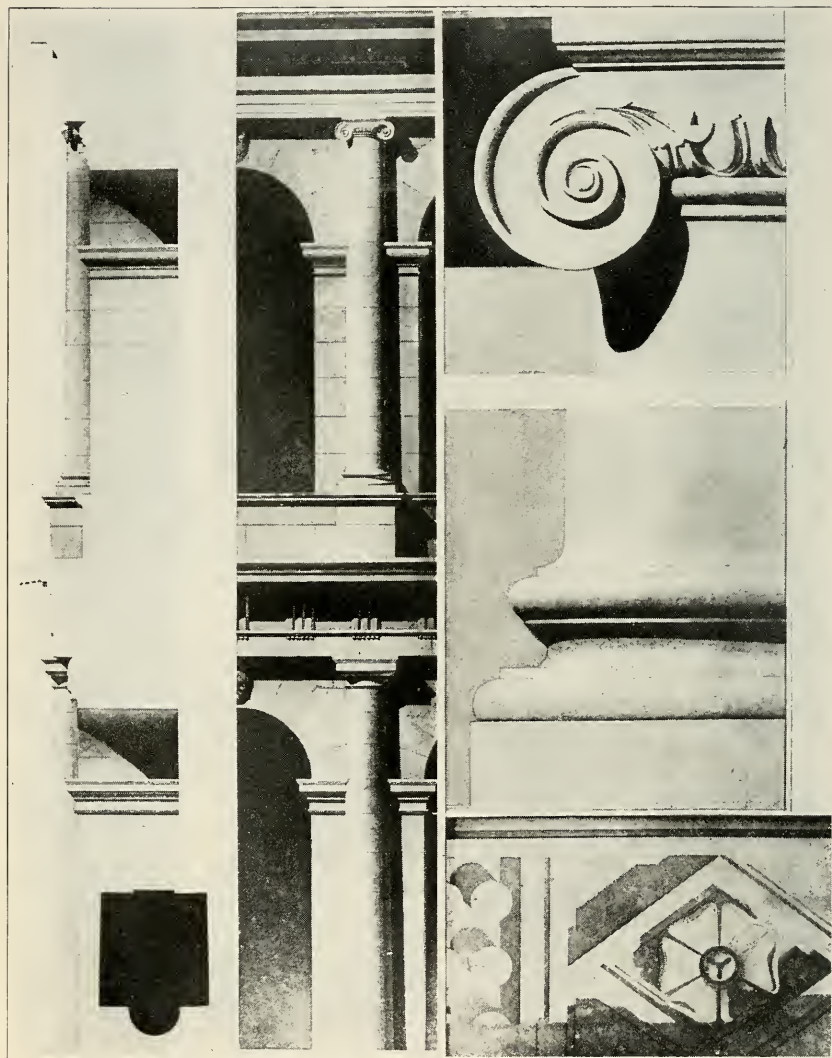


FIG. 26. IONIC CAPITAL, THEATRE OF MARCELLUS, FROM D'ESPOUY

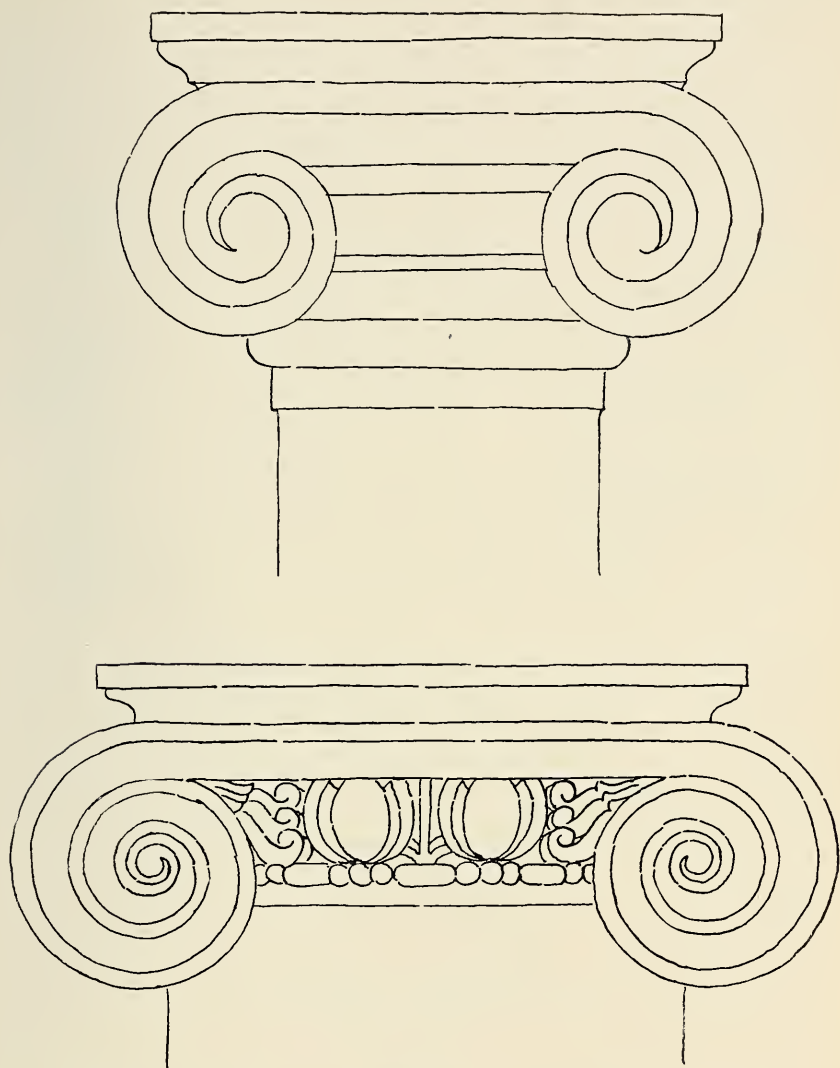


FIG. 27. IONIC CAPITALS, AFTER STEGMANN AND VON GEYMÜLLER

Above: Capital from the Canonica di San Biagio at Montepulciana.
Below: Capital from Church of S. Agostino—Monte Savino.

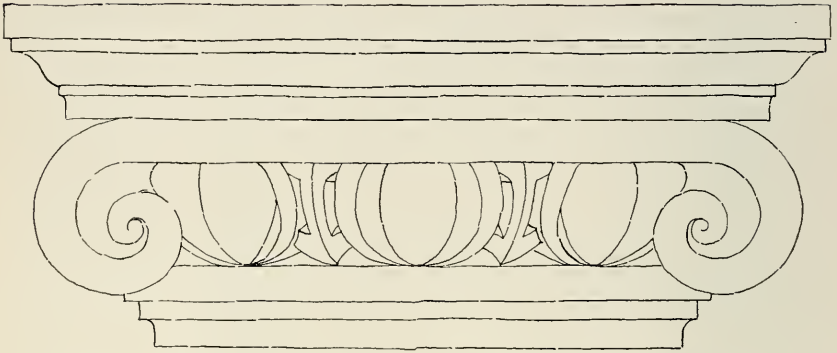


FIG. 28. IONIC CAPITAL FROM PALAZZO NOBILE AT MONTEPULCIANO,
AFTER STEGMANN AND VON GEYMÜLLER

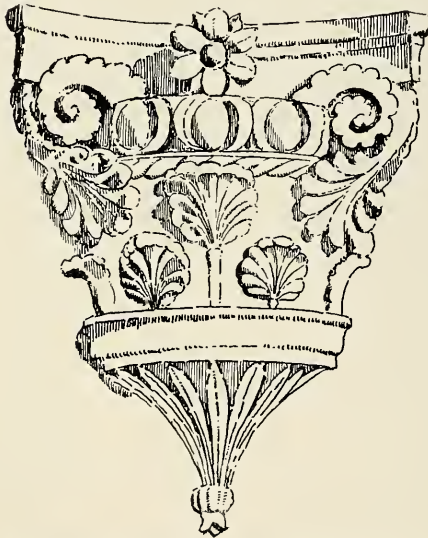


FIG. 29. CAPITAL FROM FLORENCE, AFTER STEGMANN AND VON GEYMÜLLER

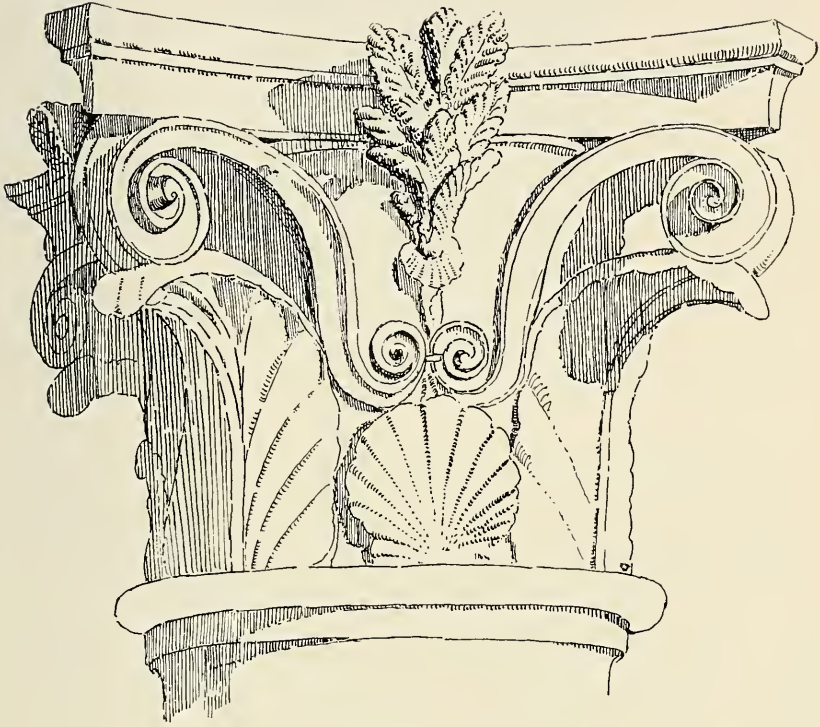


FIG. 30. CAPITAL FROM PALAZZO PICCOLOMINI, PIENZA,
AFTER STEGMANN AND VON GEYMÜLLER



FIG. 31. EGYPTIAN BELL-SHAPED CAPITAL, AFTER STURGIS, WHO CREDITS PRISSE



FIG. 32. CAPITAL FROM THE SACRED ENCEINTE, EPIDAUROS, FROM D'ESPOUY

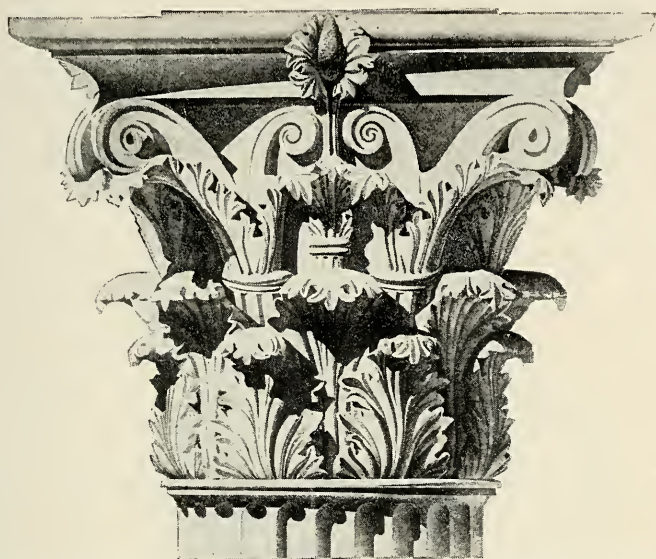


FIG. 33. CAPITAL FROM THE TEMPLE OF VESTA, ROME, FROM D'ESPOUY



FIG. 34. DETAILS OF VOLUTES FROM CAPITAL, TEMPLE OF VESTA, ROME,
AFTER D'ESPOUY

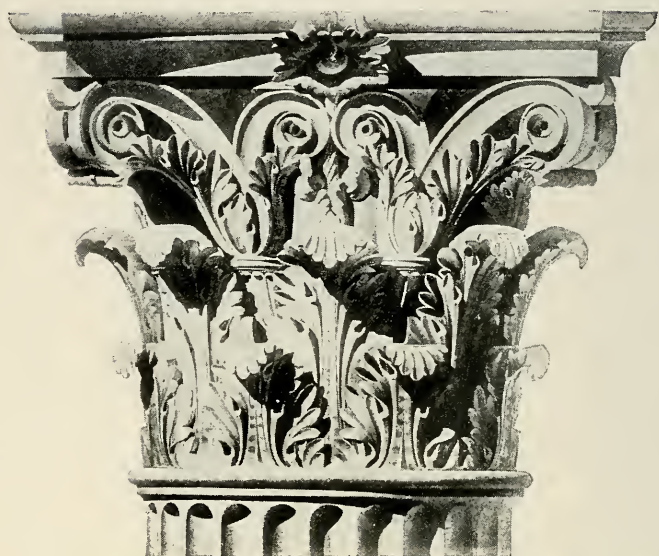


FIG. 35. CAPITAL FROM TEMPLE OF MARS ULTOR, ROME, FROM D'ESPOUY



FIG. 36. CAPITAL FROM TEMPLE OF JUPITER STATOR, ROME, FROM D'ESPOUY

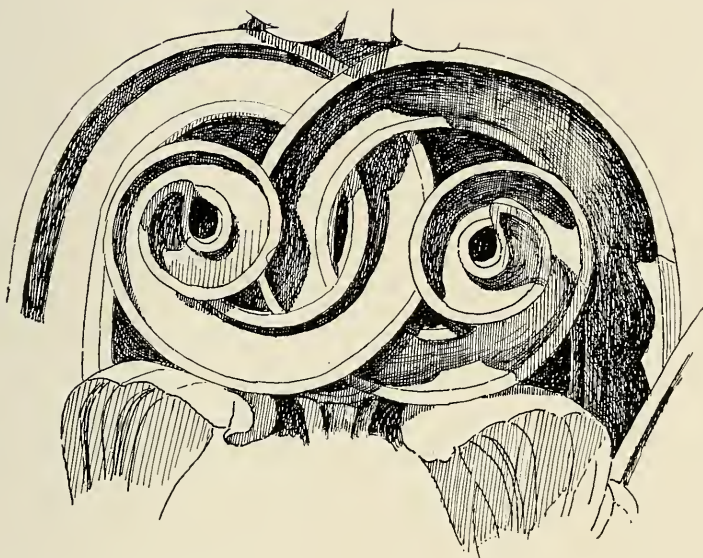


FIG. 37. DETAIL FROM CAPITAL, TEMPLE OF JUPITER STATOR, ROME,
AFTER D'ESPOUY

V. THE VOLUTE IN LATER CAPITALS

9. *Early Christian and Byzantine.*—While the use of the volute upon classic capitals has long been appreciated, its use upon the capitals of the Middle Ages has not been so generally remarked. In this connection it has seemed necessary to point out something of the trend of voluted forms in architecture between the time of the fall of the Roman Empire in the west and the culmination of the Gothic Period. This has been done principally because the volute has so long and frequently been considered simply a classic element, confined to classic periods or classic revivals. It is thought that by pointing out the continued and unbroken use of the volute in one form or another during the Middle Ages, this use, together with the well known popularity of voluted forms during the classic periods and the Renaissance, will serve to establish beyond a doubt the original contention and purpose in presenting the study, namely, that the volute is a universal element in nature, consequently in art, and that as such it persists and will continue to persist as long as man lives in his present environment. The tracing of voluted forms during the Middle Ages requires no very strong written argument and a pictorial presentation is perhaps as effective here as in the case of the classic examples.

Of the type of orders in use at Rome during the Empire only the voluted types, that is, the Ionic, Corinthian, and Composite, appear to have been used by the builders of the early Christian churches. Of the voluted forms, the Ionic was less used than its relatives, the Corinthian and the Composite. These two types were produced by thousands in every Roman province even as far away as Britain. This general class of capitals is called by the name Corinthianesque, for the Corinthian-like variety far surpassed the Composite in its frequency of use, and served as the prototype of the Gothic crocket capitals of a later period.

It has already been noted that capitals of the Corinthian, bell, or basket-type have been used with and without voluted decoration from the time of the Ptolemaic Period in Egypt to the present day, and from the early Christians, Byzantines, and subsequent church-

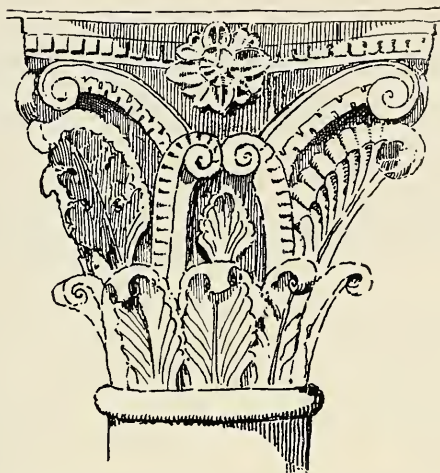


FIG. 38. CAPITAL FROM CHURCH OF PIERRE LE MOUTIER, NIÈVRE, FRANCE,
AFTER MARCOU

building peoples the volute received its full measure of attention. With the second century, the artists sacrificed Roman types for freer versions, and from these early types it is possible to trace the complete transition to the foliated Gothic capital of the eleventh century. However, Roman forms persist in some localities even down to the eleventh century, a good example of a free version of a Roman variety occurring upon the church of St. Pierre-le-Moutier, at Nièvre, France (Fig. 38).

10. *Romanesque and Gothic.*—Forms more medieval in character, yet at the same time retaining the volutes, are shown in Fig. 39, a capital from the nave of the Cathedral of Toulouse, or Fig. 40, a capital from the nave of the church at Aignan, both of which retain the volute as an essential part of their make up. The Romanesque capital from the Cathedral of Bayeaux (Fig. 41) represents a development midway between the Toulouse type and the example from St. Martin de Champs, Paris (Fig. 42). The volute in this latter type has become the Gothic crocket* and persists not only in the capitals but also in the finials of the period. In this connection it should be noted that the crocket proper (circa 1192 B.C.) did not come into existence

* Bond, "Gothic Architecture in England." p. 429.

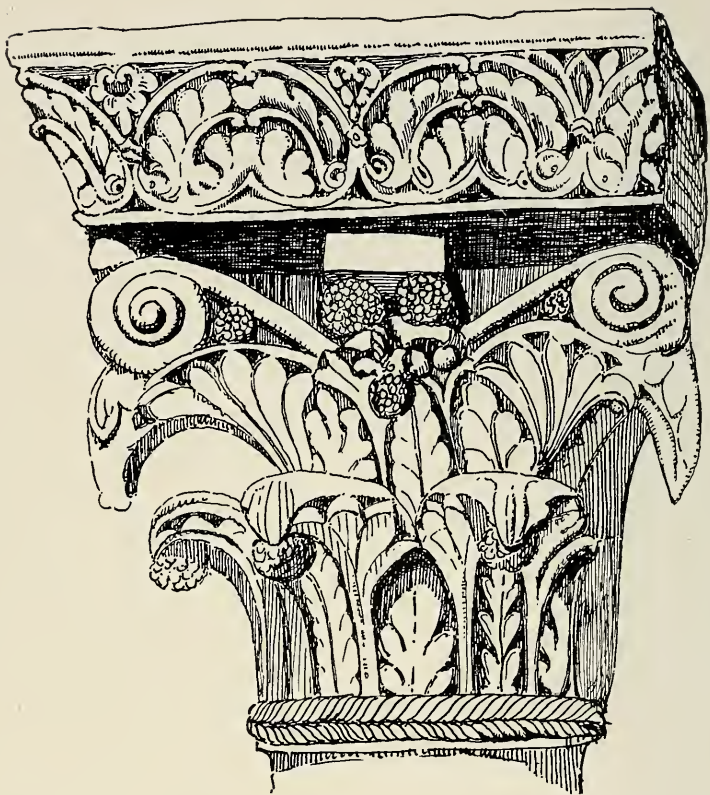


FIG. 39. CAPITAL FROM THE NAVE, CATHEDRAL OF TOULOUSE, FRANCE,
AFTER MARCOU

until after the so-called "crocket" capital, which was derived from the classic Corinthian type as indicated, and which gave rise to the "crocket" proper, but the term "crocket" capital is convenient, although historically incorrect.*

From the Saint Martin de Champs capital it is but a step to the "wind-blown" Salisbury capital (Fig. 44), where the volute almost loses all resemblance to its early prototypes in the curly whorls of foliage. This type marks the culminating phase of the voluted capital during the Gothic period.

* For crocket see Fig. 43.

Another type of capital used in Byzantine, Romanesque, and Gothic art was the cubical block type, and even upon this capital, although there is no structural place for voluted forms, the volute often persists in the decoration, as will be noted in the Sancta Sophia capital (Fig. 45), the St. Peter's, Northampton, capital (Fig. 46), and the capital from the Musée d'Arles (Fig. 47), a Gothic version of the cubical block type.



FIG. 40. CAPITAL FROM THE NAVE OF THE CHURCH OF AIGNAN, FRANCE,
AFTER MARCOU

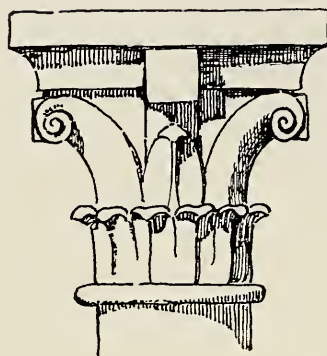


FIG. 41. CAPITAL FROM BAYEAUX CATHEDRAL, FRANCE,
AFTER HAMLIN

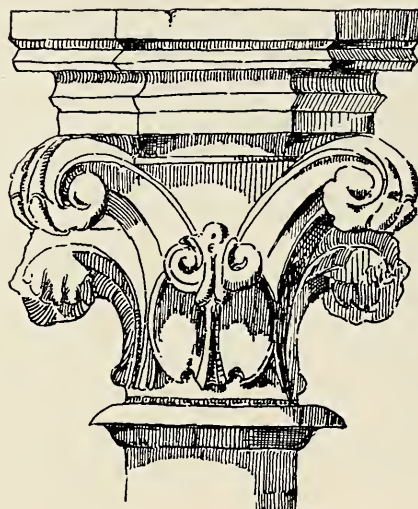


FIG. 42. CAPITAL FROM ST. MARTIN DE CHAMPS, PARIS, FRANCE,
AFTER HAMLIN

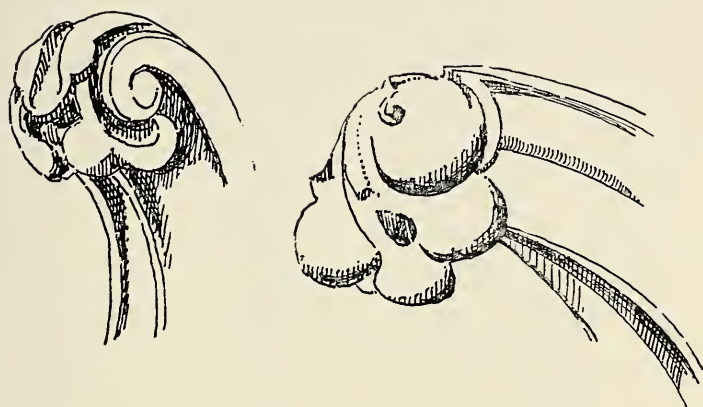


FIG. 43. FINIAL CROCKETS, A SKETCH

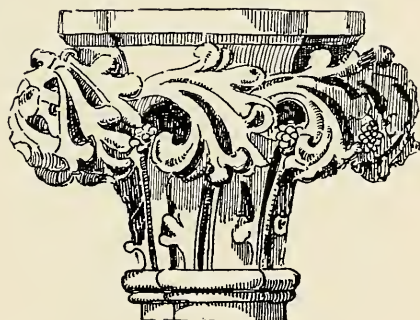


FIG. 44. CAPITAL FROM SALISBURY CATHEDRAL, ENGLAND,
AFTER HAMLIN

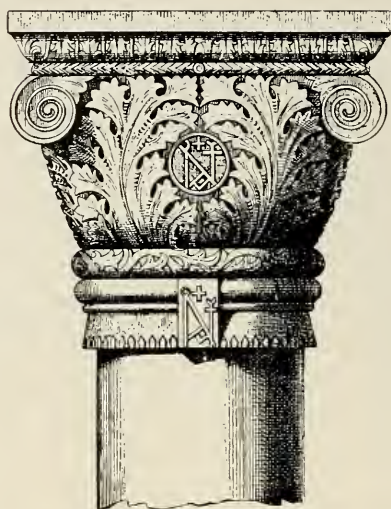


FIG. 45. BYZANTINE CAPITAL, SAINT SOPHIA, CONSTANTINOPLE,
FROM PULGHER

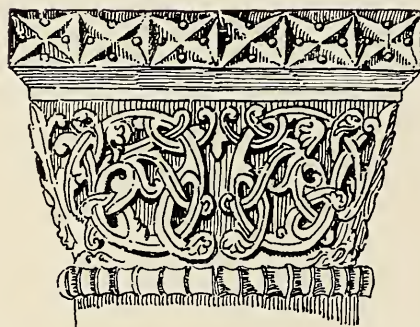


FIG. 46. CAPITAL FROM ST. PETER'S, NORTHAMPTON, ENGLAND,
AFTER BOND

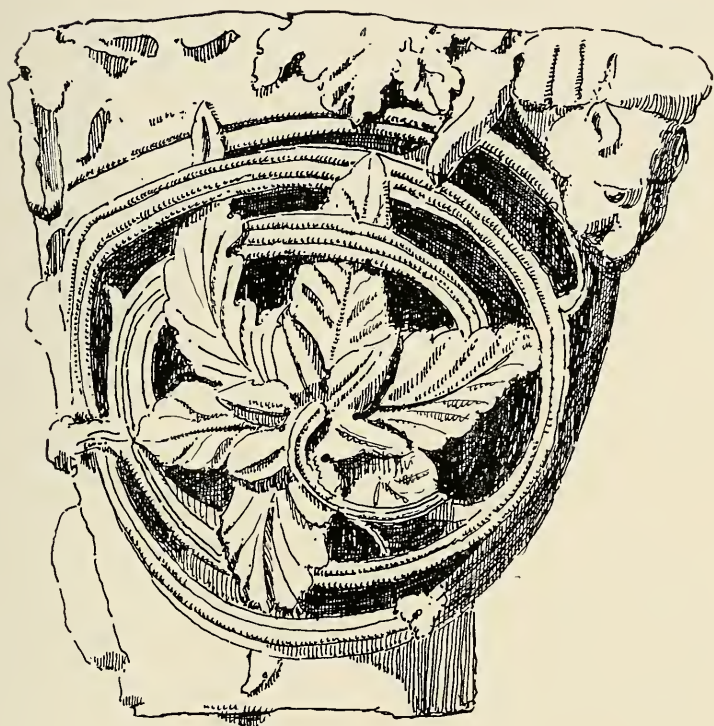


FIG. 47. CAPITAL FROM MUSÉE D'ARLES, AFTER MARCOU

VI. THE VOLUTE IN OTHER STRUCTURAL FORMS

11. *Consoles, Modillions, and Brackets.*—Aside from these considerations of the use of the volute upon the capital, some attention should be paid to its use in other structural forms, where it has persisted with as marvelous an insistence as in the capital. Among these forms employing the volute as a motif may be mentioned consoles, modillions, and transitional brackets.

The employment of the volute as a motif for supporting members hinges upon its ability to make a graceful transition from the vertical to the horizontal, and vice versa. In this connection the spiral motif is usually a double one, employing two spirals winding in opposite directions, as will be noted in Fig. 53. A form thus composed serves as a bracket horizontally or vertically, and in this capacity has served its purpose from the time of the Erechtheum doorway down through the ages to modern times.

The voluted console form has likewise been employed as a keystone upon such structures as the Roman triumphal arches and the like. A similar bracket-like use is the employment of the form as modillions upon the entablatures of the Corinthian and Composite orders.

In addition to these well known uses in a structural manner, volutes have been employed to make transitions in line in situations where their structural significance becomes void. An example of this use occurs in the great scrolls employed in a buttress-like fashion, but for purely æsthetic purposes, upon the Church of Santa Maria della Salute at Venice. Upon other Renaissance buildings voluted inverted bracket forms have been used to make the transition, upon the façade, from the clerestory wall to the walls of the side aisles, well known examples of which use are to be found upon the various Jesuit churches in Italy.

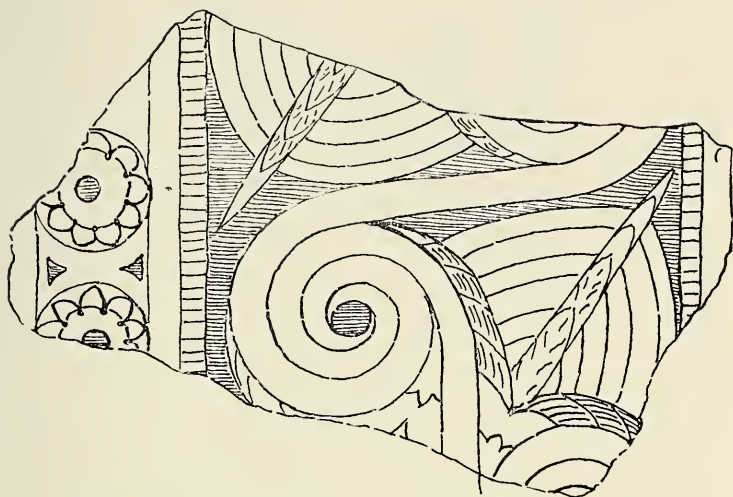


FIG. 48. SPIRALS ON FRAGMENT FROM PALACE AT TIRYNS,
AFTER DÜRM



FIG. 49. ROMAN RINCEAU, A SKETCH

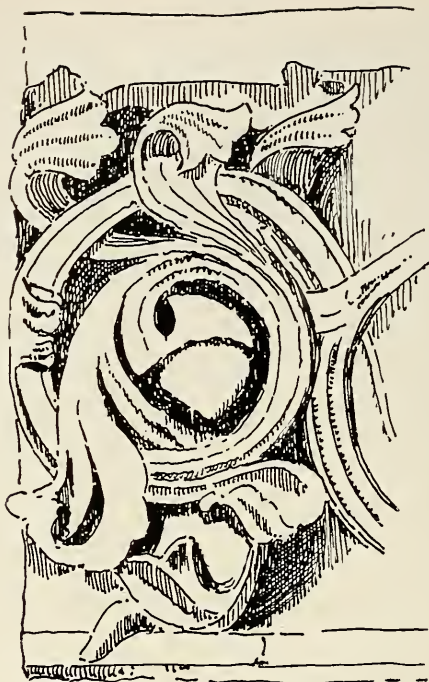


FIG. 50. GOTHIC SPIRAL CARVING, AFTER MARCOU

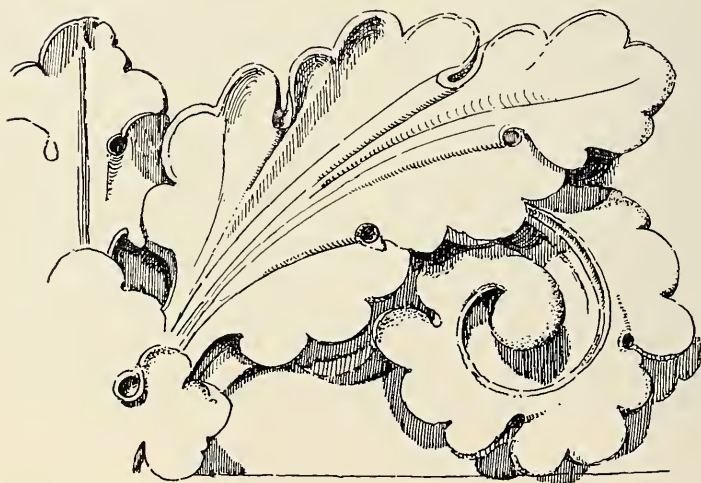


FIG. 51. SPIRAL CARVED DETAIL FROM CATHEDRAL OF PARIS, FRANCE,
AFTER MARCOU



FIG. 52. SPIRAL CARVED BYZANTINE ORNAMENT, FROM CHURCH OF SAINT MARK, VENICE, AFTER DEHLI

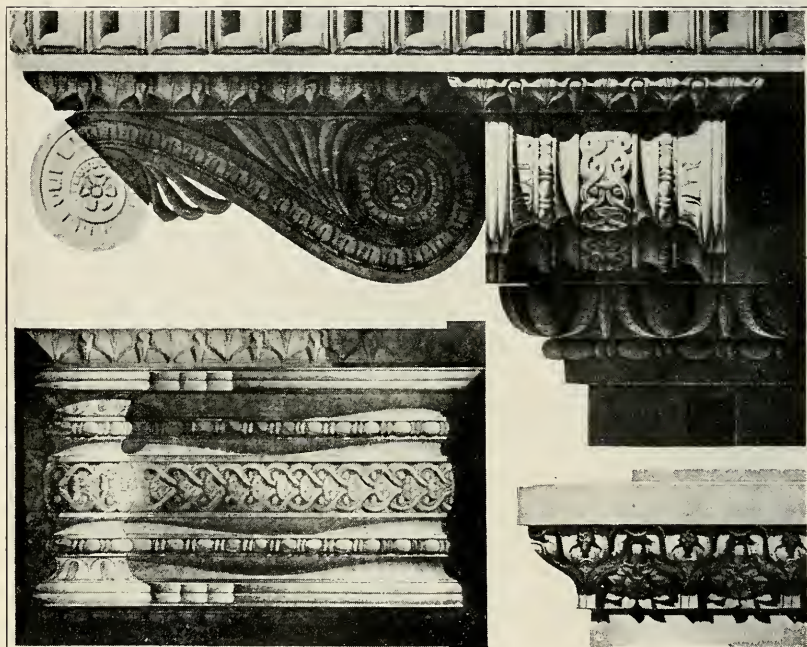


FIG. 53. MODILLION FROM TEMPLE OF CONCORD, ROME,
FROM D'ESPOUY

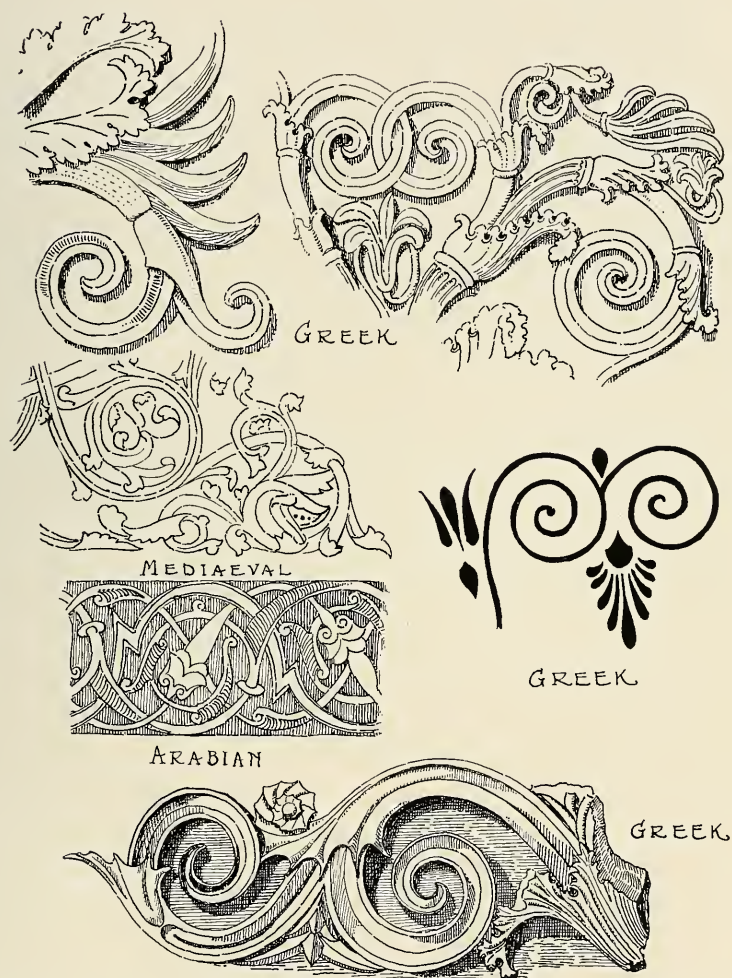


FIG. 54. VARIOUS SPIRAL ORNAMENTS, SKETCHES FROM VARIOUS SOURCES

VII. THE VOLUTE IN ARCHITECTURAL DECORATION

12. *Use from the Early Times to the Renaissance.*—The use of spiral decorative forms as architectural decoration in Egypt, Assyria, and early Greece, has already been mentioned. In its primitive use the spiral figured in decoration either as running ornament or as all-over decoration (see Fig. 48). These same motifs persisted in Greek architectural decoration of a later period, appearing in various forms and manifestations (see Fig. 54), and were subsequently handed down to Rome, where they were favorites, used especially as running ornament, but rarely in all-over patterns (See Figs. 10 and 49).

From Rome the spiral, as running ornament, found its way into the hands of the early Christians and Byzantines (Fig. 52), perpetuated itself during Romanesque times (Fig. 46), and culminated during the Gothic Period (Figs. 50 and 51). Its revival as running ornament or as a terminal motif during the Renaissance was a matter of course. It was accorded the same place as other classic forms.

VIII. CONCLUSION

13. *Summary.*—The general conclusions to be drawn from the foregoing discussion may briefly be summarized as follows:

(1) The volute is one of the most universally occurring curves in nature: it represents a principle in nature, a fundamental law of growth. It is dynamic, infinite, and mysterious in its significance; and hence has from the earliest times had a great appeal to mankind, and has entered deeply into the art expression of all ages.

(2) The volute as an art motif has been developed from natural forms, rather than from a technomorphic inspiration. At the same time the curve as it occurs in art is not purely realistic or naturalistic; but is rather the natural curve idealized and conventionalized into an abstract, ideal form; even the very earliest spiral forms found in art are conventional, geometrical.

(3) Because of the universality of its appeal to the mind of man, the volute as an art motif has a wide geographical distribution, being found in all countries at all times under one guise or another. It appears to have developed independently, although not, perhaps, originated independently, in many cultural centers.

(4) From the evidence available it appears probable that there was little or no Assyrian influence in the development of the Ionic order, but that the spiral motif as found in Ionic art is a product of Ægean culture, and did not come from the East.

(5) The Ionic capital, if the general laws of architectural evolution are borne in mind, appears likely to have been developed from some primitive structural form; and the "log end" theory as formulated in the text offers the most plausible explanation of its origin. The evolution of the capital can be divided into four distinct steps:

- (a) Log ends with painted concentric circles as ornament.
- (b) Wooden bolster-block with painted concentric circles or tangent spirals as ornament.

- (c) Stone block capital with "scratched-in" spirals.
- (d) Stone capital with sculptured volutes and well defined echinus.

(6) The volute continues today a favorite motif in architecture and art partly because, so to speak, its use has acquired a momentum that would be difficult to overcome even if it were not still an inspiring natural form, but far more largely because of its intrinsic beauty and appeal. From classic times it has persisted down through the middle ages to the present day, its treatment and application constantly changing with the development of art and architecture, but its intrinsic form and significance remaining ever the same; and it will persist in the future so long as mankind continues to appreciate the truth and beauty of natural form, and to respond to the subtle appeal of the infinite.



FIG. 55. MAP OF GREEK CONQUESTS AND MIGRATIONS, AFTER WEBSTER

APPENDIX

LIST OF IMPORTANT IONIC EXAMPLES

1. Archaic Capitals from Athens.
2. Archaic "Bolster" Capital from Delos.
3. Capital from Messa, Island of Lesbos.
4. Capital from Neandria in the Troad, Asia Minor, VII Century B.C.
5. Archaic Capitals from Delos.
6. Naxian Votive Column, Delphi.
7. Temple of Apollo, Naukratis, Egypt, 650 B.C.
8. Temple of Hera (Old), Samos, VI Century B.C.
9. Archaic Temple of Diana (Artemis), Ephesus, 560 B.C.
10. Temple upon the Illisus, Athens, 484 B.C.
11. Temple of Niké Apteros, Athens, 438 B.C.
12. Propylaea at Athens (Internal Order), 437-32 B.C.
13. Temple of Apollo Epicurius, Bassæ, 430 B.C.
14. Erechtheum, 420-393 B.C.
15. Various Lycian Tombs, V. and IV Centuries B.C.
16. Mausoleum of Halicarnassos, 354 B.C.
17. Temple of Dionysus, Teos, 350 B.C.
18. The Philipeion, Olympia, 338 B.C.
19. Temple of Apollo Didymæus, near Miletus, 335-320 B.C.
20. Temple of Diana (Artemis), Ephesus, 330 B.C.
21. Temple of Athena Polias, Priene, (near Miletus), 320 B.C.
22. The Cybele's Temple, Sardis, IV Century B.C.
23. Altar of Zeus, Pergamum, 197-159 B.C.
24. Temple of Fortuna Virilis, Rome, 100 B.C.
25. Theatre of Marcellus, Rome, 23-13 B.C.
26. Colosseum, Rome, 70 A.D.
27. Temple of Saturn, Rome, 285 A.D.

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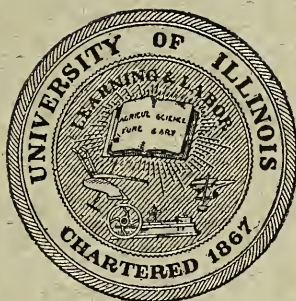
THE THERMAL CONDUCTIVITY AND DIFFUSIVITY OF CONCRETE

BY

A. P. CARMAN

AND

R. A. NELSON



BULLETIN No. 122

ENGINEERING EXPERIMENT STATION

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UNIVERSITY OF ILLINOIS
ENGINEERING EXPERIMENT STATION

BULLETIN No. 122

APRIL, 1921

THE THERMAL CONDUCTIVITY AND
DIFFUSIVITY OF CONCRETE

BY

A. P. CARMAN

PROFESSOR OF PHYSICS

AND

R. A. NELSON

ASSISTANT IN PHYSICS

ENGINEERING EXPERIMENT STATION

PUBLISHED BY THE UNIVERSITY OF ILLINOIS, URBANA

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THERMAL CONDUCTIVITY AND DIFFUSIVITY OF CONCRETE

I. INTRODUCTION

1. *Object of the Investigation.*—The object of this investigation was to obtain the absolute thermal conductivity of a number of standard concrete mixtures. The diffusivity, or thermometric conductivity, has also been calculated from the specific heat, the density, and the thermal conductivity. The investigation was undertaken in response to inquiries for information as to such constants from engineers, received by the Engineering Experiment Station of the University of Illinois, but, apart from the need of such constants in present engineering problems, the increasingly numerous uses of concrete make determination of these physical constants for such a common material desirable.

In this investigation, it has been considered important to describe in detail the material for which the absolute thermal conductivity has been determined. During the last ten years the composition and methods of preparation of concrete mixtures have been studied and standardized, and the present investigators have had the advantage of dealing with concrete mixtures which can be described much more definitely than was possible a few years ago. The results of only a few determinations of the absolute thermal conductivity of concrete are recorded in the literature of the subject,* and to a large extent these lack definiteness in regard to the composition and method of preparation of the material, so that it is impossible to make more than a very general comparison of the results here recorded with those previously obtained.

* Proceedings of National Association of Concrete Users, Vol. VII, article by C. L. Norton. 1911.

"Thermo-conductivity of Heat Insulators," W. Nusselt. Engineering, Vol. 87, p. 1. Jan. 1909.

"A Study of the Heat Transmission of Building Materials" by A. C. Willard and L. C. Lichty. Univ. of Ill. Eng. Exp. Sta. Bul. No. 102, 1917.

Nusselt gives a single determination for "neat" cement, and he did not set a high value on this determination. The results of Norton and of Willard and Lichty will be noticed later in this bulletin.

In addition to determinations of the thermal conductivity of concrete, similar determinations for white marble were also made. The method used in the determinations for marble was the same as had been used for concrete. This not only gives an independently determined value of this constant for marble, but also links the determinations of thermal conductivity for concrete with those for a substance for which a number of values of the thermal conductivity are given in the literature on the subject.*

2. *Acknowledgments.*—In the preparation of the concrete specimens, we have received valuable help from the Department of Theoretical and Applied Mechanics of the University of Illinois. Professor A. N. Talbot, head of the department, has kindly advised with reference to descriptive terms for the material tested. To Mr. H. F. Gonnerman, formerly of the same department, thanks are due for advice and aid in the preparation of the large number of concrete cylinders. Mr. C. C. Schmidt, Assistant in Physics, aided Mr. Nelson, during the summer of 1920, in the preparation of the concrete cylinders and in the experimental work.

* The most extended series of determinations for marble are those of Professor B. O. Pierce and Mr. R. W. Wilson, published in the Proceedings of the American Academy of Arts and Sciences, Vol. 34, p. 3, 1898. They used a wall method, while the method used here was, as will be described, a cylinder method.

II. PRINCIPLES AND METHODS OF MEASUREMENT

3. *Definitions and Units.*—The fundamental fact of heat conductivity is that, for steady flow, the quantity of heat flowing in a unit of time through a plate varies directly as the difference of temperature between the faces of the plate, directly as the cross section of the plate, and inversely as the thickness of the plate. This law, which was first stated clearly by Joseph Fourier, the famous French military engineer and mathematician, is expressed by the formula

$$Q = k \frac{t_2 - t_1}{l} ST$$

In this equation, Q is the quantity of heat, $t_2 - t_1$ the difference of temperature at the two faces, S the area of the cross section, T the time of flow, and l the distance of flow, or the thickness. The quantity k is a constant, which is a property of the material of which the body is composed, and is called the *thermal conductivity* of the material. The fraction $\frac{t_2 - t_1}{l}$ is the fall of temperature per unit distance, and is called the *temperature gradient*.

The unit of thermal conductivity clearly depends upon the chosen units of length, area, time, and heat. In the following calculations a unit based on the centimeter, second, and gram-calorie, called here “the c.g.s. physical unit,” has been used; but the results are stated also in a unit based on the foot, hour, and British thermal unit, called here “the British engineering unit.” These units are defined as follows:

(a) *The C. G. S. Physical Unit* of thermal conductivity corresponds to the flow of one gram-calorie in one second, when the flow is steady, through a section of a plate of the substance in question one square centimeter in area, the thickness of the plate being one centimeter, and the difference between the temperatures of the faces of the plate one degree centigrade.

(b) *The British Engineering Unit* of thermal conductivity corresponds to the flow of one British thermal unit (B.t.u.) in one hour, when the flow is steady, through a section of a plate of the substance in question one square foot in area, the thickness of the plate being one foot, and the difference between the temperatures of the faces of the plate one degree Fahrenheit.

The conversion of results given in either of the above units into other units is a matter of simple calculation.

Upon this subject of units of thermal conductivity the following quotation is made from the excellent text-book of Ingersoll and Zobel entitled "The Mathematical Theory of Heat Conduction": "There is probably no subject in which the confusion of units is greater than that of heat conduction. While the physicist uses the metric or c.g.s. unit—that is, the gram-calorie per second, per square centimeter of area, for a temperature gradient of a degree centigrade per centimeter—there is no such uniformity of practice among engineers. The steam engineer refers his observations to the B.t.u. per hour, per square foot, per degree Fahrenheit, per inch in thickness, while the refrigerating engineer prefers the day as the unit of time rather than the hour, and the electrical engineer uses various systems, based frequently on the kilowatt, as representing the rate of heat flow. There are also numbers of other units, some of them making use of the idea of thermal resistance, analogous to electrical resistance, and therefore being reciprocally related to conductivity. These various engineering units have been introduced to simplify the computation of heat losses in various types of problems, and on these grounds perhaps justify their existence; but from the standpoint of the present work they are, with one or two exceptions, not usable. This is because, in a large majority of the cases we shall have occasion to consider, it is not the conductivity but the *diffusivity*, or thermometric conductivity, which enters directly into the computations, and this latter is too complex a unit to use profitably with a mixture of English and metric systems, or an English system involving two different units of length—for example, feet and inches, as in common engineering practice. Only two, then, of the many heat-conduction units lend themselves readily to our purpose—the B.t.u. per hour, per square foot, for a temperature gradient of a degree Fahrenheit per foot, and the metric unit. But the former is practically never used (the gradient being expressed in degrees per *inch* in the common engineering unit), while the latter is becoming of more general use every day, so we shall confine our units and calculations to the metric system, giving in many cases, however, the English equivalents."

Thermal conductivity expressed in the units defined above, applies to the condition of steady flow, that is, the condition existing when the temperature at each point through the plate is not changing, and the

quantity of heat entering at one face is equal to the quantity emitted at the opposite face.

There is another constant, which is particularly important to the concrete engineer—namely, that which expresses the rate of flow of temperature for a material, or the *thermal diffusivity*. This evidently depends not only on the thermal conductivity, but also on the amount of heat required to raise the temperature of unit volume of the material one degree: that is, upon the density and the specific heat of the material. Thus, if D is the diffusivity, ρ the density, s the specific heat, and k the thermal conductivity of the material,

$$D = \frac{k}{\rho s}$$

In this investigation the thermal diffusivity has been calculated for the mixtures for which the thermal conductivity was determined, and also for the marble.

4. *Method of Measuring Conductivity.*—It is evident that the quantities required for the determination of thermal conductivity are Q , the quantity of heat passing under steady flow conditions at right angles through a surface of predetermined area S , and the quantity $\frac{t_2 - t_1}{l}$, or temperature gradient. For poor heat conductors, such as concrete and marble, the method most commonly employed for the measurement of the quantities has been the “plate” or “wall” method. In this method an essential condition is to have steady flow through a flat plate of uniform thickness, and the lines of flow at right angles to the faces of the plate; this condition is practically realized for the middle part of an extended disk. The heat may be generated electrically in a flat coil, and the quantity of heat can be easily calculated from the electrical energy consumed. Another method of obtaining the quantity of heat Q is to absorb the transmitted heat in a water jacket, and measure the rise in temperature of the water. Still another method is that used by Lees and Charlton* in which the transmitted heat was radiated from a standard plate under constant conditions, the constants for this radiated heat having been determined by a separate experiment. The method of determining thermal conductivity which

* Phil. Mag. No. 5, Vol. 41, p. 495, June, 1898.

has been used in this investigation can be described as the "cylinder method." It is the same general method as was used in the determination of the thermal conductivity of fire-clay made in this laboratory in 1909.* A long cylinder of the substance is used, and the heat is generated electrically in a coil placed in a hole running axially through the cylinder. Near the ends of the cylinder the flow of heat is, of course, not radial, but for some length near the middle of a long cylinder we can assume radial flow. Experiments show that this assumption is justified. The amount of heat generated per unit length of the middle part of the cylinder is easily calculated from measurements of the electromotive force, E , and the current, I , flowing in the coil.

To measure the temperature gradient of the heat flow, small "probing" holes are placed in the cylinder, parallel to the axis, and extending from one end to the middle of the cylinder where the temperature is to be measured. In most of the cylinders used, there were three such holes, one near the coil hole along the axis, one near the outer surface of the cylinder, and one about half way between. The temperatures can be read by thermocouples placed in these holes. Then, knowing the radial distances r_1 and r_2 , and the corresponding temperatures t_1 and t_2 , the temperature gradient is determined directly. Fig. 1 shows the arrangement and location of the coil and the thermocouples.

Fourier's equation for the stationary flow of heat becomes, in the case of a long cylinder, with source along the axis,

$$Q = \frac{2 \pi k l}{\log r_2/r_1} (t_1 - t_2) T \quad \dagger$$

The equation for the thermal conductivity then becomes

$$k = \frac{Q}{2 \pi l (t_1 - t_2) T} \log \frac{r_2}{r_1}$$

All the quantities on the right-hand side of this equation can be directly measured, and thus the absolute thermal conductivity can be determined.

* Thermal Conductivity of Fire Clay at High Temperatures." Univ. of Ill. Eng. Exp. Sta. Bul. No. 36, 1909.

† Ingersoll and Zobel, "The Mathematical Theory of Heat Conduction," p. 27.

5. *Method of Measuring Diffusivity.*—Diffusivity has already been defined as thermal conductivity divided by the product of specific heat and density. To determine the diffusivity of a material it is, therefore, necessary to determine the specific heat and the density as well as the conductivity.

In the present investigation the “method of mixture” with water was used to determine the specific heat. The concrete was broken into pieces containing from $1\frac{1}{2}$ to 2 cubic inches and about 300 grams of these pieces were used in each determination. These were heated to about 100 deg. C. for four hours in an oven to be sure that all portions were at a uniform temperature.

To determine the specific gravity large pieces of the dried concrete were covered with a thin coating of paraffin, and weighed in air, and in water.

III. COMPOSITION AND PREPARATION OF THE CONCRETE CYLINDERS

6. *Materials and Proportions Used.*—In making the concrete test cylinders, “Universal” portland cement was used. The sand and gravel were of dolomitic limestone from a pit at Attica, Indiana. The specific gravity of the stone was about 2.65. The gravel weighed 99 pounds per cubic foot, loose, and the sand 110 pounds per cubic foot. Sieve analyses were made of the sand and gravel, using “Tyler standard series.” The results of the analyses are shown in Table 1. The sand was coarse and well suited for making concrete, and the gravel was well graded.

After consideration of various mixtures of sand and gravel, the proportion of 55 per cent sand to 45 per cent gravel, by weight, was chosen, though the proportioning was actually done by loose volumes. These proportions lie between the coarser mixtures that are frequently used and the mixtures that have larger percentages of sand; they gave an easily worked mixture. The aggregate of this mixture had a weight of 125 lbs. per cubic foot, and this corresponds to 25 per cent of voids.

Three consistencies were used in the concrete mixtures, the normal consistency, which will be referred to as 100 per cent water content, and two others, with 10 per cent and 20 per cent additional water, respectively, which will be referred to as 110 per cent and 120 per cent water content. The normal consistency was such that freshly molded concrete in the form of a cylinder 8 inches in diameter by 12 inches long, with a 1:4 mix, would slump $\frac{1}{2}$ inch to 1 inch when the form was removed.

In Table 2 will be found the proportions of the different constituents for the mixtures tested: these proportions are stated on a volume basis. In the table the quantity of each constituent in one cubic foot of the mixed concrete is also given.

The forms were removed from the cylinders after the concrete had set 24 hours. The cylinders were then stored in damp sand for two weeks, and afterwards removed to a dry room. They were all thoroughly dry when tested. The appearance of the cross sections of broken concrete cylinders is shown in Fig. 2.

TABLE 1

SIEVE ANALYSIS OF SAND AND GRAVEL USED IN CONCRETE CYLINDERS TESTED
(Percentages are based upon weight)

Sieve Size	Size of Square Opening		Per Cent of Sample Coarser than a Given Sieve	
	In.	Mm.	Sand	Gravel
100 mesh	0.0058	0.147	99.0	100.0
48 mesh	0.0116	0.295	97.1	100.0
28 mesh	0.0232	0.59	81.6	100.0
14 mesh	0.046	1.17	52.0	100.0
8 mesh	0.093	2.36	33.6	100.0
4 mesh	0.185	4.70	8.6	99.4
$\frac{3}{8}$ in.	0.37	9.4	0.0	97.2
$\frac{3}{4}$ in.	0.75	18.8	0.0	49.8
1.5 in.	1.5	38.1	0.0	0.0

TABLE 2

COMPOSITION OF CONCRETE MIXTURES USED IN CYLINDERS TESTED

Mixture Cement: Aggregate	Ratios Cement: Sand: Gravel	Relative Water Content Per Cent	Volumes in Cu. Ft.			
			Cement Cu. Ft.	Sand Cu. Ft.	Gravel Cu. Ft.	Water Cu. Ft.
1:2	1-1.2-1.1	100	0.50	0.620	0.567	0.282
		110				0.310
		120				0.338
1:3	1-1.9-1.7	100	0.33	0.62	0.567	0.225
		110				0.248
		120				0.270
1:4	1-2.4-2.3	100	0.25	0.62	0.567	0.196
		110				0.216
		120				0.235
1:5	1-3.1-3.0	100	0.20	0.62	0.567	0.176
		110				0.194
		120				0.211
1:7	1-4.3-4.0	110	0.143	0.62	0.567	0.171
		120				0.186
		110	0.111	0.62	0.567	0.160
1:9	1-5.6-5.1	110				0.175
		120				0.186
		100	1.00			0.384
"Neat"		100				0.384
		110				0.423

IV. TESTING PROCEDURE

7. *Description of Test Specimens and Apparatus Used.*—The cylinders were 24 inches long and $7\frac{1}{2}$ inches in diameter. The central hole for the heating coil was $1\frac{1}{2}$ inches in diameter. The holes for inserting the thermocouples were made by placing $\frac{5}{32}$ -inch rods in the fresh concrete parallel to the axis, and at different radial distances. A section of the cylinder and the heating coil is shown in Fig. 1.

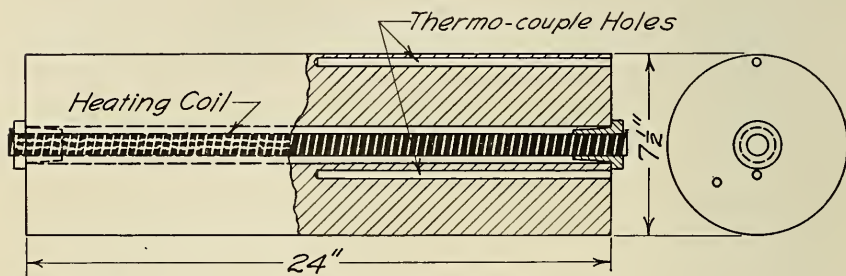


FIG. 1. SECTIONAL AND END VIEWS OF TEST CYLINDER, SHOWING LOCATION OF HEATING COIL AND THERMOCOUPLES

The heating coils were made by winding "Chromel C"* ribbon 0.25 inch by 0.025 inch on hard porcelain insulator tubes 24 inches long. In order to center the heating coils, tapered collars for the ends were made of portland cement and plaster of Paris. The outer surfaces of the concrete cylinders were covered with a very thin coat of plaster of Paris to make the emissivity uniform over the surface of the cylinder. The current for the heating coil was supplied by a motor-generator set equipped with a General Electric special voltage regulator, the motor of the set being driven by the alternating current from the University mains, which is of fairly constant voltage. The regulator kept the voltage of the 110 D.C. generator constant to within less than one-half of one per cent. It was found necessary to heat the cylinders from 14 to 16 hours before the temperature became constant so that observations could be taken. A Thwing thermocouple recorder

* This is a nickel-chromium alloy resistance metal made by the Hoskins Manufacturing Company of Detroit, Michigan.

was used to indicate when a steady condition of temperature was reached.

Fig. 3 shows the method of insulating the hot junction of the thermocouple and centering it in the hole; the outside glass protecting tube fitted closely in the holes in the cylinder. Copper-constantan thermocouples were used, and were calibrated by first of all determining the readings at three fixed temperatures, the boiling point of water,

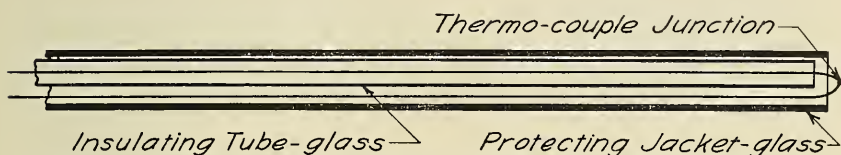


FIG. 3. METHOD OF INSULATING AND CENTERING HOT JUNCTION OF THERMOCOUPLES

the boiling point of naphthalene, and the melting point of lead. These three points having been determined, the complete calibration curve for each couple was drawn by comparison with the calibration curve for a standard thermocouple, that had been carefully calibrated. A Wolff potentiometer was used to measure the electromotive force. The general arrangement of the apparatus and circuits are shown in Figs. 4 and 5.

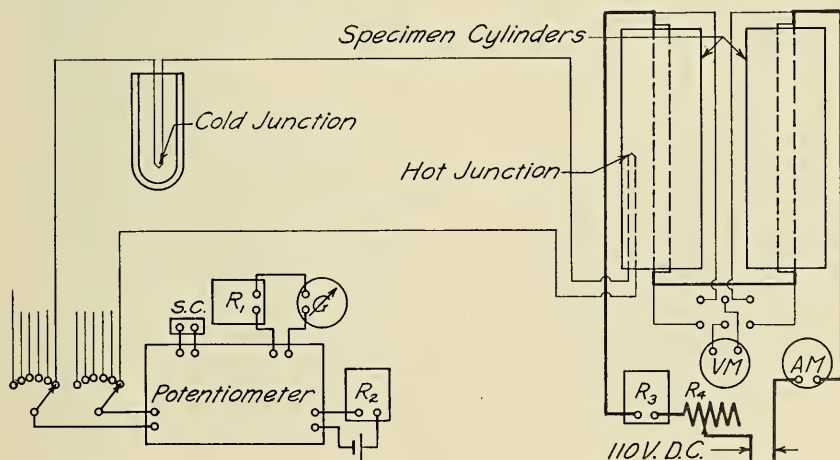


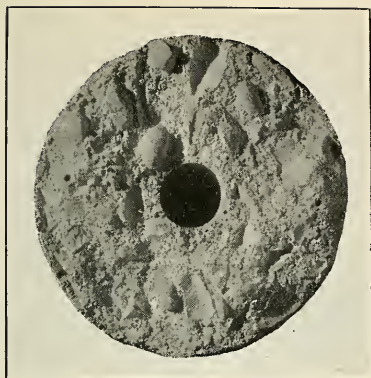
FIG. 4. GENERAL ARRANGEMENT OF APPARATUS AND ELECTRICAL CIRCUITS

8. *Testing Procedure.*—Temperature readings were taken for most of the cylinders at three radial distances from the axis. On the first tests a thermocouple was used in each hole and temperatures were taken simultaneously, but it was afterwards found that one thermocouple could be used, and changed from one hole to the other, without affecting the accuracy of the observations. Temperature readings were taken over a range of five centimeters axial length at the middle of the cylinder. Preliminary tests had shown that for this portion of the cylinder there was practically no variation in temperature parallel to the axis; that is, the flow of heat was truly radial through this mid-portion of the cylinder. The current through the heating coils and the potential differences were read at the beginning and at the end of each set of observations. In most of the determinations a length of heating coil of 59 cm. was used, but for a few cases the length was 57.5 cm. Before taking any test temperature readings the cylinders were first given a preliminary heating to a temperature of over 100 deg. C. in order to dry them out; this was found to be necessary in order to obtain consistent readings.

After the tests were completed the cylinders were broken as near the middle as possible and the radial distances of the thermocouple holes from the cylinder axes were measured. In calculating the thermal conductivity, these measured radial distances were used, with the corresponding temperatures.

Fig. 2 shows sectional views of the cylinders when broken, and the probing holes for measuring the temperature can be seen. Fig. 6 shows a collection of cylinders which were tested. Results of experiments on fifty-one concrete cylinders are given in the tables.

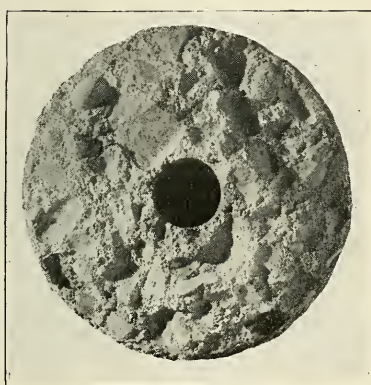
9. *Tests on a Marble Cylinder.*—In addition to the tests on the concrete cylinders, some tests were made on a cylinder of marble, of similar dimensions. White Alabama marble was used, the sample having been purchased from the Peoria Marble Works of Peoria, Illinois. The grain of this marble was of a fine sugary texture and the specimen is described as being of a "very good grade" of marble. Chemical analysis showed that it was composed principally of calcium carbonate, with a small amount of magnesium carbonate. The tests were made both for thermal conductivity and for diffusivity. Before testing, in order to free the marble from moisture, it was heated to 130 deg. C. in a large oven for four hours. In carrying out the con-



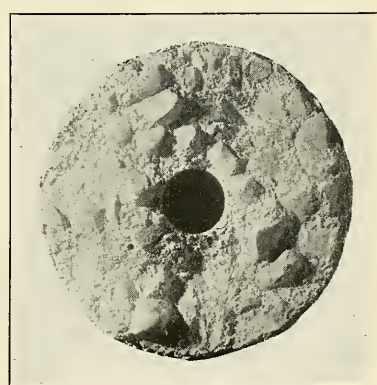
1-2 Mixture



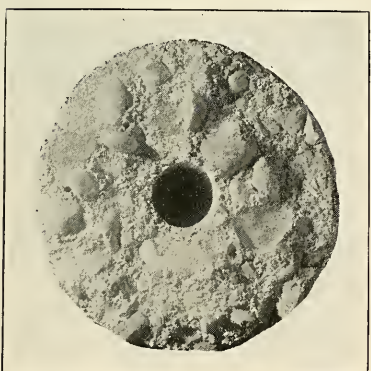
1-3 Mixture



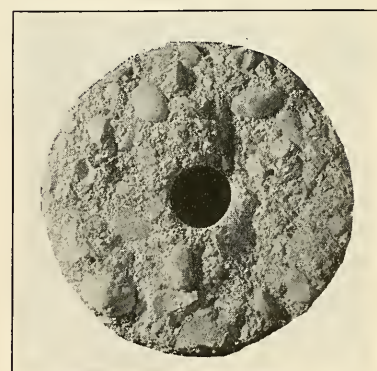
1-4 Mixture



1-5 Mixture



1-7 Mixture



1-9 Mixture

FIG. 2. CROSS-SECTIONAL VIEWS OF BROKEN CONCRETE CYLINDERS

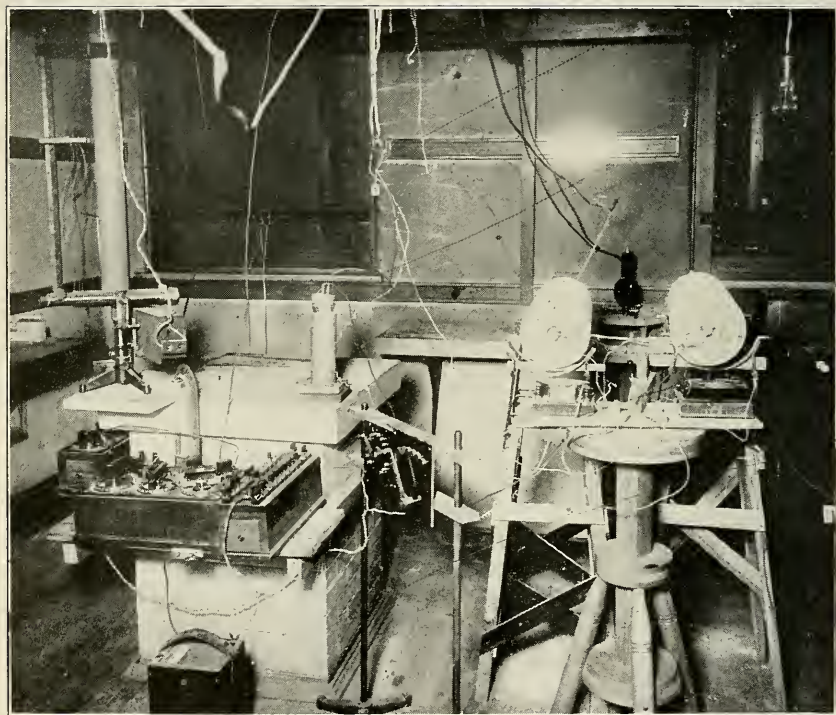


FIG. 5. GENERAL VIEW OF APPARATUS



FIG. 6. GENERAL VIEW OF BROKEN TEST CYLINDERS

ductivity tests the cylinder was first heated to about 50 deg. C. and readings taken; the temperature was then increased each day, and observations made, until a temperature of about 200 deg. C. was reached; the specimen was then allowed to cool, and was again tested in the same manner. At about 235 deg. C. the cylinder cracked in several places, and observations were discontinued. The results of the conductivity tests are given in Table 11; the mean values for the conductivity are given in Table 13.

For purposes of comparison, in Table 3 are given figures for specific gravity, specific heat, thermal conductivity, and diffusivity for the sample tested, and also for another sample of a somewhat similar marble, the values given in the latter case having been taken from results already published by Pierce and Wilson.*

TABLE 3
PROPERTIES OF MARBLE

	Pierce and Wilson's "American White"	"Alabama White"
Specific gravity.....	2.72	2.71
Specific heat.....	0.214	0.213
Thermal conductivity.....	0.00596	0.00614
Diffusivity.....	0.0102	0.0106

* Proc. Am. Acad. Arts and Sciences, Vol. 36, 1900.

V. RESULTS OF OBSERVATIONS AND DETERMINATIONS

10. *Explanation of Tables.*—The results of the observations and calculations are shown in Tables 4 to 11. Tests were made on fifty-one concrete cylinders, including three cylinders of “neat” cement, and on one cylinder of “White Alabama” marble. The first column gives the identification mark for the cylinder; the second column gives the “relative water content” as defined in Section III, page 12; the third column gives the number of days, at the time of test, since the cylinder was cast; the fourth and fifth columns give the radial distances r_1 and r_2 of the probing holes at which the temperatures t_1 and t_2 , given in the eighth and ninth columns, were measured; the sixth and seventh columns give respectively the current I in amperes, and the electromotive force E in volts of the heating coil, for calculating the heat Q ; the tenth and eleventh columns give the values of k , the thermal conductivity in “c.g.s.” and “British engineering” units as defined in Section II, page 7; the material of the cylinder is indicated in the caption. The proportions of the concrete mixtures are given in Table 2.

In Table 12 are given values for densities, specific heats, conductivities, and diffusivities, for the different mixtures of concrete, and for the marble. These values for concrete are given for one relative water content only, namely, 110 per cent; it was considered that thus good average values would be obtained, and that in any case the diffusivity would not show much variation with variation of the relative water content.

In calculating the diffusivities the conductivities determined for a range of temperature of from 100 deg. C. to 200 deg. C. were used. This was done because it was felt that these results were more reliable, both on account of the larger number of readings taken over this range, and because the readings were somewhat more consistent, owing to the larger temperature differences.

TABLE 4
THERMAL CONDUCTIVITY OF NEAT CEMENT

Cylinder Mark	Relative Water Content Per Cent	Age Days	r ₁ cm.	r ₂ cm.	I Amp.	E Volts	t ₁ deg. C.	t ₂ deg. C.	k c. g. s. Physical	k British Engineering
1H	100	28	3.493	6.055	15.67	27.30	253.7	152.5	0.00150	0.363
			3.493	9.508			253.7	91.5	0.00170	0.411
1H	100	120	3.493	6.055	17.02	29.7	345.7	212.1	0.00135	0.327
			6.055	9.508			212.1	123.0	0.00165	0.399
			3.493	9.508			345.7	123.0	0.00147	0.356
1H	100	150	3.493	6.055	7.48	12.15	91.0	67.6	0.00138	0.334
			6.055	9.508			67.6	49.0	0.00142	0.344
			3.493	9.508			91.0	49.0	0.00140	0.339
3H	110	28	2.244	4.958	13.99	24.40	286.6	148.8	0.00126	0.305
			2.244	8.799			286.6	89.2	0.00153	0.370
4H	110	28	2.990	6.378	13.99	25.00	217.1	112.9	0.00164	0.397

TABLE 5
THERMAL CONDUCTIVITY OF CONCRETE
Mixture 1:2

Cylinder Mark	Relative Water Content Per Cent	Age Days	r ₁ cm.	r ₂ cm.	I Amp.	E Volts	t ₁ deg. C.	t ₂ deg. C.	k c. g. s. Physical	k British Engineering
1E	110	28	2.394	4.811	16.41	26.10	197.2	138.8	0.0033	0.80
			2.394	8.662			197.2	103.4	0.0038	0.92
1E	110	120	2.394	4.811	19.00	32.95	321.8	229.4	0.0031	0.75
2E	110	28	2.161	6.131	16.37	28.40	239.3	145.3	0.0033	0.80
			6.131	9.255			145.3	104.9	0.0031	0.75
			2.161	9.255			239.3	104.9	0.0033	0.80
2E	110	120	2.161	6.131	19.00	33.35	354.2	210.8	0.0031	0.75
2E	110	140	2.161	6.131	7.52	12.21	76.4	56.3	0.0031	0.75
			6.131	9.255			56.3	48.0	0.0029	0.70
			2.161	9.255			76.4	48.0	0.0030	0.73
3E	100	28	2.313	7.390	15.64	24.80	194.2	115.6	0.0037	0.90
			2.313	7.567			194.2	113.9	0.0037	0.90
			2.313	7.390	16.18	25.25	221.0	132.5	0.0035	0.85
			2.313	7.567			221.0	128.2	0.0034	0.82
3E	100	120	2.313	7.390	17.02	29.70	265.4	151.1	0.0034	0.82
			2.313	7.567			265.4	142.4	0.0032	0.77
3E	100	140	2.313	7.390	7.55	12.39	68.7	49.5	0.0037	0.90
			2.313	7.567			68.7	48.9	0.0036	0.87
3E	100	150	2.313	7.390	19.13	32.80	334.7	175.4	0.0030	0.73
			2.313	7.567			334.7	174.3	0.0031	0.75
4E	120	28	4.860	6.372	16.18	28.00	170.4	137.4	0.0024	0.58
			6.372	9.270			137.4	110.2	0.0040	0.97
			4.860	9.270			170.4	110.2	0.0031	0.75
5E	110	28	3.978	9.297	16.36	26.00	174.7	105.2	0.0033	0.80
5E	110	120	4.423	9.297	15.08	25.61	155.8	107.8	0.0038	0.92
			3.978	9.297			170.1	107.8	0.0034	0.82
6E	100	28	2.528	7.105	16.79	27.80	245.2	151.4	0.0033	0.80
			2.528	7.936			245.2	134.1	0.0031	0.75
			2.528	7.105	16.79	28.30	260.0	161.4	0.0035	0.85
			2.528	7.936			260.0	143.1	0.0030	0.73
6E	100	120	2.528	7.105	15.09	26.35	220.9	140.5	0.0034	0.82
			2.528	7.936			220.9	121.8	0.0030	0.73
7E	100	28	3.003	5.493	16.18	28.25	189.4	144.7	0.0040	0.97
			3.003	9.123			189.4	112.9	0.0043	1.04

TABLE 6
THERMAL CONDUCTIVITY OF CONCRETE
Mixture 1:3

Cylinder Mark	Relative Water Content Per Cent	Age Days	r ₁ cm.	r ₂ cm.	I Amp.	E Volts	t ₁ deg. C.	t ₂ deg. C.	k c. g. s. Physical	k British Engineering
1D	110	28	2.594	6.776	15.80	24.85	175.4	115.1	0.0040	0.97
			2.594	6.776	19.72	30.35	289.6	171.5	0.0031	0.75
			6.776	9.196			171.5	132.5	0.0030	0.73
2D	100	28	2.594	9.196			289.6	132.5	0.0031	0.75
			2.467	4.904	15.80	27.50	196.2	138.7	0.0034	0.82
			2.467	8.713			196.2	104.1	0.0038	0.92
3D	110	28	2.467	4.904	19.75	34.55	312.5	217.3	0.0032	0.77
			4.904	8.713			217.3	155.2	0.0041	0.99
			2.461	8.713			312.5	155.2	0.0035	0.85
4D	110	28	2.420	7.097	15.09	23.45	159.1	107.5	0.0048	1.16
			2.420	7.097	16.00	26.00	199.8	133.0	0.0044	1.06
			2.420	9.024			199.8	105.7	0.0038	0.92
5D	110	28	2.220	8.841	15.10	26.15	185.2	98.2	0.0040	0.97
			2.220	4.197	16.00	27.50	222.9	168.9	0.0034	0.82
			4.197	8.841			168.9	116.1	0.0040	0.97
5D	110	28	2.220	8.841			222.9	116.1	0.0037	0.90
			2.307	5.852	17.12	29.75	234.7	164.8	0.0044	1.06
			5.852	9.235			164.8	126.2	0.0039	0.94
5D	110	120	2.307	9.235			234.7	126.2	0.0042	1.01
			2.307	5.852	14.94	24.00	181.7	121.7	0.0036	0.87
			5.852	9.235			121.7	94.0	0.0038	0.92
5D	110	140	2.307	9.235			181.7	94.0	0.0037	0.90
			2.307	5.852	7.46	12.02	72.0	54.0	0.0031	0.75
			5.852	9.235			54.0	47.0	0.0039	0.94
6D	110	28	2.307	9.235			72.0	47.0	0.0033	0.80
			4.478	9.252	17.14	26.55	168.4	120.5	0.0044	1.06
			2.474	9.252			228.1	120.5	0.0036	0.87
6D	110	120	2.474	4.478	14.94	24.40	169.5	135.9	0.0043	1.04
7D	120	28	2.535	6.225	16.43	28.45	226.5	147.1	0.0034	0.82
			6.225	9.255			147.1	118.6	0.0037	0.90
			2.535	9.255			226.5	118.6	0.0036	0.87
7D	120	120	2.535	6.225	13.83	24.00	179.3	121.1	0.0033	0.80
			6.225	9.255			121.1	95.1	0.0035	0.85
			2.535	9.255			179.3	95.1	0.0033	0.80
7D	120	140	2.535	6.225	7.51	12.25	70.8	54.5	0.0033	0.80
			6.225	9.255			54.5	46.6	0.0032	0.77
			2.535	9.255			70.8	46.6	0.0032	0.77
8D	100	28	2.253	4.953	16.17	25.60	188.8	138.9	0.0042	1.01
			4.953	7.819			138.9	110.1	0.0042	1.01
			2.253	7.819			188.8	110.1	0.0042	1.01
9D	100	28	2.360	4.230	16.77	28.80	248.9	194.8	0.0034	0.82
			4.230	9.131			194.8	127.7	0.0036	0.87
			2.360	9.131			248.9	127.7	0.0036	0.87
9D	100	120	2.360	4.230	16.79	29.10	259.8	200.1	0.0031	0.75
			4.230	9.131			200.1	132.2	0.0038	0.92
			2.360	9.131			259.8	132.2	0.0033	0.80
9D	100	140	2.360	4.230	13.83	24.65	183.8	145.0	0.0034	0.82
			4.230	9.131			145.0	103.6	0.0042	1.01
			2.360	9.131			183.8	103.6	0.0038	0.92
9D	100	140	2.360	4.230	7.73	12.50	71.9	60.5	0.0033	0.80
			4.230	9.131			60.5	46.9	0.0036	0.87
			2.360	9.131			71.9	46.9	0.0035	0.85
11D	120	120	2.653	5.539	12.88	21.90	143.6	103.5	0.0033	0.80
			2.653	9.073			143.6	85.0	0.0038	0.92
			2.653	5.539	14.56	25.00	189.9	137.0	0.0033	0.80
11D	120	140	2.653	9.073			189.9	105.9	0.0034	0.82
			2.653	5.539	13.83	23.60	173.7	126.5	0.0033	0.80
			2.653	9.073			173.7	98.5	0.0035	0.85
11D	120	140	2.653	5.539	7.26	11.77	66.1	52.8	0.0031	0.75
			2.653	9.073			66.1	46.1	0.0034	0.82
			2.653	9.073	10.17	17.10	106.2	65.6	0.0034	0.82
11D	120	140	2.653	9.073	13.96	23.90	176.9	97.5	0.0033	0.80
			2.653	5.539	18.19	31.00	268.0	181.6	0.0031	0.75
			2.653	9.073			268.0	124.4	0.0031	0.75
			2.653	5.539	18.26	31.10	283.5	195.0	0.0031	0.75
			2.653	9.073			283.5	139.6	0.0031	0.75

TABLE 7
THERMAL CONDUCTIVITY OF CONCRETE
Mixture 1:4

Cylinder Mark	Relative Water Content Per Cent	Age Days	r_1 cm.	r_2 cm.	I Amp.	E Volts	t_1 deg. C.	t_2 deg. C.	k c. g. s. Physical	k British Engineering
1C	110	28	2.616	5.038	15.28	24.05	161.4	119.2	0.0037	0.90
			5.038	8.293			119.2	94.2	0.0047	1.14
			2.616	8.293			161.4	94.2	0.0041	0.99
			2.616	5.038	19.81	32.61	322.5	240.5	0.0033	0.80
			5.038	8.293			240.5	168.8	0.0029	0.70
2C	100	28	2.616	8.293			322.5	168.8	0.0031	0.75
			2.498	4.406	15.58	26.75	176.8	136.2	0.0038	0.92
			4.406	6.693			136.2	111.7	0.0046	1.11
			2.498	6.693			176.8	111.7	0.0041	0.99
			2.440	6.455	15.61	24.1	192.9	116.9	0.0031	0.75
3C	110	28	6.455	7.395			116.9	107.8	0.0036	0.87
			2.440	7.395			192.9	107.8	0.0032	0.77
3C	110	120	2.440	6.455	15.00	25.95	199.6	133.8	0.0037	0.90
			6.455	7.395			133.8	121.4	0.0028	0.68
			2.440	7.395			199.6	121.4	0.0036	0.87
3C	110	140	2.440	6.455	7.52	12.21	65.2	50.8	0.0040	0.97
			2.440	7.395			65.2	50.1	0.0043	1.04
			2.440	6.455	19.14	32.81	317.3	196.0	0.0036	0.87
3C	110	150	2.440	6.455	19.14	32.81	317.3	196.0	0.0036	0.87
4C	110	28	2.639	3.679	16.41	26.00	198.3	177.8	0.0044	1.06
			3.679	9.469			177.8	104.1	0.0035	0.85
			2.639	9.464			198.3	104.1	0.0037	0.90
5C	110	28	2.700	5.852	16.41	28.50	207.7	148.0	0.0039	0.94
			2.700	9.125			207.7	122.4	0.0043	1.04
			5.569	9.103	15.90	27.55	158.5	122.1	0.0037	0.90
6C	100	120	5.569	9.103	14.96	25.9	142.4	108.5	0.0036	0.87
			4.588	9.103			159.9	108.5	0.0033	0.80
			2.494	6.998	15.84	27.85	234.2	143.2	0.0032	0.77
7C	100	28	6.998	8.725			143.2	125.5	0.0035	0.85
			2.494	8.725			234.2	125.5	0.0033	0.80
			2.494	6.998	14.93	26.20	216.2	128.9	0.0030	0.73
8C	120	28	2.494	8.725			216.2	118.5	0.0032	0.77
			2.232	5.966	15.21	26.35	232.5	147.8	0.0030	0.73
			5.966	9.467			147.8	114.0	0.0035	0.85
8C	120	120	2.232	9.467			232.5	114.0	0.0032	0.77
			2.232	5.966	14.93	26.05	229.0	136.8	0.0028	0.68
10C	120	28	2.563	5.588	16.01	27.35	238.6	152.9	0.0026	0.63
			5.588	9.075			152.9	113.8	0.0035	0.85
			2.563	9.075			238.6	113.8	0.0029	0.70
10C	120	120	2.563	5.588	17.61	31.15	273.4	186.9	0.0033	0.80
			5.588	9.075			186.9	133.6	0.0033	0.80
			2.563	9.075			273.4	133.6	0.0033	0.80
			2.563	9.075	14.92	26.25	216.2	118.3	0.0034	0.82
			2.563	5.588	7.53	12.31	66.2	56.8	0.0051	1.23
10C	120	140	5.588	9.075			56.8	47.2	0.0031	0.75
			2.563	9.075			66.2	47.2	0.0041	0.99

TABLE 8
THERMAL CONDUCTIVITY OF CONCRETE
Mixture 1:5

Cylinder Mark	Relative Water Content Per Cent	Age Days	r ₁ cm.	r ₂ cm.	I Amp.	E Volts	t ₁ deg. C.	t ₂ deg. C.	k c. g. s. Physical	k British Engineering
1B	110	28	2.531	8.890	72.3	7.12	244.7	128.0	0.0035	0.84
1B	110	28	2.531	8.890	90.0	8.71	360.1	176.4	0.0034	0.82
3B	110	28	2.651	3.298	12.05	19.00	116.0	105.1	0.0037	0.90
			3.298	7.595			105.0	80.8	0.0041	0.99
			2.651	7.595			116.0	80.8	0.0045	1.09
4B	110	28	2.610	6.014	12.05	20.95	101.2	83.4	0.0035	0.85
			2.610	8.510			101.2	71.5	0.0040	0.97
5B	110	28	2.512	4.666	11.83	20.30	122.2	96.7	0.0037	0.90
			4.666	8.522			96.7	75.3	0.0035	0.85
			2.512	8.522			122.2	75.3	0.0033	0.80
			2.512	4.666	19.80	29.65	290.8	233.6	0.0041	0.99
			4.666	8.522			233.6	170.9	0.0036	0.87
			2.512	8.522			290.8	170.9	0.0039	0.94
			2.512	4.666			154.6	115.8	0.0041	0.99
			4.666	8.522			115.8	96.2	0.0040	0.97
			2.512	8.522			154.6	96.2	0.0040	0.97
6B	100	28	2.474	6.064	16.02	25.4	210.7	146.7	0.0039	0.94
			6.064	9.181			146.7	114.5	0.0034	0.82
			2.474	9.181			210.7	114.5	0.0036	0.87
6B	100	120	2.474	6.064	14.93	26.10	211.0	138.8	0.0032	0.77
			6.064	9.181			138.8	110.0	0.0037	0.89
			2.474	9.181			211.0	110.0	0.0034	0.82
7B	120	28	2.308	7.043	15.08	26.70	235.3	129.6	0.0028	0.68
			7.043	8.550			129.6	116.3	0.0038	0.92
			2.308	8.550			235.3	116.3	0.0029	0.70
7B	120	120	2.308	7.043	14.92	26.01	232.9	126.3	0.0027	0.65
			7.043	8.550			126.3	110.6	0.0031	0.75
			2.308	8.550			232.9	110.6	0.0027	0.65
			2.308	7.043	7.73	12.54	80.3	54.8	0.0028	0.68
			7.043	8.550			54.8	49.0	0.0021	0.51
			2.308	8.550			80.3	49.0	0.0027	0.65
7B	120	150	2.308	7.043	19.14	32.70	375.5	188.9	0.0025	0.61
			7.043	8.550			188.9	157.7	0.0025	0.61
			2.308	8.550			375.5	157.7	0.0025	0.61
8B	120	28	4.788	5.586	15.09	26.40	163.6	154.3	0.0043	1.04
			5.586	9.288			154.3	114.4	0.0033	0.80
			4.788	9.288			163.6	114.4	0.0034	0.82
8B	120	120	4.788	5.586	14.94	25.91	162.9	145.7	0.0025	0.60
			5.586	9.288			145.7	107.4	0.0034	0.80
			4.788	9.288			162.9	107.4	0.0030	0.73
9B	120	28	2.304	6.754	15.64	27.45	238.5	136.8	0.0029	0.70
			6.754	9.187			136.8	109.0	0.0031	0.75
			2.304	9.187			238.5	109.0	0.0030	0.73
9B	120	120	2.304	6.754	17.61	30.61	293.0	162.1	0.0028	0.68
			6.754	9.187			162.1	133.4	0.0036	0.87
			2.304	9.187			293.0	133.4	0.0030	0.73
9B	120	140	2.304	6.754	7.73	12.60	78.2	51.7	0.0026	0.63
			6.754	9.187			51.7	46.3	0.0035	0.85
			2.304	9.187			78.2	46.3	0.0027	0.65

TABLE 9
THERMAL CONDUCTIVITY OF CONCRETE
Mixture 1:7

Cylinder Mark	Relative Water Content Per Cent	Age Days	r ₁ cm.	r ₂ cm.	I Amp.	E Volts	t ₁ deg. C.	t ₂ deg. C.	k c. g. s. Physical	k British Engineering
1F	110	28	5.224	7.641	16.72	27.15	167.4	137.8	0.0036	0.87
			3.395	7.641			190.1	137.8	0.0043	1.04
2F	110	28	4.599	8.178	16.71	28.85	178.4	128.8	0.0036	0.87
3F	120	28	2.468	5.463	15.76	24.40	216.3	142.0	0.0027	0.65
			5.463	7.058			142.0	119.4	0.0028	0.68
			2.468	7.058			216.3	119.4	0.0027	0.65
3F	120	120	5.463	7.058	15.61	27.10	152.4	130.3	0.0032	0.77
4F	120	28	5.724	8.096	15.76	27.10	148.8	117.5	0.0031	0.75
			3.787	8.096			173.7	117.5	0.0031	0.75
5F	110	28	5.190	8.772	15.55	23.25	141.9	114.1	0.0044	1.06
			3.050	8.772			194.3	114.1	0.0031	0.75
5F	110	120	3.050	5.190	15.62	27.30	196.0	155.0	0.0037	0.90
			5.190	8.772			155.0	120.6	0.0043	1.04
			3.050	8.772			196.0	120.6	0.0040	0.97
5F	110	140	3.050	5.190	7.47	12.15	65.0	55.5	0.0041	0.99
			5.190	8.772			55.5	48.3	0.0043	1.04
			3.050	8.772			65.0	48.3	3.0037	0.90
6F	120	28	2.296	6.491	15.70	27.20	224.7	148.7	0.0037	0.90
			6.491	9.174			148.7	110.8	0.0026	0.63
			2.296	9.174			224.7	110.8	0.0033	0.80

TABLE 10
THERMAL CONDUCTIVITY OF CONCRETE
Mixture 1:9

Cylinder Mark	Relative Water Content Per Cent	Age Days	r ₁ cm.	r ₂ cm.	I Amp.	E Volts	t ₁ deg. C.	t ₂ deg. C.	k c. g. s. Physical	k British Engineering
1G	110	28	2.362	4.828	16.1	25.3	234.0	155.4	0.0024	0.58
2G	110	28	3.642	5.232	16.08	27.70	190.7	161.0	0.0035	0.85
			5.232	7.976			161.0	126.6	0.0035	0.85
			3.642	7.976			190.7	126.6	0.0035	0.85
2G	110	120	3.642	5.232	15.38	26.30	172.4	150.3	0.0044	1.06
			5.232	7.976			150.3	119.6	0.0037	0.90
			3.642	7.976			172.4	119.6	0.0040	0.97
3G	110	28	3.315	5.595	16.00	24.00	194.9	150.8	0.0030	0.73
			5.595	7.495			150.8	130.1	0.0035	0.85
			3.315	7.495			194.9	130.1	0.0031	0.75
3G	110	120	3.315	7.495	15.27	26.65	174.5	118.5	0.0038	0.92
3G	110	150	3.315	5.595	7.55	12.44	58.8	53.4	0.0060	1.45
			5.595	7.495			53.4	50.0	0.0054	1.30
			3.315	7.495			58.8	50.0	0.0058	1.41
3G	110	150	3.315	7.495	19.13	32.65	259.4	172.5	0.0038	0.92
4G	110	28	4.589	4.978	15.97	26.50	170.0	164.2	0.0038	0.92
			4.978	7.611			164.2	133.4	0.0038	0.92
			4.587	7.611			170.0	133.4	0.0038	0.92
4G	110	120	4.589	4.978	15.27	26.85	159.9	154.6	0.0042	1.02
			4.978	7.611			154.6	121.7	0.0035	0.85
			4.589	7.611			159.9	121.7	0.0036	0.87
5G	120	28	2.316	5.234	15.55	23.20	228.8	162.8	0.0029	0.70
			5.234	9.134			162.8	112.9	0.0026	0.63
			2.316	9.134			228.8	112.9	0.0027	0.65

TABLE 11
THERMAL CONDUCTIVITY OF "ALABAMA WHITE" MARBLE

r ₁ cm.	r ₂ cm.	I Amp.	E Volts	t ₁ deg. C.	t ₂ deg. C.	k c. g. s. Physical	k British Engineering
2.890	6.401	7.25	11.71	58.7	51.5	0.0063	1.52
6.401	8.911			51.5	48.3	0.0059	1.43
2.890	6.401	9.53	16.10	79.9	67.4	0.0065	1.57
6.401	8.911			67.4	62.3	0.0066	1.60
2.890	6.401			79.9	62.3	0.0066	1.60
2.890	6.401	10.18	17.20	88.6	71.6	0.0055	1.33
6.401	8.911			71.6	65.1	0.0061	1.48
2.890	6.401			88.6	65.1	0.0056	1.36
6.401	8.911	12.87	22.05	101.6	91.5	0.0062	1.50
2.890	6.401			128.4	101.6	0.0055	1.33
2.890	6.401			128.4	91.5	0.0059	1.43
2.890	6.401	13.83	23.50	146.7	114.0	0.0053	1.28
6.401	8.911			114.0	99.9	0.0055	1.33
2.890	6.401			146.7	99.9	0.0052	1.26
2.890	6.401	13.96	24.00	147.9	110.4	0.0048	1.16
6.401	8.911			110.4	96.8	0.0054	1.31
2.890	6.401			147.9	96.8	0.0049	1.19
2.890	6.401	14.58	24.90	160.3	124.1	0.0053	1.28
6.401	8.911			124.1	107.9	0.0050	1.21
2.890	6.401			160.3	107.9	0.0052	1.26
6.401	8.911	18.19	31.11	147.0	119.4	0.0045	1.09
2.890	6.401			219.6	147.0	0.0041	0.99
2.890	6.401			219.6	119.4	0.0042	1.02
6.401	8.911	18.27	31.25	160.4	133.3	0.0047	1.14
2.890	6.401			235.2	160.4	0.0041	0.99
2.890	8.911			235.2	133.3	0.0042	1.02

TABLE 12
DIFFUSIVITY OF CONCRETE AND MARBLE

Mixture	Relative Water Content Per Cent	Density		Conductivity		Specific Heat		Diffusivity	
		gms. per c. c.	lbs. per cu. ft.	c. g. s. Physical	British Engi- neering	Per deg. C.	Per deg. F.	c. g. s. Physical	British Engi- neering
"Neat"	110	1.83	114	0.00147	0.356	0.278	0.153	0.00289	0.0204
1-2	110	2.26	141	0.00344	0.832	0.216	0.119	0.00705	0.0492
1-3	110	2.28	142	0.00379	0.917	0.218	0.121	0.00762	0.0533
1-4	110	2.29	143	0.00352	0.852	0.218	0.121	0.00705	0.0493
1-5	110	2.29	143	0.00323	0.782	0.217	0.120	0.00650	0.0455
1-7	110	2.23	139	0.00384	0.929	0.227	0.126	0.00758	0.0530
1-9	110	2.16	135	0.00352	0.852	0.223	0.124	0.00732	0.0509
Marble		2.71	169	0.00613	1.483	0.213	0.118	0.01059	0.0739

VI. SUMMARY OF RESULTS AND CONCLUSIONS

11. *Summary of Results.*—In the following tables may be found a summary of the detailed observations and calculations contained in Tables 4 to 11.

Table 13 gives the average thermal conductivities of the different mixtures tested, at different temperatures; the values found in the table have been arrived at by collecting and averaging the values given in Tables 4 to 11.

Table 14 gives the average thermal conductivities, at different temperatures, for mixtures in which different amounts of water were

TABLE 13
AVERAGE THERMAL CONDUCTIVITIES OF DIFFERENT MIXTURES OF CONCRETE,
AND OF MARBLE, AT DIFFERENT TEMPERATURES
(Averaged from Tables 3 to 10.)

Mixture By Volumes		50° C. to 100° C. 120° F. to 212° F.		100° C. to 200° C. 212° F. to 390° F.		200° C. to 300° C. 390° F. to 570° F.	
Cement: Aggregate	Cement: Sand: Gravel	k c. g. s. Physical	k British Engi- neering	k c. g. s. Physical	k British Engi- neering	k c. g. s. Physical	k British Engi- neering
"Neat"		0.00140	0.339	0.00163	0.394	0.00140	0.339
1-2	1-1.2-1.1	0.00326	0.789	0.00344	0.832	0.00318	0.770
1-3	1-1.9-1.7	0.00335	0.811	0.00379	0.917	0.00318	0.770
1-4	1-2.4-2.3	0.00413	0.995	0.00352	0.852	0.00328	0.794
1-5	1-3.1-3.0	0.00327	0.791	0.00323	0.782	0.00334	0.808
1-7	1-4.3-4.0	0.00400	0.968	0.00384	0.929		
1-9	1-5.6-5.1	0.00574	1.39	0.00352	0.852		
Marble		0.00614	1.49	0.00493	1.19		

TABLE 14
VARIATION OF CONDUCTIVITY WITH RELATIVE WATER CONTENT
(Summarized from Tables 4 to 9.)

Mix- ture	Relative Water Content Per Cent	k in c. g. s. Physical Units			Mix- ture	Relative Water Content Per Cent	k in c. g. s. Physical Units		
		50° C. to 100° C.	100° C. to 200° C.	200° C. to 300° C.			50° C. to 100° C.	100° C. to 200° C.	200° C. to 300° C.
1:2	100	0.00365	0.00322	0.00320	1:5	100		0.00353	
	110	0.00300	0.00332	0.00310		110		0.00380	
	120		0.00317			120	0.00381	0.00305	0.00270
1:3	100	0.00347	0.00365	0.00340	1:7	110	0.00402	0.00387	
	110	0.00343	0.00391			120		0.00300	
	120	0.00353	0.00345	0.00310	1:9	110	0.00573	0.00359	
1:4	100		0.00357			120		0.00273	
	110	0.00415	0.00373	0.00322					
	120	0.00410	0.00316						

TABLE 15
EFFECT OF AGE ON CONDUCTIVITY
(Summarized from Tables 4 to 9.)

Mixture	Relative Water Content Per Cent	Age Days	k c. g. s. Physical	Mixture	Relative Water Content Per Cent	Age Days	k c. g. s. Physical
1:2	110	28	0.00335	1:5	120	28	0.00330
		120	0.00331			120	0.00297
1:3	110	28	0.00398	1:7	110	28	0.00380
		120	0.00365			120	0.00380
1:4	110	28	0.00376	1:9	110	28	0.00340
		120	0.00337			120	0.00387

used; the values have been arrived at by averaging the values given in Tables 5 to 10.

Table 15 gives the average thermal conductivities for mixtures of different ages. The values have been obtained for only a few of the mixtures, and for one water content only in each case.

12. *General Conclusions.*—From Table 13 it can be seen that the neat cement had a much lower thermal conductivity than any of the sand and gravel concrete mixtures; in fact, the thermal conductivity of the neat cement is scarcely half that of the 1:2 mixture.

In the case of the sand and gravel concrete mixtures, the figures in the table also show that there is practically no difference in thermal conductivity due to the relative “richness” or “leanness” in cement of a mixture, at any rate for the range of temperature of 100 deg. C. to 200 deg. C. The values, as previously noted, are probably more accurate for this range than for lower temperatures, on account of the number and character of the observations.

From the values given in Table 12 for the densities of the various materials it can be calculated that the voids in the sand and gravel concrete mixture range from 16 to 20 per cent; while in the case of the neat cement the percentage of voids is about 42. It seems probable that the proportion of solid material to voids to a large extent determines the conductivity, and this accounts for the fact that the thermal conductivity of the neat cement is so much lower than that of the concrete mixtures, and that the conductivities of the different mixtures are so nearly the same. The same table shows that the thermal conductivity of a stone, like marble, is much greater than that of a concrete mixture.

The figures given in Table 14 appear to indicate that as far as consistency is concerned the maximum thermal conductivity occurs with a relative water content of about 110 per cent; for relative water contents of 100 or 120 per cent the thermal conductivity is generally lower.

From Table 15 it can be seen that age has little if any effect on the thermal conductivity of concrete. If there is any change, there is a slight decrease in thermal conductivity with age, but this is small, and may be due to very small changes in absolute moisture.

Most of the cylinders cracked at temperatures under 300 deg. C., and that fact limited the range of the investigation. In general, the richer mixtures of concrete cracked at lower temperatures than the leaner mixtures. The results indicate that there is very slight, if any, change of conductivity with change of temperature, for concrete; for marble, there is a marked decrease in conductivity with rise of temperature.

In Table 16 are reproduced the results of the experiments of Professor C. L. Norton, to which reference has already been made.*

TABLE 16
EARLIER DETERMINATIONS OF CONDUCTIVITY OF CONCRETE
(From experiments of Professor C. L. Norton.)

Temperatures—degrees C.	Mixture	k—c. g. s. Physical Unit
35	Stone 1-2-5	0.00216
50	Stone 1-2-4	0.00216
50	Not stamped	0.00110 to 0.00160
200	Cinder 1-2-4	0.00081
400	Stone 1-2-4	0.0021
500	Stone 1-2-4	0.0022
1000	Stone 1-2-4	0.0023
1100	Stone 1-2-4	0.0027
	Stone 1-2-4	0.0029

Professor Norton's method at the lower temperatures was, as he names it, the "flat-plate" method. For the higher temperatures he used a cylinder of concrete cast about a steel bar which was heated by the passage of a heavy electric current. He gives practically no details, describing his investigation "in outline only." The table indicates a small increase of thermal conductivity with increase of temperature. As the methods employed in his determinations at

* Proceedings of National Association of Concrete Users, Vol. VII, article by C. L. Norton, 1911.

lower temperatures are not the same as those for higher temperatures, the results are not very conclusive. The values of absolute conductivity are considerably lower than those found in the present investigation, but it is impossible to identify the mixtures used.

Willard and Lichty* give for the thermal conductivity of a 1:2:4 concrete mixture the value 8.3 "per 1-inch thickness per sq. ft. per 1 deg. F."; this is equivalent to 0.00296 in the c.g.s. physical units. The method of determination employed was a "hot-air box method," a method specially useful for their purpose in testing materials used for the walls of buildings.

From the present investigation, for the more commonly used concrete mixtures, that is, those with proportions of cement to aggregate of 1:3 to 1:7, the following average values of thermal conductivity and thermal diffusivity appear established: for the c.g.s. physical unit system, for the range of temperature between 50 deg. C. and 200 deg. C., the average thermal conductivity is 0.00366, and the average thermal diffusivity, 0.00719; for the British engineering unit system, for the range of temperatures between 120 deg. F. and 390 deg. F., the average thermal conductivity is 0.904, and the average thermal diffusivity, 0.0503. These values are for thoroughly dry concrete, of the stone-concrete mixture described.

While the values for such physical constants as thermal conductivity and thermal diffusivity, for a material like concrete, are necessarily averages, and subject to the variation of averages, yet they are probably as definite as other physical constants for structural materials, and particularly so when the average values are obtained for a considerable number of specimens, as in this investigation.

* "A Study of the Heat Transmission of Building Materials," Univ. of Ill. Eng. Exp. Sta. Bul. No. 102, 1917.

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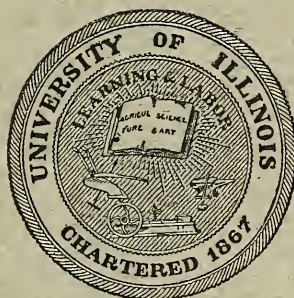
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STUDIES ON COOLING OF FRESH CONCRETE IN FREEZING WEATHER

BY

TOKUJIRO YOSHIDA



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BULLETIN No. 123

MAY, 1921

STUDIES ON COOLING OF FRESH CONCRETE
IN FREEZING WEATHER

BY
TOKUJIRO YOSHIDA
STUDENT IN THE GRADUATE SCHOOL
OF THE UNIVERSITY OF ILLINOIS

291

ENGINEERING EXPERIMENT STATION

PUBLISHED BY THE UNIVERSITY OF ILLINOIS, URBANA

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STUDIES ON COOLING OF FRESH CONCRETE IN FREEZING WEATHER *

I. INTRODUCTION

1. *Preliminary.*—The practice of placing concrete in freezing weather renders important a knowledge of the rate at which this material will cool and the effect of the various methods of protecting the freshly placed concrete from the cold. The experiments herein recorded furnish test data on the length of time required for concrete of a given temperature to lose its heat and become cold enough to freeze when it is exposed to temperatures lower than the freezing point of water. The values of two thermal constants, diffusivity and the ratio of emissivity to coefficient of thermal conductivity, were determined for freshly placed concrete. Some experiments were made on the protective effects of coverings. While a number of applications of the experimental data are presented, no originality can be claimed for the underlying mathematical theory, which dates back, of course, to the time of Fourier. It is not considered that these solutions will give complete data regarding concreting in freezing weather under various conditions; however, it is hoped that they will throw some light on the behavior of fresh concrete at low temperatures and will indicate the necessity of protection or other precautions.

2. *Acknowledgments.*—The tests reported herein were made in the Engineering Experiment Station of the University of Illinois. The writer is indebted to PROFESSOR A. N. TALBOT, PROFESSOR H. F. GONNERMAN, MR. F. E. RICHART, MR. R. A. NELSON, and MR. W. H. BRAMAN.

3. *Discussion Regarding Assumptions.*—Every calculation in engineering is based upon some assumption. This is not strange when we consider that the principles and laws of physics and chemistry upon which engineering is founded are based upon fundamental hypotheses or principles.

* This bulletin presents the principal results of the investigation made in 1920 on the flow of heat through fresh concrete by Tokujiro Yoshida, Assistant Professor of Civil Engineering, Kyushu Imperial University, Fukuoka, Japan, who was in residence as a graduate student at the University of Illinois.

Mathematicians may be satisfied by proving that the solution of a problem is possible, but the engineer cannot be satisfied until he obtains the particular numerical value which is applicable to an actual case. In this fact is found the reason for the difference between the view points of physicists or mathematicians and engineers.

Engineering problems are usually very complicated. There are many conditions which cannot be expressed mathematically, or which are entirely unknown. So, if the results obtained by physicists or mathematicians are to be applied to engineering, many assumptions must be made, on the one hand, in order that the conditions may be expressed physically or mathematically, while, on the other hand, the calculations must be made as simple as possible for actual use. In general, the nearer the assumption is to the actual fact, the more accurate will be the result based on the assumption; but no matter how accurate the result, if the time spent in calculation is too long the method is often not acceptable to the engineer. When a method of calculation or an engineering formula is used, the assumptions on which it is based must first be known, and then these assumptions compared with the actual conditions affecting the case under consideration. It is an important function of engineering science to conduct researches into conditions affecting engineering problems, in order that assumptions may be made which will approximate the facts of the case and at the same time render possible the introduction of methods of calculation that are reasonably simple.

In the following problems a number of assumptions have been made in order to simplify calculations as well as to cover many conditions that are unknown. It is important that this discussion with reference to assumptions be borne in mind by the reader.

II. EXPERIMENTS ON COOLING OF FRESH CONCRETE AND RISE IN TEMPERATURE DURING SETTING OF CONCRETE AND MORTAR

4. *Apparatus for Measuring Temperature.*—The general scheme of the experiments on the cooling of fresh concrete involved the exposing of one surface of a specimen to a low temperature, all other surfaces being carefully insulated from the cold, and the measuring of the temperature of the concrete at various distances from the exposed surface. After a study of several methods for the accurate measurement of these temperatures, a system of thermocouples was chosen as most suitable for the purpose. Fig. 1 shows the general layout of the apparatus for measuring temperature.

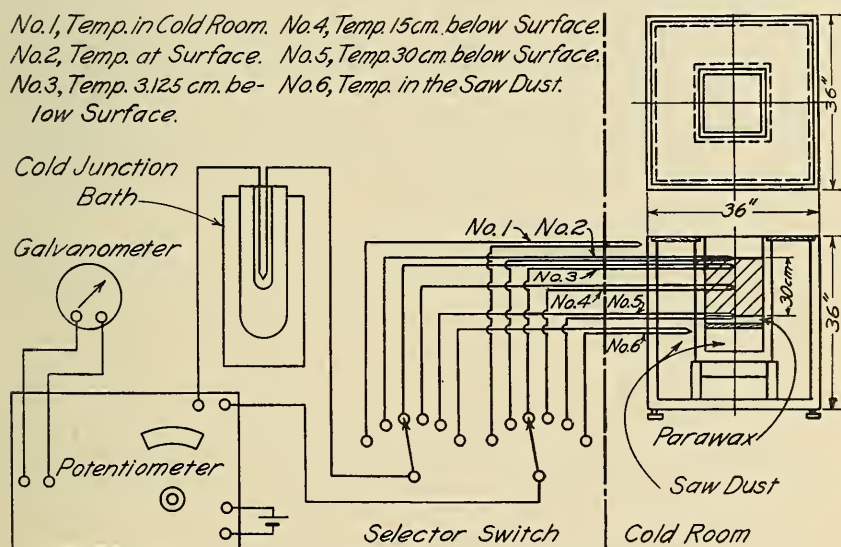


FIG. 1. LAYOUT OF TEMPERATURE-MEASURING APPARATUS

By the use of a number of thermocouples connected to a sensitive potentiometer the temperatures at various points were quickly and accurately indicated in terms of electrical units, which were later converted into degrees centigrade for use in making computations. Since the range of temperatures to be measured in this investigation was relatively small, and hence the electromotive force in the ther-

thermocouple circuit also small, the use of the deflection method was considered inadvisable, because in this method the readings are not independent of the resistance in the circuit and are liable to errors due to changes in resistance. The potentiometer was found to be sufficiently accurate for the purpose in view, and since the principle involved embodied the measurement of the electromotive force with zero current in the circuit, thus making the readings independent of the external resistance, it was adopted. In the potentiometer method the precision of reading is a function of the accuracy with which the initial balance is attained. The accuracy of the initial balance was greatly increased in the present case by using a sensitive galvanometer.

The wires used in the thermocouple were of copper and of a copper-nickel alloy known under the trade name of constantan. These metals were chosen because they are homogeneous and because the magnitude of the induced electromotive force per degree difference in temperature between the hot and cold junction is large. The wires were No. 20 B. & S. gage, and were cotton covered. This small size of wire was used to minimize the effect of radiation to or from the couple, and the effect of the depth of its immersion. To prevent the serious effect of moisture or dew, between the cold storage room and the room in which the instruments were arranged the wires were all carefully insulated with rubber tape.

The junctions of the couples were all formed by fusing the two wires together in an oxyacetylene gas flame, making a button about the size of a pin head at the junction. A strong joint was made in this manner.

To guard against the effect of water in the concrete the wires leading to the hot junctions of the thermocouples were insulated with glass tubing and the joints themselves were put into melted paraffin so that the paraffin filled the tubing about two inches and covered the surface of the wires with a thin film at the joint.

At the cold junction the joint of the wires was put into a test tube filled with transformer oil, the wires being insulated from each other by glass tubing. A thermos bottle filled with ice shavings and water was used as the cold bath of the test tube.

The thermocouples were calibrated by means of a mercury thermometer which was graduated to one-tenth of one degree centigrade. The oil bath used for constant temperature in calibrating was a small test tube which was inserted into a thermos bottle. The calibration curves for all the couples were practically the same. Fig. 2 shows

the calibrating apparatus and a typical calibration curve. The discontinuity of the curve near the zero point is due to the instrumental error of the potentiometer used.

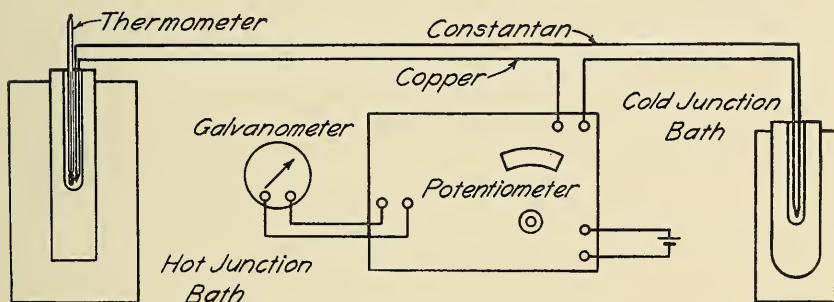
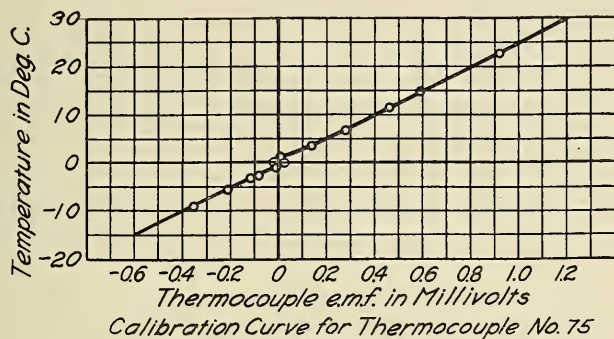


FIG. 2. CALIBRATING APPARATUS AND A TYPICAL CALIBRATION CURVE

The potentiometer gave a direct reading of 0.01 millivolt. To increase the accuracy of the balance of electromotive force, the galvanometer of the instrument was replaced by a D'Arsonval galvanometer.

As the precision of the thermocouple in use is about 25 deg. C. per millivolt, the precision attainable is 0.25 deg. C. It was found possible to repeat the readings to within 0.005 millivolt, or 0.13 deg. C. when the hot junction was placed in a constant temperature oil bath.

5. *Concrete Materials.*—The materials used were similar in character and quality to those used in other concrete and reinforced

concrete specimens made and tested by the Engineering Experiment Station in recent years.

Universal portland cement was used in all specimens. It fulfilled the requirements of the specification of the American Society for Testing Materials.

The sand used came from pits at Attica, Indiana, and passed a $\frac{1}{4}$ -in. screen. It weighed 111 lb. per cubic ft. The sieve analysis of this sand, using the sieves commonly known as the Tyler Standard Sieves, is given in Table 1.

Clean gravel from the same pits was used. It passed a $1\frac{1}{2}$ -in. screen and was retained on a $\frac{1}{4}$ -in. screen. The weight of this gravel was 99 lb. per cubic ft. A sieve analysis of the material is shown in Table 1.

Concrete of three proportions, 1-3-6, 1-2-4, and 1-1-2, by volume, was tested as indicated in Table 2. To insure accuracy in proportioning, the materials for each specimen were weighed out separately and mixed.

TABLE 1
SIEVE ANALYSIS OF SAND AND GRAVEL
(TYLER STANDARD SIEVES USED)

Material	Sieve Size	Size of Square Openings, inches	Percentage Passing*
Sand.....	$\frac{3}{8}$ in.	0.371	100
	No. 4	0.185	91
	" 8	0.093	66
	" 14	0.046	49
	" 28	0.0231	19
	" 48	0.0116	3
Gravel.....	" 100	0.0058	1
	$1\frac{1}{2}$ in.	1.49	100
	$\frac{3}{4}$ "	0.742	57
	$\frac{3}{8}$ "	0.371	11
	No. 4	0.185	5
	" 8	0.093	3

* Each value represents the average of four tests.

The fineness modulus* and surface modulus* of these concrete aggregates are 5.98 and 4.69 respectively. From these figures it appears that the materials are rather coarse, but they are undoubtedly similar to the materials used in common practice. It is believed that the experimental data are applicable to any other well proportioned concrete.

* See Discussion on Proportioning of Concrete, by Professor D. A. Abrams and Professor A. N. Talbot. Proceedings of A. S. T. M., 1919. p. 477-485.

TABLE 2
PROPERTIES OF THE CONCRETE USED IN THE EXPERIMENTS

Proportion by Volume	Water-Cement Ratio	Slump Test on 6 by 12-in. Cylinders, inches	Twenty-eight Day Compression Tests of 6 by 12-in. Cylinders, lb. per sq. in.*
1-2-4 (all except Experiments Nos. 13, 14, 18, and 20)	1.01	2	1300
1-2-4 (Experiments Nos. 13 and 18) wet consistency.	1.26	9	700
1-2-4 (Experiments Nos. 14 and 20) dry consistency.	0.81	0	2200
1-3-6 medium consistency	1.29		800
1-1-2 medium consistency. . . .	0.72	..	2900

* The cylinders were stored in damp sand at an average temperature of 70 deg. F. Each value is the mean of tests on three cylinders.

6. *Mixing of Concrete.*—The amount of water used in mixing the concrete was designed to produce the wettest concrete which could satisfactorily be used in freezing weather, except in Experiments Nos. 13 and 18, and 14 and 20, in which very wet and very dry mixtures, respectively, of 1-2-4 concrete were used. An idea of the mobility of these concretes may be obtained from the slump tests made on 6-in. by 12-in. cylinders noted in Table 2, as well as from the properties of the concrete aggregates and the water-cement ratio (ratio of volume of water to volume of cement) also shown in Table 2.

The mixing of the concrete was done by hand. The dry cement and sand were first mixed to a uniform color and spread out in a thin layer in a large mixing pan; the stone was then added, and the whole mass turned with shovels until thorough incorporation of the dry materials was secured; water was then added, and the materials turned until thoroughly mixed.

The temperature of the concrete thus made was practically uniform and was the same as the temperature of the room in which the concrete was mixed.

7. *General Features of the Experiments.*—Sixteen experiments were made to obtain data on the cooling of fresh concrete exposed to freezing temperatures. One experiment was made on concrete which was 31 days old.

Experiments Nos. 1-6, 10, and 13-15 were made with the purpose of determining the thermal constants of fresh concrete which are necessary in applying the theory of heat conduction to the cooling of fresh concrete, and others were made to obtain information on the effect of protection of the concrete surface. Experiment No. 17 was made to determine the thermal constants of concrete 31 days old. These experiments were made at the plant of the Smith Ice and Cold Storage Company at Champaign, Illinois.

The size of specimen, the concrete mixtures, and the conditions of the surface are given in Table 3.

TABLE 3
DATA OF THE SPECIMENS

Experiment No.	Proportion by Volume	Depth of the Specimen* cm.	Condition of the Surface
1	1-3-6	50	No protection at the surface under a still air condition.
2	1-2-4	50	"
3	1-1-2	50	"
4	1-3-6	30	"
5	1-2-4	30	"
6	1-1-2	30	"
7	1-2-4	30	Surface cooled by air current from a fan.
8	1-2-4	30	Surface covered with a board $\frac{3}{4}$ -in thick.
9	1-2-4	30	Surface covered with canvas.
10	1-2-4	20	No protection at the surface.
11	1-2-4	20	Surface covered with a board.
12	1-2-4	20	Surface covered with canvas.
13	1-2-4	30	No protection at the surface.
14	1-2-4 (wet consistency)	30	"
15	1-2-4 (dry consistency)	25	"
16	1-2-4	10	Surface covered with a board.
17	1-2-4	30	No protection at the surface.

* All the specimens are 12-in. square in horizontal section.

8. *Preparation of Test Specimens.*—The mold for the test specimens was a wooden box 12 in. square and from 12 to 24 in. deep, in

inner dimensions. It was made of 1½-in. pine boards. This mold was placed inside another wooden box 3 ft. square and 3 ft. deep. The space between the two was filled with dry sawdust to prevent cooling of the fresh concrete at the sides and at the bottom of the mold.

The inner surface of the inner box was coated with Parowax to prevent the absorption of water from the concrete by the wood.

As each specimen was being poured, the thermocouples were inserted one by one through small holes in the side of the mold and embedded in the concrete, their joints being at specified depths below the upper surface and in the vertical axis of the specimen. A thermocouple was also embedded in the sawdust to measure its cooling. The temperatures of the concrete, the boxes, and the sawdust were practically the same as the temperature of the room at the beginning of all experiments.

The specimen thus made was removed to the adjoining cold storage room about twenty minutes after the water was added to the concrete mixture. The wires of the thermocouples were carried across the door sill from the cold room to the room in which the measuring instruments were arranged. The temperature of the cold room was measured by a thermocouple. Fig. 1 shows in diagrammatic form the general arrangement of the apparatus.

9. *Phenomena of Experiments.*—In experiments Nos. 1, 2, and 3, specimens 50 cm. in depth, of 1-3-6, 1-2-4, and 1-1-2 concrete, respectively, were tested. From the data of these experiments Fig. 3 has been plotted. It gives the curves of cooling at the several depths below the exposed surface of the specimen. The abscissas give the elapsed time from the date the specimen was placed in the cold room. The temperature of the cold room is also shown in the diagram. The effect of the heat produced during the setting of the concrete is noticeable in the figure even in the case of the 1-3-6 mixture.

In these first three experiments the thermocouple measuring the temperature of the sawdust at the underside of the mold always showed a lower temperature than that at the bottom of the specimens, and it was clear that there was a flow of heat through the bottom of the mold from the specimen to the sawdust. The rise in temperature during the setting of the concrete was also marked with so great a depth of specimen. For these reasons, the data obtained from these three experiments were not used in calculating the thermal constants.

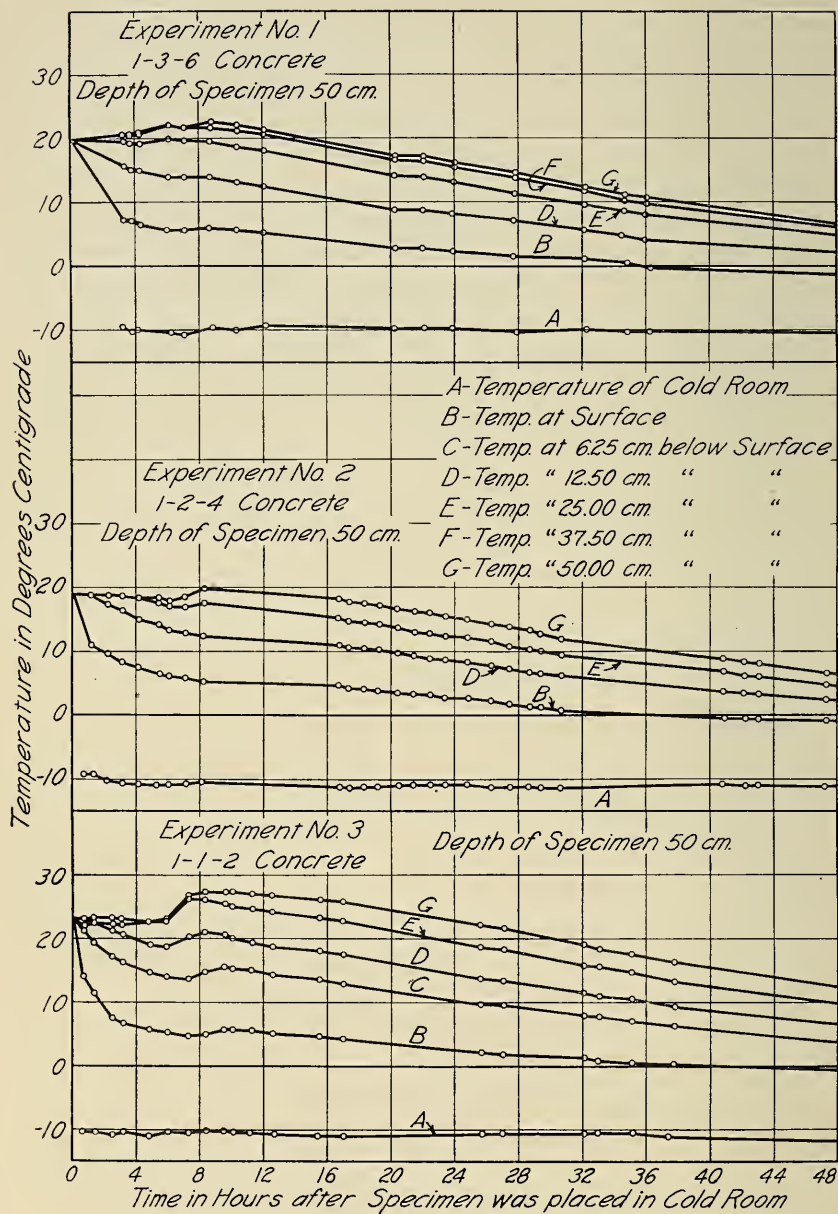


FIG. 3. CURVES OF COOLING, EXPERIMENTS 1, 2, AND 3

In experiments Nos. 4, 5, and 6, specimens 30 cm. thick and of 1-3-6, 1-2-4, and 1-1-2 concrete, respectively, were used. This depth was used in order to reduce the effect of the rise in temperature produced in the setting of the concrete, and also to minimize the flow of heat through the bottom of the mold. For the latter reason further insulation was obtained by filling in the bottom of the mold with a 3-in. layer of sawdust, placing upon this a $\frac{3}{4}$ -in. pine board, and adding a $1\frac{1}{2}$ -in. layer of Parowax. The inner sides of the mold were also given a covering of Parowax $\frac{1}{8}$ in. thick. With these precautions the thermocouple in the sawdust below the bottom of the mold showed practically the same temperature as that of the bottom of the specimen, and it was considered that the bottom of the mold was thus kept sufficiently impervious to the flow of heat. This condition was assumed in calculating the thermal constants of the concrete from the test data. The curves of cooling for these three specimens are plotted in Fig. 4.

The specimens used in experiments Nos. 7, 8, and 9 were of the same material as that used in experiment No. 5; they differed in surface conditions.

In experiment No. 7 the surface of the specimen was subjected to an air current produced by an ordinary electric fan operating at its medium speed. The distance between the fan and the concrete was about one foot, the direction of the current making an angle of about 30 degrees with the surface. The velocity of the air current was about ten miles per hour. It was intended by this method to obtain information on the effect of wind on the cooling of fresh concrete. The cooling of the concrete surface was very striking. The time required for the freezing temperature to penetrate the surface in this experiment was about one-tenth of that under a still air condition. It is clear that the evaporation of water from the surface had a very great effect on the cooling of the surface; within the mass the effect was slight. Fig. 5 shows the results of the experiment.

In experiment No. 8 the surface of the specimen was covered with a pine board 1.9 cm. ($\frac{3}{4}$ in.) thick. The small opening between the edge of the board and the box was sealed with Parowax. This experiment was made to determine the effect of wood forms on the cooling of fresh concrete. The board protected the surface very well. The results are plotted in Fig. 5.

In experiment No. 9 the top of the inner and outer boxes was covered with 10-oz. duck. There was about a 3-in. air space between

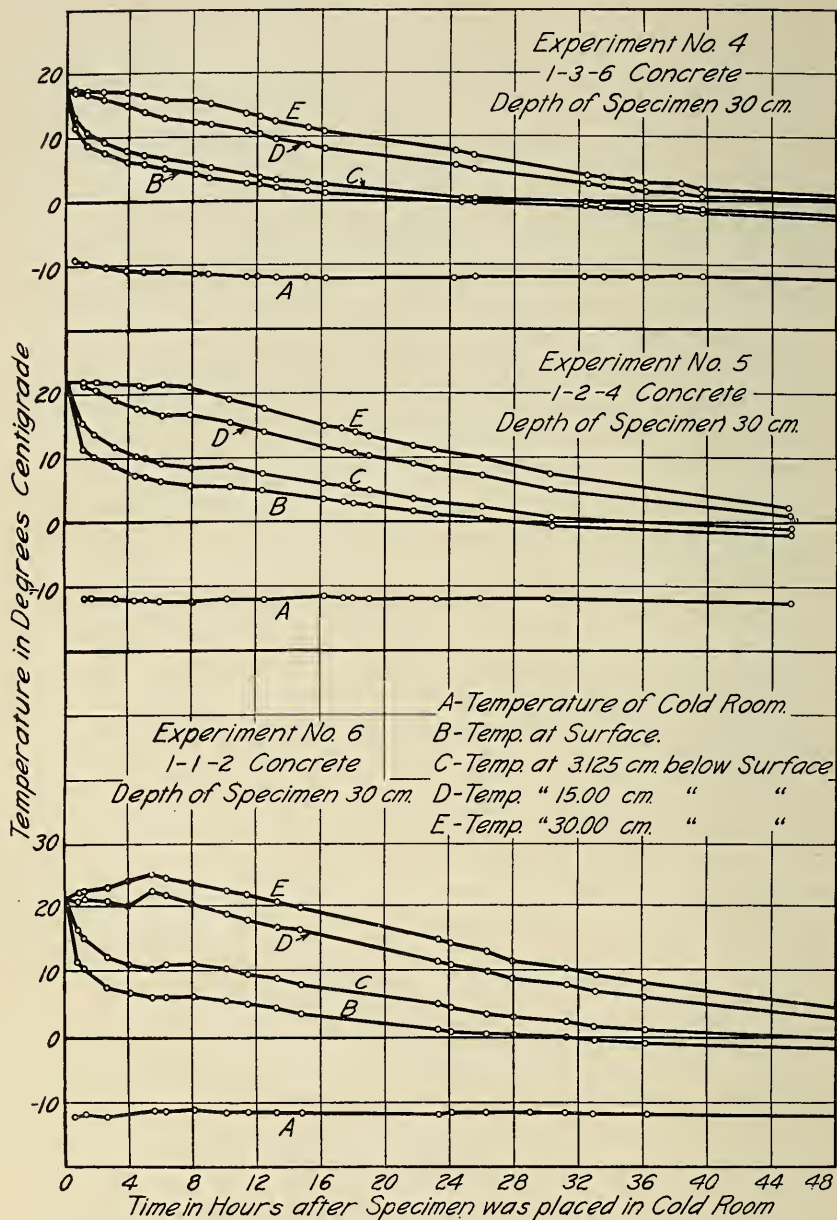


FIG. 4. CURVES OF COOLING, EXPERIMENTS 4, 5, AND 6

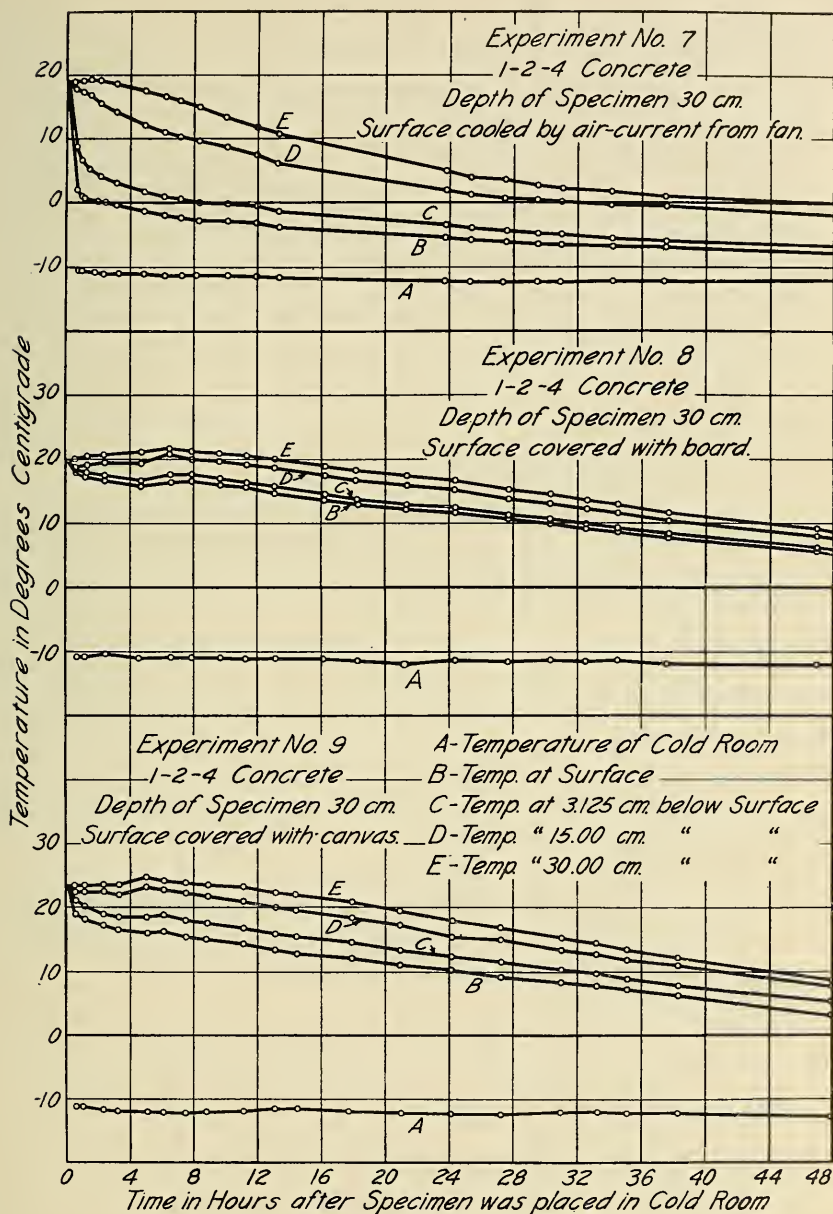


FIG. 5. CURVES OF COOLING, EXPERIMENTS 7, 8, AND 9

the canvas and the surface of the specimen. This experiment was made to investigate the effect of protection by using canvas in cold weather. The curves of cooling are shown in Fig. 5. The protection is seen to be about the same as with the $\frac{3}{4}$ -in. board.

The specimens used in experiments Nos. 10, 11, and 12 were 20 cm. in depth. The depth of the inner box was changed to 16 in.; other conditions were the same as before. Experiment No. 10 was made as a check on No. 5; the conditions of the specimen were the same as in No. 5 except for the depth. The conditions in experiments Nos. 11 and 12 were the same as in Nos. 8 and 9 except as to the depth of the specimen. The curves of cooling are shown in Fig. 6.

The two experiments (Nos. 13 and 14) made to investigate the effect of the amount of water in the mixture upon the cooling of fresh concrete had conditions of specimen the same as No. 4 except for the consistency of the mixture.

In experiment No. 13, 1-2-4 concrete of wet consistency was tested. The concrete was so wet that after making the specimen its surface was covered with water to the depth of about $\frac{3}{16}$ -in. After six hours in the cold room at an average temperature of -12.2 deg. centigrade, the surface was covered with laitance about $\frac{3}{8}$ -in. thick which had the appearance of soft chocolate ice cream. The thermocouple which was set at the surface was at first also covered with the laitance. It may be that the laitance offered some protection to the cooling of the surface. The curves of cooling are shown in Fig. 7.

In experiment No. 14 a concrete of very dry consistency was tested; much difficulty was experienced in placing this concrete. Fig. 7 shows the results of the experiment.

In experiment No. 15 the depth of the specimen was 25 cm., other conditions being the same as in experiment No. 5; this experiment was made as a check on the results of experiments Nos. 5 and 10. Fig. 7 shows the results.

In experiment No. 16 the specimen was 10 cm. in depth, and was covered with a board at the surface; other conditions were the same as in experiment No. 11. The curves of cooling are shown in Fig. 8.

For experiment No. 17, to determine the thermal constants of concrete 31 days old, a specimen in the form of a 30-cm. (12-in.) cube of 1-2-4 concrete was made in the Concrete Laboratory at the University. Two thermocouples were used at both the surface and the bottom and one at the center. Two days after the specimen was made

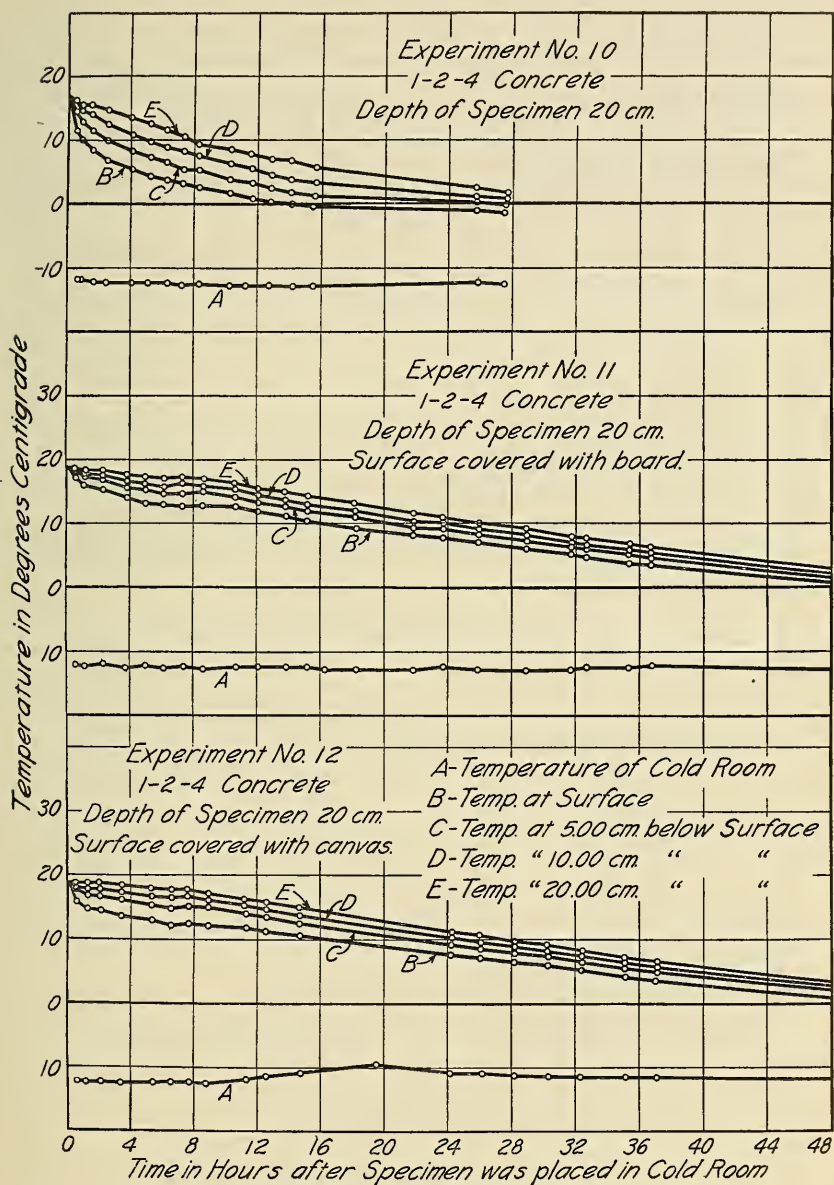


FIG. 6. CURVES OF COOLING, EXPERIMENTS 10, 11, AND 12

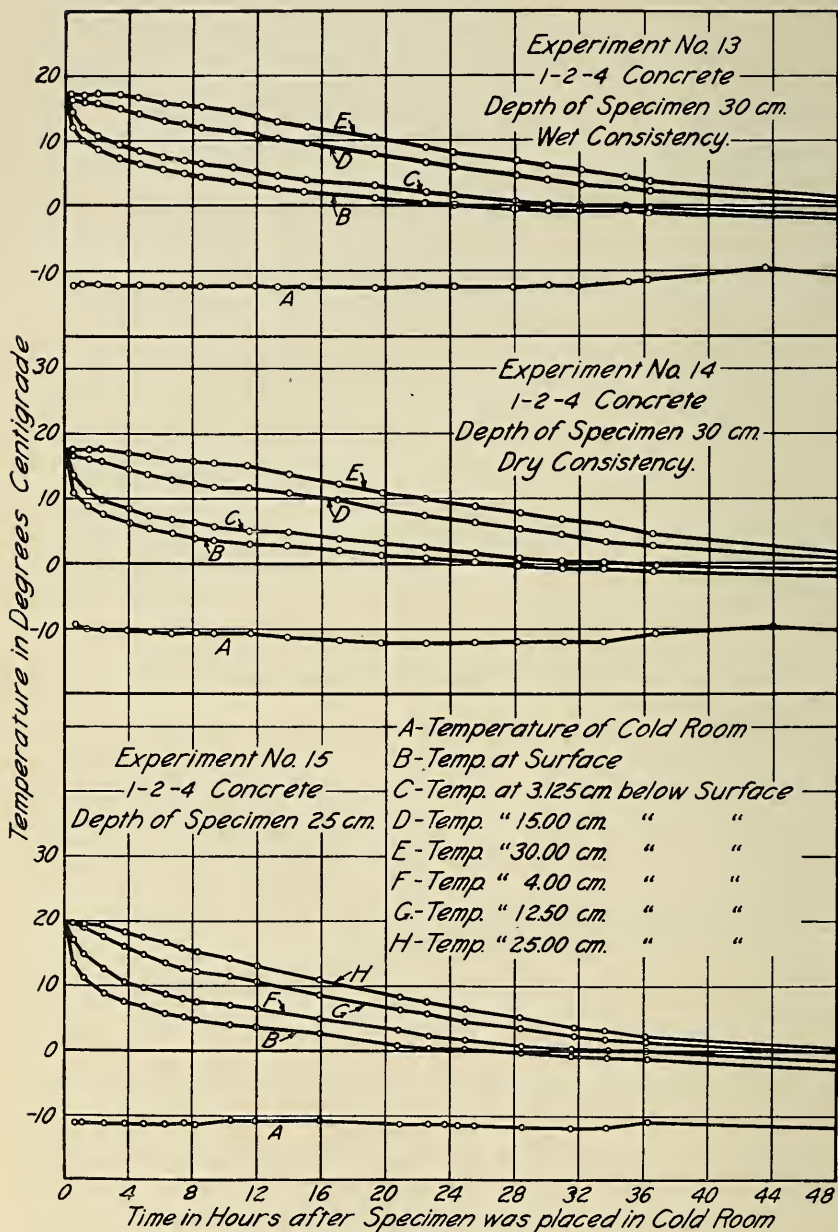


FIG. 7. CURVES OF COOLING, EXPERIMENTS 13, 14, AND 15

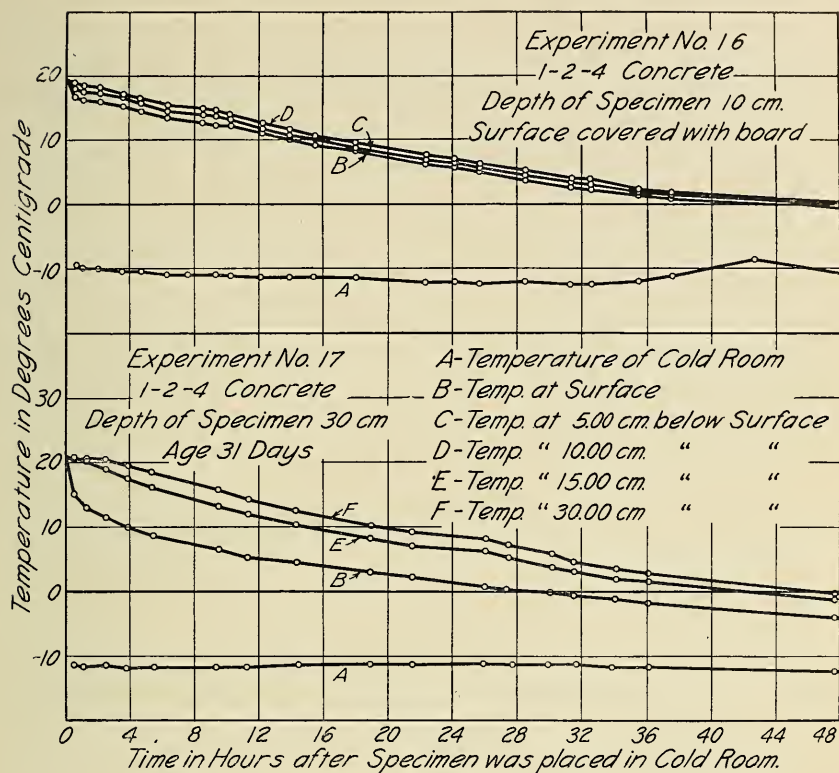


FIG. 8. CURVES OF COOLING, EXPERIMENTS 16 AND 17.

the forms were taken off, and the cube was stored in damp sand. After 24 days it was taken out of the sand and dried in the laboratory for one week. It was then put in the wooden mold, the small spaces between the specimen and walls of the mold being filled with Parowax.

The specimen and the temperature measuring apparatus were arranged as in experiment No. 1. Before it was carried into the cold room the temperature of the specimen at different points was about the same, and the mean was taken as the initial temperature of the specimen. The curves of cooling are shown in Fig. 8.

10. *Rise in Temperature During Setting of Concrete and Mortar.*

—The chemical combination of water with portland cement is an exothermic reaction, the heat evolved being sufficient to raise materially the temperature of concrete and mortar during the period of setting and hardening.

In experiments Nos. 18-26, this rise in temperature of concrete and mortar was measured in order to study the relative importance of these phenomena in the preceding experiments. These experiments were made in the Concrete Laboratory of the University. The apparatus for measuring the temperature was the same as previously described. The water-cement ratio of the four different mixtures of neat cement, 1-1 mortar, 1-2 mortar, and 1-3 mortar was 0.38, 0.48, 0.60, and 0.85, respectively.

The concrete or mortar to be tested was poured into a wooden mold with inner dimensions for holding a 12-in. cube. The mold was made of 1½-in. pine boards. Two thermocouples were embedded to measure the temperature, one at the center of the cube and the other at three in. away from the center. The mold was put in the center of the large box, and the space around all six surfaces

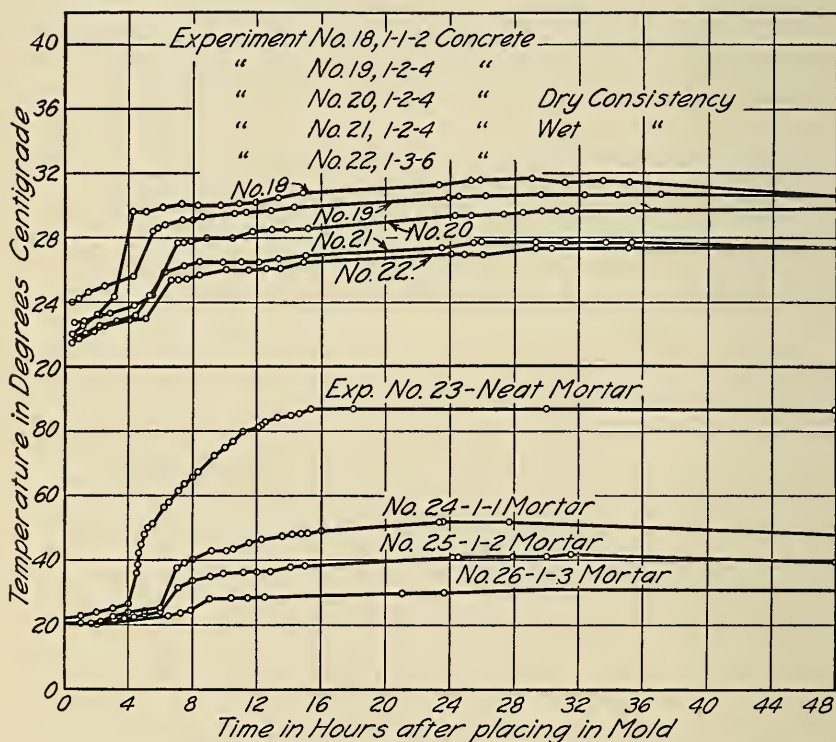


FIG. 9. DIAGRAMS SHOWING RISE IN TEMPERATURE DURING SETTING OF CONCRETE AND MORTAR

of the mold was filled with sawdust about 8 in. thick, to prevent the cooling of the specimen.

The initial temperature of the fresh concrete or mortar was the same as that of the laboratory. The two thermocouples showed about the same temperature, and the temperature at the center is plotted in Fig. 9. After 48 hours the rise in temperature was very small in all cases.

The total rise in temperature, the rate of increase, and the time interval before the maximum temperature was reached were all variable, depending upon the proportions of the mixture, the amount of water used in the mixing, and the initial temperature of the concrete or mortar. The curves in the figure, however, show very clearly that considerable heat is produced during the setting of concrete or mortar at ordinary temperatures, and its effect is very noticeable from 6 to 12 hours after the pouring of the concrete.

11. *Calculation of the Thermal Constants of Fresh Concrete.*—In order to apply the theory of heat conduction to concreting in cold weather, it is necessary to determine certain thermal properties of the concrete from the preceding experiments. Even in a homogeneous material such thermal properties are variable and may be considered constant only for a certain range in temperature; in fresh concrete the properties will be found to vary with other conditions as well as temperature. It is the purpose here to determine certain thermal properties for a limited range of temperatures near freezing, and for purposes of application these properties will be termed “thermal constants.”

From the time that water is added to the concrete materials, a chemical reaction begins with an accompanying rise in temperature, and the mass solidifies after a few hours under ordinary temperatures. In temperatures below about 10 deg. C. (50 deg. F.) the setting takes place very slowly and produces a small amount of heat, while if the temperature falls below 0 deg. C. (32 deg. F.) the fresh concrete will freeze and little setting can be expected. Therefore, even though the effect of heat produced in the setting of concrete be neglected, the coefficient of thermal conductivity of fresh concrete will vary from time to time according to the degree of its setting and its temperature. The specific heat and density of the material also vary considerably with different conditions of setting, as well as with the mix of the concrete and the temperature.

When the surface of the concrete is worked and finished, there is more or less water remaining on it. Therefore, the cooling at the surface of concrete is due not only to radiation and convection, but also to evaporation of water, which, of course, depends upon the humidity and other conditions of the surrounding air.

It is obviously impossible to give certain and definite values of the thermal constants of fresh concrete. However, it is considered useful to determine fair average values of the constants which may be used for practical problems. The execution of concrete work in general practice does not compare in refinement with the measurement of temperature to one degree centigrade or of a time interval to one minute. It is usually possible, however, to make observations with an accuracy of within a few degrees in temperature or a few hours in time, and this usually will be sufficient for the application of information regarding concreting in freezing weather. Hence, the constants determined here should be useful in applying the mathematical theory of heat conduction to practical problems.

The method used in calculating the thermal constant involves three things: the use of well established fundamental formulas applying to various materials, a rigorous definition of the assumptions which may be made regarding the conditions in these experiments, and a reduction of the fundamental equations to special forms which satisfy the assumptions and provide the desired working formulas.

In deriving the fundamental formulas, a body will be considered which has infinite length and breadth and a finite thickness. For convenience in calculation the two surfaces are considered to be exposed to air at zero temperature, although the results of the analysis may be readily applied with any other air temperatures. The C. G. S. system of units is used in the following treatment.

Let v = the temperature at any point in the body at a distance x from one of the surfaces at any time, t ,

v_0 = the initial temperature of the body, that is, the temperature at all points at the time $t = 0$,

k = the diffusivity* of the body,

$$= \frac{(\text{coefficient of thermal conductivity})}{(\text{density}) (\text{specific heat})}$$

$$h = \frac{(\text{emissivity})^*}{(\text{coefficient of thermal conductivity})}$$

* For definitions and a more detailed treatment of the analysis, see Carslaw, "Fourier's series and Integrals." 1906.

where emissivity of the body is the rate of loss of heat by radiation and convection, under certain conditions of air, per square centimeter of surface, per degree difference in temperature between body and air, assuming Newton's law of cooling.

Then the equation of temperature in the body is

$$\frac{\partial v}{\partial t} = k \frac{\partial^2 v}{\partial x^2} \quad 0 < x < l$$

As the initial and boundary conditions we have,

$$v = v_0 \quad \text{for } t = 0$$

$$-\frac{\partial v}{\partial x} + h v = 0 \quad \text{at } x = 0,$$

$$\frac{\partial v}{\partial x} + h v = 0 \quad \text{at } x = l,$$

The solution* of these equations is

$$v = 2v_0 \sum_{n=1}^{\infty} e^{-k\alpha_n^2 t} \frac{\alpha_n \cos \alpha_n x + h \sin \alpha_n x}{(\alpha_n^2 + h^2)l + 2h} \int_0^l (\alpha_n \cos \alpha_n x + h \sin \alpha_n x) dx \quad (1)$$

in which α_n is the n th positive root of the equation

$$\tan \alpha l = \frac{2\alpha h}{\alpha^2 - h^2}$$

excluding the root $\alpha = 0$.

At $x = 0$, we have

$$v_s = 2v_0 \left\{ e^{-k\alpha_1^2 t} \frac{\alpha_1}{(\alpha_1^2 + h^2)l + 2h} \int_0^l (\alpha_1 \cos \alpha_1 x + h \sin \alpha_1 x) dx + \right. \\ \left. e^{-k\alpha_2^2 t} \frac{\alpha_2}{(\alpha_2^2 + h^2)l + 2h} \int_0^l (\alpha_2 \cos \alpha_2 x + h \sin \alpha_2 x) dx + \dots \dots \dots \right\}$$

At $x = \frac{l}{2}$, since

* Carslaw, "Fourier's Series and Integrals," page 274, 1906.

$$\tan \alpha_n l = \frac{2 \alpha_n h}{\alpha_n^2 - h^2}$$

$$\tan \frac{\alpha_n l}{2} = + \frac{h}{\alpha_n} \quad \text{when } n \text{ is odd,}$$

$$\tan \frac{\alpha_n l}{2} = - \frac{\alpha_n}{h} \quad \text{when } n \text{ is even.}$$

$$\text{Thus, for } x = \frac{l}{2}$$

$$(\alpha_n \cos \alpha_n x + h \sin \alpha_n x) = \alpha_n \left\{ \cos \frac{\alpha_n l}{2} + \frac{h}{\alpha_n} \sin \frac{\alpha_n l}{2} \right\}$$

$$= \alpha_n \cos \frac{\alpha_n l}{2} \left(1 + \frac{h}{\alpha_n} \tan \frac{\alpha_n l}{2} \right)$$

$$= \frac{\alpha_n}{\cos \frac{\alpha_n l}{2}} \quad \text{when } n \text{ is odd, and}$$

$$= 0 \quad \text{when } n \text{ is even.}$$

Thus

$$v_{\frac{l}{2}} = 2 v_0 \left\{ e^{-k \alpha_1^2 t} \frac{\frac{\alpha_1}{\cos \frac{\alpha_1 l}{2}}}{(\alpha_1^2 + h^2)l + 2h} \int_0^l (\alpha_1 \cos \alpha_1 x + h \sin \alpha_1 x) dx + \dots \right\}$$

As $k \alpha_n^2 t$ increases with n , if t is chosen great enough, we shall obtain a close approximation by using only the first term in each of these series.

Thus

$$\frac{v_s}{v_{\frac{l}{2}}} = \cos \frac{\alpha_1 l}{2} \dots \dots \dots (2)$$

From this result we find α_1 ; and h follows from the equation

$$\tan \frac{\alpha_1 l}{2} = \frac{h}{\alpha_1} \dots \dots \dots (3)$$

And k may be found from either of the equations

$$e^{-k\alpha_1^2 t} = \frac{v_s}{2v_o} \frac{(\alpha_1^2 + h^2)l + 2h}{\alpha_1 \left\{ \sin \alpha_1 l + \frac{h}{\alpha_1} (1 - \cos \alpha_1 l) \right\}} \dots \dots \dots (4)$$

$$e^{-k\alpha_1^2 t} = \frac{v_l}{2v_o} \frac{\left\{ (\alpha_1^2 + h^2)l + 2h \right\} \cos \frac{\alpha_1 l}{2}}{\alpha_1 \left\{ \sin \alpha_1 l + \frac{h}{\alpha_1} (1 - \cos \alpha_1 l) \right\}} \dots \dots \dots (5)$$

Thus, if the values of v_s and v_l are measured at a certain time t , the thermal constants k and h can be determined by equations (2) to (5).

To apply the experimental data to equations (2) to (5), the following assumptions have been made:

- (1) The diffusivity of fresh concrete is assumed a constant.
- (2) The emissivity of fresh concrete is defined as a constant rate of loss of heat by radiation, convection, and evaporation of water on the surface (that is, the effect of all agents which have relation to the conditions of the surface) per square centimeter of surface under still air conditions per degree difference in temperature between body and air.
- (3) The bottom of the specimen is assumed to be impervious to heat, i. e. there is no transference of heat across the bottom plane of the specimen.
- (4) The cooling in the vertical axis of the specimen is assumed to be due entirely to the cooling at the surface of the specimen.
- (5) The initial temperature of the concrete, i.e. the temperature of the concrete just before the specimen was put into the cold room, is assumed constant throughout the specimen.
- (6) For convenience of calculation, the mean temperature of the cold room until the time considered in the calculation is taken as the constant temperature of the cold room.
- (7) The heat produced during setting clearly retards the cooling of fresh concrete. However, with a comparatively small specimen, in a cold temperature, this effect is not so noticeable.

It is assumed that this effect is included in the constant, diffusivity, of the concrete.

Now let

$-v_2$ = the mean temperature of the cold room until the time t .

v_1 = the initial temperature of the concrete.

Then the difference between the initial temperature of the concrete and the mean temperature of the cold room until the time considered is $v_0 = v_1 + v_2$.

As the cooling of the body is dependent upon the difference of temperature, it may be considered that the concrete of temperature v_0 cools under the air at 0 deg. C.

It has been assumed that there is no flow of heat across the bottom plane of the specimen. If we imagine a wall having the thickness equal to twice the depth of the specimen and the temperature of air on both sides of the wall is zero, then there will be no transference of heat across its middle plane because of the symmetrical condition of the wall, and hence this plane could be made of a material impervious to heat without altering the conditions. Therefore, we may consider that the specimen is one-half of that wall, and if we take the thickness of the wall as twice the depth of the specimen, equations (2) to (5) apply to the specimens of the experiments until the freezing temperature penetrates the surface. Hence the thermal constants k and h may be calculated by these equations.

When the freezing of fresh concrete begins latent heat must be taken into account. The problem is very complicated and it is not considered here.

The results of the calculation of the thermal constants k and h are given in Table 4.

For 1-2-4 concrete the mean values of k and h , calculated for the time when the freezing temperature penetrated the surface in experiments Nos. 5, 10, and 15 are 0.00626 and 0.0436, and the means of the values calculated at different times in the three experiments are 0.00628 and 0.0457 respectively.

For 1-3-6 concrete, the values of k and h calculated for the time when the temperature of the surface of the specimen reached zero are 0.00626 and 0.0428, and the means of the values calculated at different times are 0.00641 and 0.0466 respectively.

As the depth of the specimens used in the calculation is not large and the effect of heat produced during the setting of concrete is

not noticeable, the values 0.0063 and 0.046 seem to be safe average values of k and h respectively, for concrete of 1-2-4 and 1-3-6 mixtures having a consistency that can satisfactorily be used in cold weather. For massive concrete work the heat evolved during the setting of the concrete will retard the cooling considerably, and the values obtained will err on the side of safety.

For 1-1-2 concrete, as the heat produced during the setting of the concrete had a great effect, it was considered absurd to apply equations (2) to (5) for calculation of the thermal constants; but, of course, it is very safe to use the above values for construction in which such a rich mixture is used.

The 1-2-4 concrete of wet consistency shows lower values of the thermal constants than that of medium consistency. This is probably due to the protecting effect of the laitance on the surface in that experiment, and also to the high specific heat of water. It is considered that the values for the concrete of medium consistency will apply very safely to cases of wet consistencies until the freezing of the concrete takes place.

For the concrete of dry consistency the diffusivity is lower than that for concrete of other consistencies. The slightly higher value of h is considered to be due to the lower value of the coefficient of conductivity of this concrete.

For the concrete 31 days old the mean values of the constants k and h , calculated at different times, are 0.0083 and 0.032 respectively. In this experiment the cooling of the specimen through the sawdust was apparent, and the calculated values are considered to be higher than they should be.

12. *Effect of Wood Forms in Protecting Fresh Concrete.*—The wood used in forms is a poor conductor of heat and protects fresh concrete from cooling in cold weather. In addition to this, so-called contact or surface resistance, which is offered to the flow of heat by the discontinuity of the concrete at the forms, also serves to retard the cooling of the fresh concrete.

The coefficient of thermal conductivity of pine wood which is largely used in form work may be taken as about 0.00009 across the grain and 0.0003 with the grain.* In a board the flow of heat is not only across the fibre, but also along annular rings, depending upon

* Ingersoll and Zobel, "Mathematical Theory of Heat Conduction," page 162, 1906.

TABLE 4
VALUES OF THERMAL CONSTANTS h AND k FOR FRESH CONCRETE

Experiment No.	Mixture	Depth of Specimen cm.	Time after Placing in Cold Storage hr. min.	h	k
5	1-2-4	30	12 23	0.0454	0.00611
			23 4	0.0464	0.00653
			*28 18	0.0444	0.00624
			mean	0.0454	0.00628
10	1-2-4	20	10 20	0.0464	0.00684
			11 42	0.0472	0.00653
			*14 10	0.0470	0.00630
			mean	0.0467	0.00656
15	1-2-4	25	12 00	0.0479	0.00620
			20 46	0.0465	0.00596
			24 54	0.0467	0.00559
			*26 00	0.0393	0.00625
			mean	0.0451	0.00600
13	1-2-4 (wet consistency)	30	11 58	0.0429	0.00626
			19 20	0.0420	0.00586
			*24 15	0.0403	0.00570
			mean	0.0417	0.00594
14	1-2-4 (dry consistency)	30	11 30	0.0542	0.00513
			19 46	0.0483	0.00532
			*25 32	0.0466	0.00524
			mean	0.0497	0.00523
4	1-3-6	30	12 7	0.0480	0.00668
			12 12	0.0490	0.00628
			*24 00	0.0428	0.00626
			mean	0.0466	0.00641

* The temperature at the surface became zero at this time.

how the board was cut from the log. It is therefore obvious that the diffusivity of a board is quite variable and that the effect of a board in preventing the cooling of fresh concrete will vary considerably.

The emissivity of the same wood is about 0.0002 under still air conditions and about four times this value when exposed to a wind having a velocity of fifteen miles per hour.*

For these reasons, the more complicated mathematical solution of this case was not developed. However, in order to obtain some idea

*Harding and Willard, "Mechanical Equipment of Buildings," page 645.

of the effect of wood forms on the cooling of fresh concrete, the thickness of the board was assumed as equivalent in effect to a certain thickness of concrete.

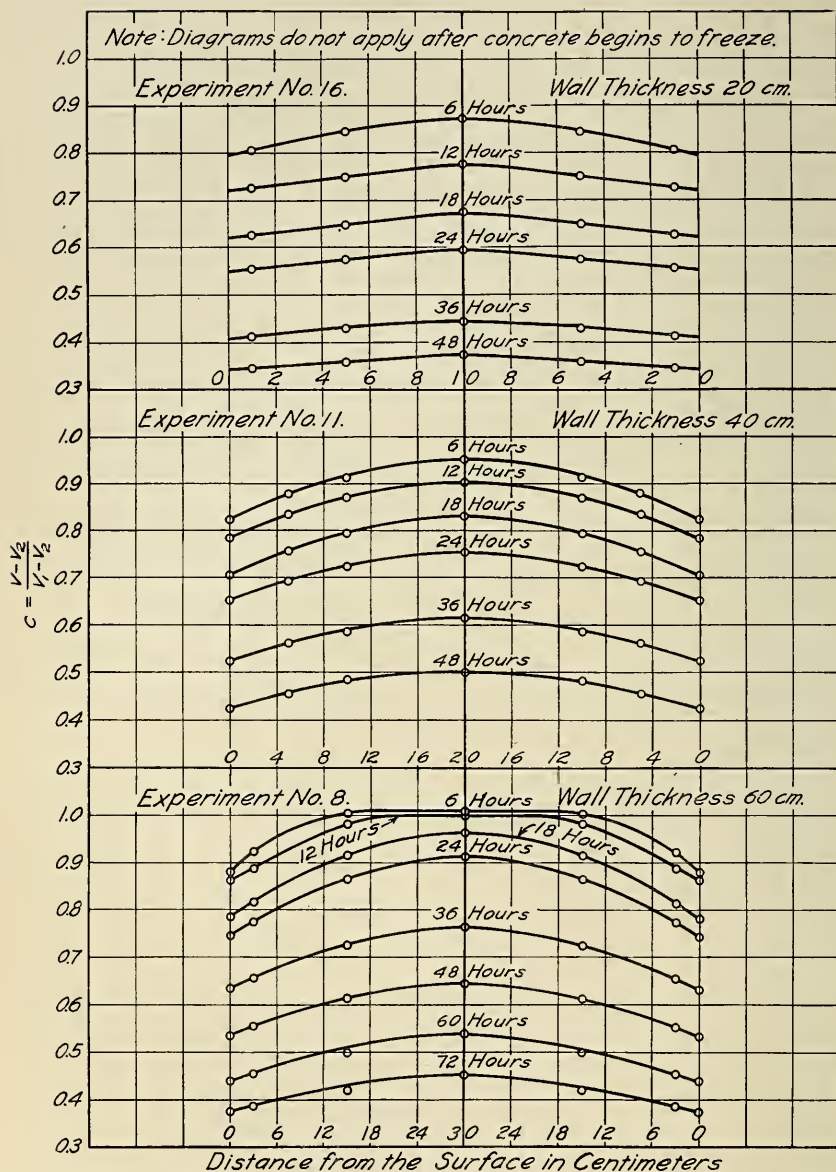
Each specimen in experiments Nos. 8, 11, and 16 may be considered as one-half of a wall having a thickness of twice the depth of the specimen and having both faces of the wall protected by boards 1.9 cm. ($\frac{3}{4}$ in.) in thickness. Curves showing temperatures at different depths from the surface of the walls for various intervals of time, taking the initial temperature difference of air and concrete as unity, are plotted in Fig. 10.

Assuming a value of h , curves of temperatures corresponding to the above figures were calculated from equation (1) and plotted for different thicknesses of wall using the value of diffusivity of concrete as $k = 0.0063$. These curves and the curves of Fig. 10 were compared to find out at what values of h and thickness of wall the former approximated the latter. By this tentative method it was found that when a thickness of 11.25 cm. for the concrete was used in place of 1.9 cm. for the board and h equal to 0.031, the curves coincided very closely for the 48-hour period, but not so well for the 12-hour period.

On the whole, considering the entire range of the experiments, a thickness of concrete five times the thickness of a wood board may be considered equivalent in effect to the board, using $h = 0.031$ for the latter.

13. *Effect of Canvas in Protecting Fresh Concrete.*—The mathematical explanation of this effect is more difficult than the above case, and from the results of experiments Nos. 9 and 12 no general conclusion could be made. However, the curves shown in Fig. 11, which are based on the results of the experiments, are believed to give some information on the protection afforded by the use of canvas.

14. *Effect of Amount of Water Used.*—The curves of temperature in Fig. 12 which are based on the results of experiments Nos. 14, 5, and 13 for very dry, medium, and very wet consistencies of concrete, respectively, show that the more water used in mixing the concrete the slower the cooling is until the freezing of the fresh concrete begins. However, when the strength of concrete and the condition of the concrete after freezing are taken into account, it will be far better and safer in mixing concrete to use the least amount of water that will produce a workable mixture.

FIG. 10. DIAGRAMS FOR COOLING OF WALLS PROTECTED BY $\frac{3}{4}$ -IN. WOOD FORMS

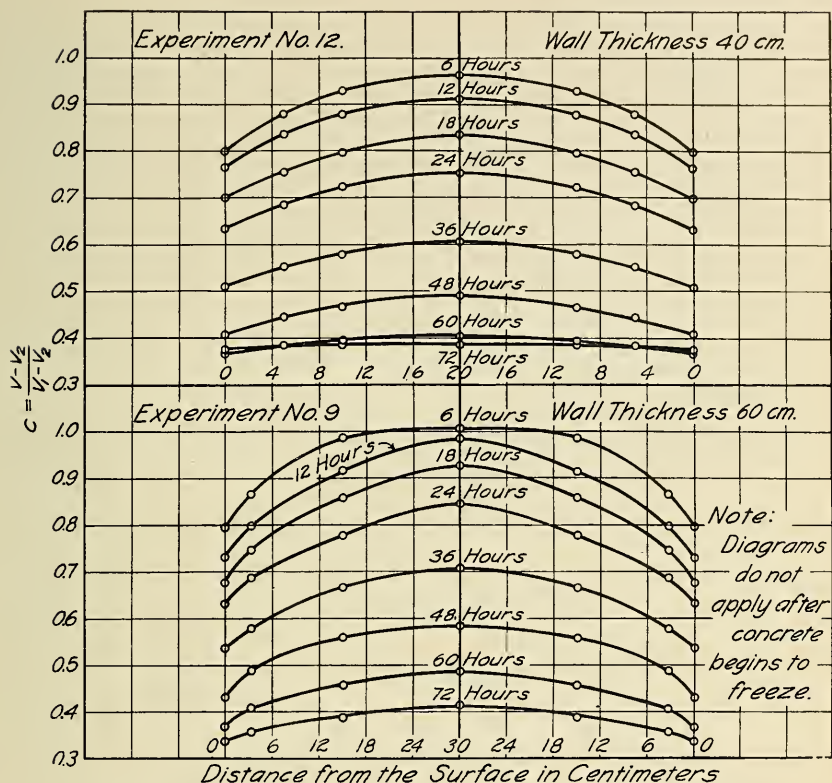


FIG. 11. DIAGRAMS FOR COOLING OF WALLS PROTECTED BY CANVAS

15. *Summary of Experimental Results.*—It is believed that the following general conclusions are instructive:

(1) For the application of the mathematical theory of heat conduction to the cooling of fresh concrete, the thermal constants, diffusivity and the ratio of the emissivity to the coefficient of conductivity, for commonly used concrete mixtures having the wettest consistency which can satisfactorily be used in cold weather and a still air condition, may be taken, in C. G. S. units, as 0.0063 and 0.046 respectively. These figures are especially safe for massive concrete work and for rich concretes.

(2) Under favorable temperature conditions in the concrete, the rise in temperature during its setting is greatest in the period between six and twelve hours after the time of mixing.

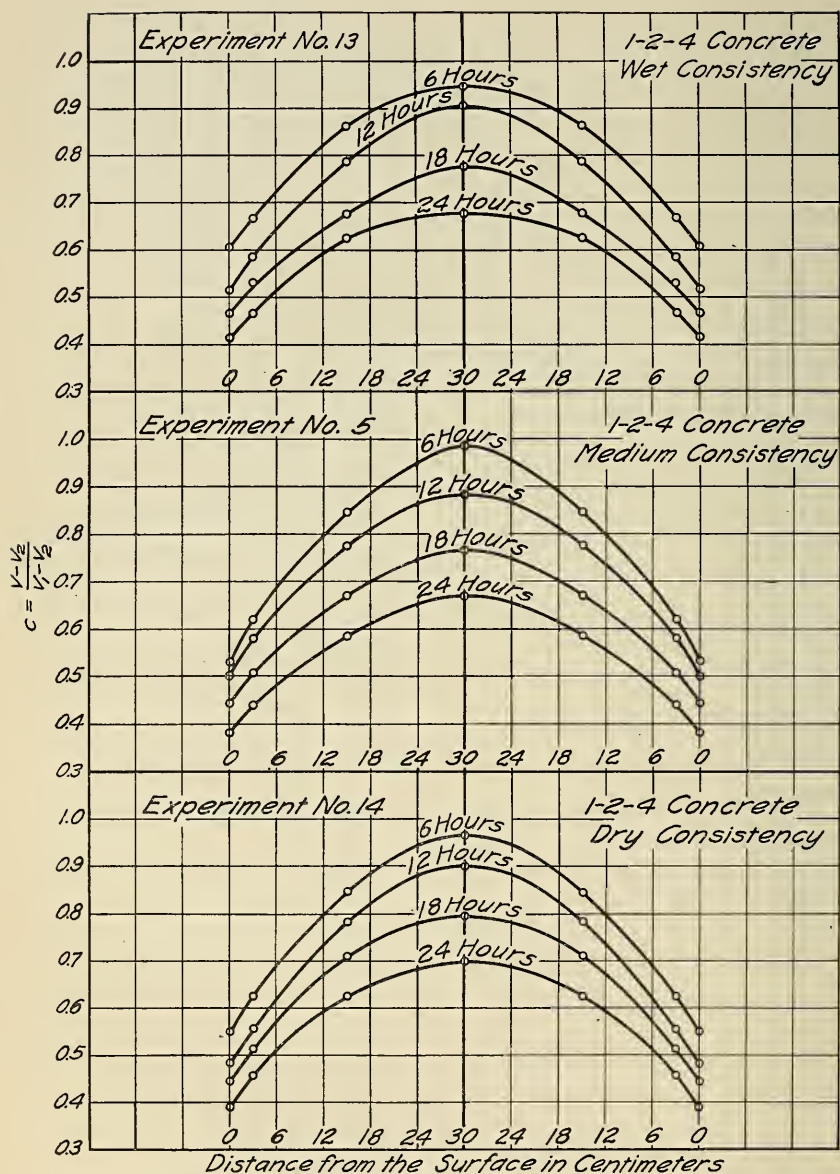


FIG. 12. DIAGRAMS FOR COOLING OF WALLS MADE OF CONCRETE OF DIFFERENT CONSISTENCIES

It is important that as much strength as possible be acquired at an early age, if the concrete is later to be subjected to low temperatures. It is, therefore, important and necessary in cold weather to protect concrete very carefully for at least twelve hours after pouring, because during that time the chemical action of cement and water is greatest, the increase in strength is very marked, the heat produced helps to retard the later cooling of the concrete, and protection during this period makes the concrete better able to resist the effect of low temperature.

(3) It is apparent from the analysis that very much more care must be taken to prevent the freezing of the concrete in relatively light structures than in massive work.

(4) Wind has very great effect on the cooling of fresh concrete. Every precaution should be taken to protect the surface from the wind.

III. ANALYTICAL APPLICATIONS

16. *Massive Concrete with One Surface Exposed.*—Consider a mass of concrete of indefinite lateral extent and having one surface exposed to the air. Consider the temperature of the ground to be the same as that of the concrete when poured. For simplicity it is assumed that the soil has the same diffusivity as the fresh concrete, as would be approximately true in ordinary cases. The difference between the initial temperature of the concrete in place and the temperature of the air during the time considered (assumed to remain constant) will be called v_0 , and the difference between the temperature of the air and that of any point in the concrete at any time t after pouring will be called v , the centigrade scale being used. Take the origin of distances at the surface of the concrete, and the positive x direction as running into the concrete.

Then the cooling is due to the radiation, convection, and evaporation of water at the surface (the constants determined from experiments taking care of evaporation). The equations for the temperature are:

$$\begin{aligned} \frac{\partial v}{\partial t} &= h \frac{\partial^2 v}{\partial x^2} & 0 < x < \infty \\ -\frac{\partial v}{\partial x} + hv &= 0 & \text{at } x = 0 \\ v &= v_0 & \text{for } t = 0 \end{aligned}$$

It is assumed that the temperature at infinity is constant and always equal to v_0 , so that

$$v = v_0 \quad \text{for } t = 0$$

The solution* of these equations is

$$\begin{aligned} v &= \frac{2v_0}{\sqrt{\pi}} \int_0^{\frac{x}{2\sqrt{kt}}} e^{-u^2} du + \frac{2v_0}{\sqrt{\pi}} e^{hx + h^2 kt} \left(\int_0^{\infty} e^{-u^2} du - \int_0^{\frac{x+2hkt}{2\sqrt{kt}}} e^{-u^2} du \right) \\ \text{or } \frac{v}{v_0} &= \theta \left(\frac{x}{2\sqrt{kt}} \right) + e^{hx + h^2 kt} \left\{ 1 - \theta \left(\frac{x}{2\sqrt{kt}} + h\sqrt{kt} \right) \right\} \dots \dots (6) \end{aligned}$$

* Carslaw, "Fourier's Series and Integrals," page 245, 1906.

$$\underline{hx + h^2 kt}$$

where,

$$\theta(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-u^2} du.$$

In the equation

v = temperature in degrees centigrade,

x = depth below the surface in cm.,

t = time in seconds,

using the values $k = 0.0063$ and $h = 0.046$ which have been determined from the experiments, equation (6) becomes

$$\frac{v}{v_0} = \theta\left(\frac{x}{0.1588\sqrt{t}}\right) + e^{0.046x + 0.0001333t} \left\{ 1 - \theta\left(\frac{x}{0.1588\sqrt{t}} + 0.00365\sqrt{t}\right) \right\} \quad (7)$$

From this equation, the temperature at any depth below the surface at any time after pouring the concrete may be obtained.

The temperature at the surface at a certain time after the concrete has been poured may be written by equation (6) as follows,

$$v = cv_0 \dots \dots \dots (8)$$

where

$$c = e^{0.048t} \left\{ 1 - \theta(0.219\sqrt{t}) \right\}$$

t being measured in hours. The numerical value of c is less than unity.

It should be noted that for the constants used the equations given will hold good only until the concrete begins to freeze.

When the surface is protected with a board, it is convenient to assume a thickness of concrete equivalent in effect to the given thickness of board in the manner described in Article 12, using $h = 0.0031$ and $k = 0.0063$. Then equation (6) will apply to this case, the temperature at the depth of the equivalent thickness of concrete giving the temperature at the surface of the concrete against the board.

17. *Thin Wall or Slab.*—In a thin wall or slab having two surfaces exposed, cooling takes place from both sides. Call v_0 the difference in temperature between the initial temperature of the concrete and the assumed temperature of the air, and v the difference in temperature between the air and any point in the concrete at a time t , the point being distant x from one of the surfaces.

Then equation (1)

$$\frac{v}{v_o} = 2 \sum_{n=1}^{n=\infty} e^{-k \alpha_n^2 t} \frac{\alpha_n \cos \alpha_n x + h \sin \alpha_n x}{(\alpha_n^2 + h^2) l + 2h} \int_0^l (\alpha_n \cos \alpha_n x + h \sin \alpha_n x) dx$$

gives the temperature in the concrete in this case. The equation may be written as

$$v = cv_o$$

where c denotes the right hand member of equation (1).

In this equation

l = the thickness of the concrete in cm.

x = the distance from one of the surfaces to the point considered in cm.

t = time in seconds after pouring the concrete.

α_n = the n th positive root of the equation

$$\tan \alpha l = \frac{2 \alpha h}{\alpha^2 - h^2}$$

$$h = 0.046$$

$$k = 0.0063$$

When there are wood forms on one or both sides of the wall or slab, a thickness of concrete equivalent in effect to the given thickness of the board must be added to the thickness of concrete in the manner described in Article 12. To apply the equation, use the values $h = 0.031$ and $k = 0.0063$ for the case of forms on both sides, and $h = 0.046$ and $k = 0.0063$ for the case with form on one side, to calculate the temperature of the unprotected side.

18. *Beam or Column of Rectangular Cross-section.*—If a thickness of concrete equivalent in effect to the given thickness of board is assumed as before, the following equation of flow of heat in a rectangular parallelepiped will give the temperature in a beam or column of rectangular cross-section with wood forms, at any time after pouring the concrete until the freezing of concrete begins. Let the initial temperature of a fresh concrete column or beam (a by b in cross section, c in length) be v_o above the temperature of the air, cooling taking place at the faces by radiation and convection into the air.

Then the following equations must be satisfied:

$$\frac{\partial v}{\partial t} = k \left\{ \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2} \right\} \quad \text{in the solid;}$$

$$-\frac{\partial v}{\partial x} + h v = 0 \quad \text{at } x = 0, y = 0, z = 0;$$

$$\frac{\partial v}{\partial x} + h v = 0 \quad \text{at } x = a, y = b, z = c;$$

$$v = v_0 \quad \text{at } t = 0.$$

The solution* of these equations is

$$\frac{v}{v_0} = 4^3 h^3 \sum_{p=0}^{\infty} \sum_{q=0}^{\infty} \sum_{r=0}^{\infty} \frac{X_{2p+1}}{a(\alpha_{2p+1}^2 + h^2) + 2h} \cdot \frac{Y_{2q+1}}{b(\beta_{2q+1}^2 + h^2) + 2h} \cdot \frac{Z_{2r+1}}{c(\gamma_{2r+1}^2 + h^2) + 2h} \cdot e^{-k(\alpha_{2p+1}^2 + \beta_{2q+1}^2 + \gamma_{2r+1}^2)t} \dots \dots \dots (9)$$

$$\text{or } v = c v_0$$

where

$$X = \cos \alpha x + \frac{h}{\alpha} \sin \alpha x$$

$$Y = \cos \beta y + \frac{h}{\beta} \sin \beta y$$

$$Z = \cos \gamma z + \frac{h}{\gamma} \sin \gamma z$$

and α, β, γ are roots of the equations

$$\tan \alpha a = \frac{2 \alpha h}{\alpha^2 - h^2}$$

$$\tan \beta b = \frac{2 \beta h}{\beta^2 - h^2}$$

$$\tan \gamma c = \frac{2 \gamma h}{\gamma^2 - h^2}$$

Use C. G. S. units and $h = 0.031$ and $k = 0.0063$

19. *Column of Circular Cross-section.*—The temperature of the fresh concrete column of circular cross-section is v_0 above the temperature of the surrounding air. Call l the radius of the column. It is assumed that both ends of the column are well protected and that cooling takes place through the convex surface of the column, so that the lines of flow of heat are radial in planes perpendicular to the axis of the column.

* Carslaw, "Fourier's Series and Integrals," page 320, 1906.

Then the equations of temperature expressed in cylindrical co-ordinates, taking the center as origin, are

$$\frac{\partial v}{\partial t} = k \left(\frac{\partial^2 v}{\partial r^2} + \frac{1}{r} \frac{\partial v}{\partial r} \right), \quad 0 < r < l$$

$$\frac{\partial v}{\partial r} + h v = 0 \quad \text{at } r = l,$$

$$v = v_o \quad \text{for } t = 0.$$

The solution* of these equations is

$$\frac{v}{v_o} = \frac{2h}{l} \sum_1^{\infty} e^{-k\alpha_n^2 t} \cdot \frac{J_o(\alpha_n r)}{(\alpha_n^2 + h^2) J_o(\alpha_n l)} \quad \dots\dots\dots (10)$$

$$\text{or } v = c v_o$$

in which $\alpha_1, \alpha_2, \dots\dots\dots$ are the roots of the equation

$$\alpha J_o'(\alpha l) - h J_o(\alpha l) = 0$$

and J_o is the Bessel's function of the first kind of zero order. The values of the thermal constants h and k may be taken as $h = 0.046$ and $k = 0.0063$, C. G. S. units being used.

When there are wood forms, a thickness of concrete equivalent in effect to the given thickness of the board must be added to the thickness of concrete in the manner described in Article 12 to apply equation (10). Call l the total radius of the column including the additional thickness equivalent to the forms, and use the values $h = 0.031$ and $k = 0.0063$. Then equation (10) will give the temperature in the column under still air conditions at any point at any time until the freezing of the concrete begins.

* Carslaw, "Fourier's Series and Integrals," Page 313, 1906.

IV. SOME APPLICATION OF EXPERIMENTAL DATA AND ANALYSIS TO WORK OF CONCRETING IN FREEZING WEATHER

20. *General Considerations.*—Heat hastens the setting and hardening of concrete; cold delays it. The effect of cold becomes noticeable in this respect when the temperature falls below 50 deg. F. (10 deg. C.), and becomes more marked with lower temperatures. At temperatures below the freezing point of water the fresh concrete will freeze.

If the temperature of the concrete when placed is low, setting will be delayed, and, if subjected to further cooling by low air temperature, the concrete may freeze before it has attained sufficient strength to withstand the stresses to which it may be subjected, or even before it has taken its set. Serious objection should be made to allowing concrete to freeze before it has set or even before it has hardened very long. The general opinion is that a freezing temperature will not injure concrete that has first had an opportunity to harden for at least 48 hours under favorable conditions, though repeated freezing and thawing at an early age may be expected to cause injury. It is better and safer to protect concrete until freezing temperatures will have no injurious effect on it than to expose it to freezing at too early an age. Accordingly, the exposure of fresh concrete to freezing is not considered here, and the discussions that follow apply only until such a time as the freezing of fresh concrete would begin.

To predict the temperature of the concrete after a certain time by the application of the theory of heat conduction, it is necessary to forecast atmospheric temperature for some time after the concrete has been deposited. The forecast of the weather bureau will generally be found of great assistance.

In general, concrete work in winter is more difficult and somewhat more expensive than in summer; but policy frequently makes work in cold weather necessary and even economical. However, in order to attain satisfactory results in this case, proper precautions have to be observed.

It is the purpose of this chapter to describe some applications of the experimental data to concreting in freezing weather. It is not considered that the solutions given are very exact, but it is hoped to provide some information concerning the precautions to be observed.

The general effect of heating the materials, the presence of the forms and other protective covering, and the use of chemicals to lower the freezing point will be considered.

21. *Concrete with One Surface Exposed.*—The larger concrete constructions such as foundations, abutments, and retaining walls fairly well protected on the sides, may be considered to have the cooling take place by radiation and convection over one exposed surface, and the equations given in Article 16, "Massive Concrete with One Surface Exposed," may be considered to apply. The cooling from the sides of the structure will be small as compared with that from the exposed surface.

From equation (7) the diagrams in Figs. 13 and 14 have been prepared; v_1 is the initial temperature of the concrete; v_2 is the assumed air temperature to which the exposed surface will be subjected; v is the temperature attained by the concrete at a given distance from the surface at the given time after the concrete was poured. As $\frac{v-v_2}{v_1-v_2} = c$ is a ratio, any thermometric scale may be used, and the diagrams are, therefore, applicable to the Fahrenheit scale. It should be borne in mind that for the constants used, the diagrams do not apply after the concrete begins to freeze.

As an example, assume the temperature of the finished concrete to be 60 deg. F., and that of the air the following night 10 deg. F. From Fig. 13 or Fig. 14 the ratio c for temperature at the surface after 12 hours is

$$c = \frac{v-v_2}{v_1-v_2} = \frac{v-10}{60-10} = 0.5,$$

$$\therefore v = 35 \text{ deg. F.}$$

After 12 hours the temperature 4 in. below the surface (from Fig. 13 or Fig. 14, ratio $c = 0.705$) will be

$$v = v_2 + 0.705 (v_1 - v_2) = 45 \text{ deg. F.}$$

To determine the time when the freezing point will be reached at the surface of the concrete for the temperatures just used, the ratio

$$\frac{v-v_2}{v_1-v_2} = \frac{32-10}{60-10} = 0.44, \text{ and}$$

from Fig. 13 the time corresponding to this ratio is 18 hours.

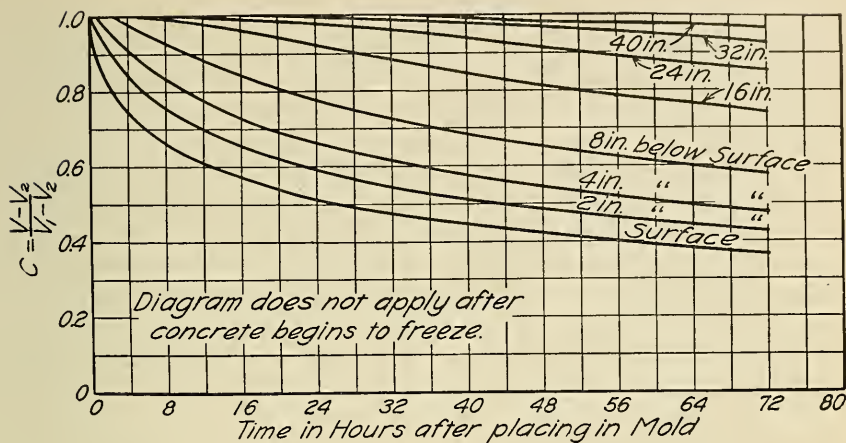


FIG. 15. DIAGRAMS FOR COOLING OF MASSIVE FOUNDATIONS *Protected by Wood Forms*

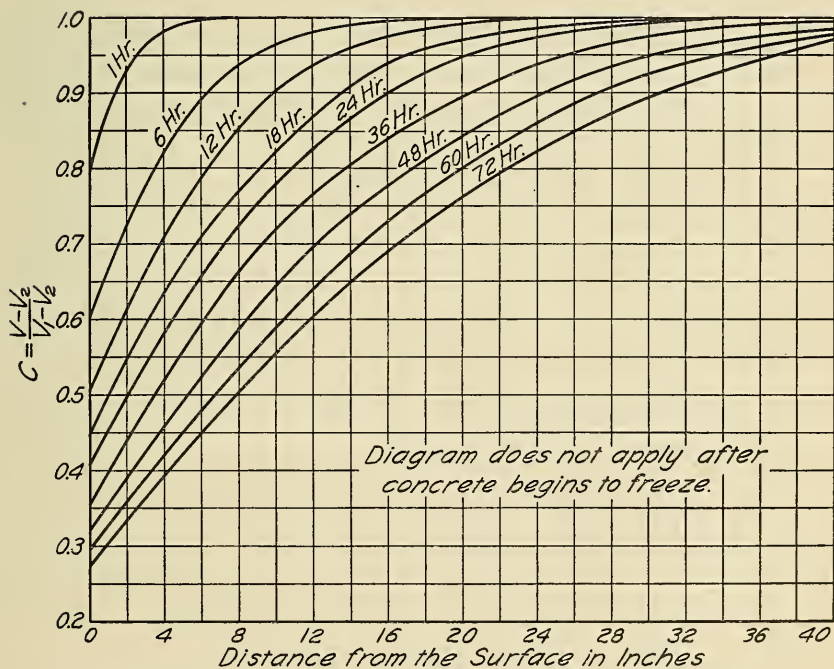


FIG. 14. DIAGRAMS FOR COOLING OF MASSIVE FOUNDATIONS

Attention is again called to the fact that the cooling is assumed to have taken place under still air conditions. The experiments showed that exposing the surface to wind produced a much more rapid cooling of the concrete at its surface, though the effects did not penetrate far below the surface.

When the surface of the concrete is protected by a covering, as for example a wooden form, the radiation and conduction at the surface is modified and a different thermal constant must be used. Figs. 15 and 16 give curves of cooling for a surface covered with boards, calculated by the method described in Article 16. To use these diagrams, consider that the board covering is equivalent in effect to an additional thickness of concrete, which may for the purpose be taken as five times the thickness of the board, and take from the diagram the ratio $\frac{v-v_2}{v_1-v_2}$ which corresponds to the total distance to the point in question, including in this distance the equivalent thickness due to the thickness of the form.

As an example assume the thickness of the board to be 1 in. This will be considered equivalent to adding 5 in. of concrete. Take v_1 as 60 deg. F. and v_2 as 10 deg. F. From Figs. 15 and 16 the ratio after 72 hours at a depth of 5 in., corresponding to the actual surface of the concrete, is 0.5. Then

$$v = 10 + 0.5 (60 - 10) = 35 \text{ deg. F.}$$

which is the expected temperature at the surface of the concrete after 72 hours. Similarly, a 2-in. board covering will show a temperature at the surface of the concrete after 72 hours of 41 deg. F. The foregoing examples assume that the thickness of the concrete is not small; little error will be found if the thickness is at least 6 in.

A covering of boards placed over the surface of concrete, such as a floor or pavement, will give much the same effect as the forms, if the covering is tight enough to prevent the circulation of air and wind.

As the experiments with a protection of canvas 3 in. from the surface of the concrete showed about the same effect as that found with a covering of $\frac{3}{4}$ -in. boards, Figs. 15 and 16 may also be expected to give values of the ratio approximating the conditions of the canvas cover.

For a covering of earth Figs. 13 and 14 may be expected to approximate the cooling conditions, the effect of a given thickness of

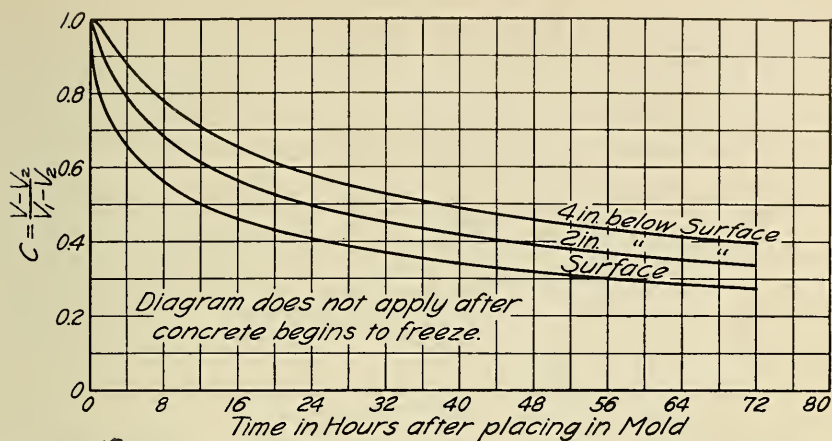


FIG. 15. DIAGRAMS FOR COOLING OF MASSIVE FOUNDATIONS PROTECTED BY WOOD FORMS

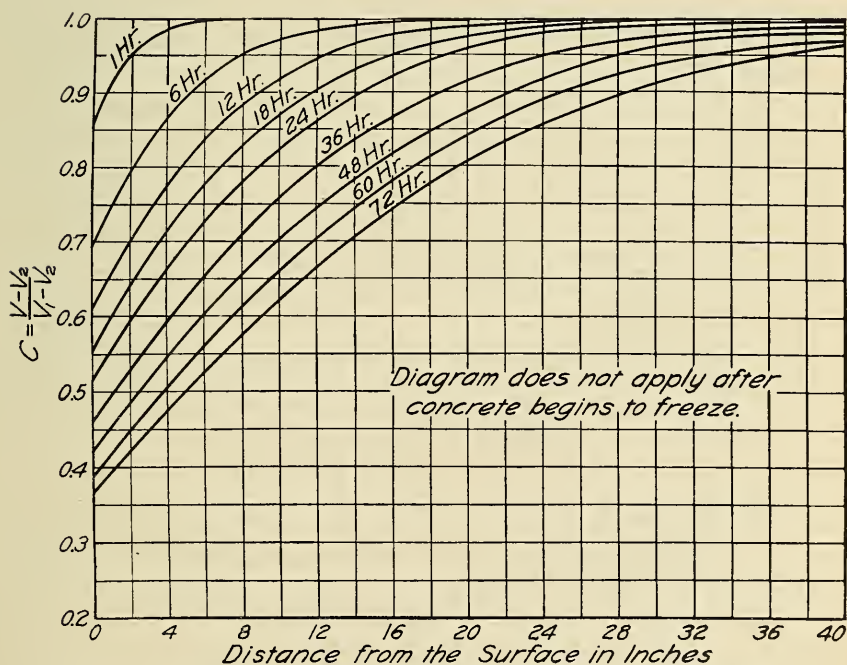


FIG. 16. DIAGRAMS FOR COOLING OF MASSIVE FOUNDATIONS PROTECTED BY WOOD FORMS

earth being taken as the same as that of an equal thickness of concrete. The temperature of the surface of the concrete will then be determined by finding a value for a distance below the surface equal to the thickness of the earth cover. This really assumes that one inch of boards gives the same protection as five inches of earth.

22. *Thin Wall or Slab.*—The concrete of temperature v_1 is deposited into the wood forms of a wall. The average temperature of the air from the time of pouring the concrete to a certain time is expected to be v_2 . It is required to predict the temperature in the wall at that time, assuming that both the top and bottom of the wall are well protected from cooling.

If the thickness of the boards of the form is $\frac{3}{4}$ in., the curves of temperatures shown in Fig. 10, which embody the results of the experiments, are directly applicable for walls having the thicknesses of 24 in., 16 in., and 8 in. respectively. As an example, assume $v_1 = 60$ deg. F., $v_2 = 10$ deg. F., and thickness of the wall = 16 in.

From Fig. 10 the ratio $c = \frac{v - v_2}{v_1 - v_2}$, for temperature at the surface of the concrete 36 hours after pouring, is

$$c = \frac{v - v_2}{v_1 - v_2} = \frac{v - 10}{60 - 10} = 0.525$$

$$\therefore v = 36 \text{ deg. F.}$$

After 36 hours the temperature at the center of the wall (from Fig. 10, $c = 0.615$) will be

$$v = 10 + 0.615 (60 - 10) = 41 \text{ deg. F.}$$

To determine the time when the freezing point will be reached at the surface of the concrete for the temperature just used, the ratio

$$c = \frac{v - v_2}{v_1 - v_2} = \frac{32 - 10}{60 - 10} = 0.44, \text{ and}$$

from Fig. 10 the time corresponding to this ratio is about 46 hours.

To obtain information as to the rate of cooling for other thicknesses of walls and other thicknesses of forms, the method described in Article 12 may be used. The temperatures at different times in walls having thicknesses of 16 in., 20 in., 24 in., and 32 in., respectively, have been calculated in the manner described in Article 17 and plotted in Figs. 17 and 18. To use these diagrams, consider that

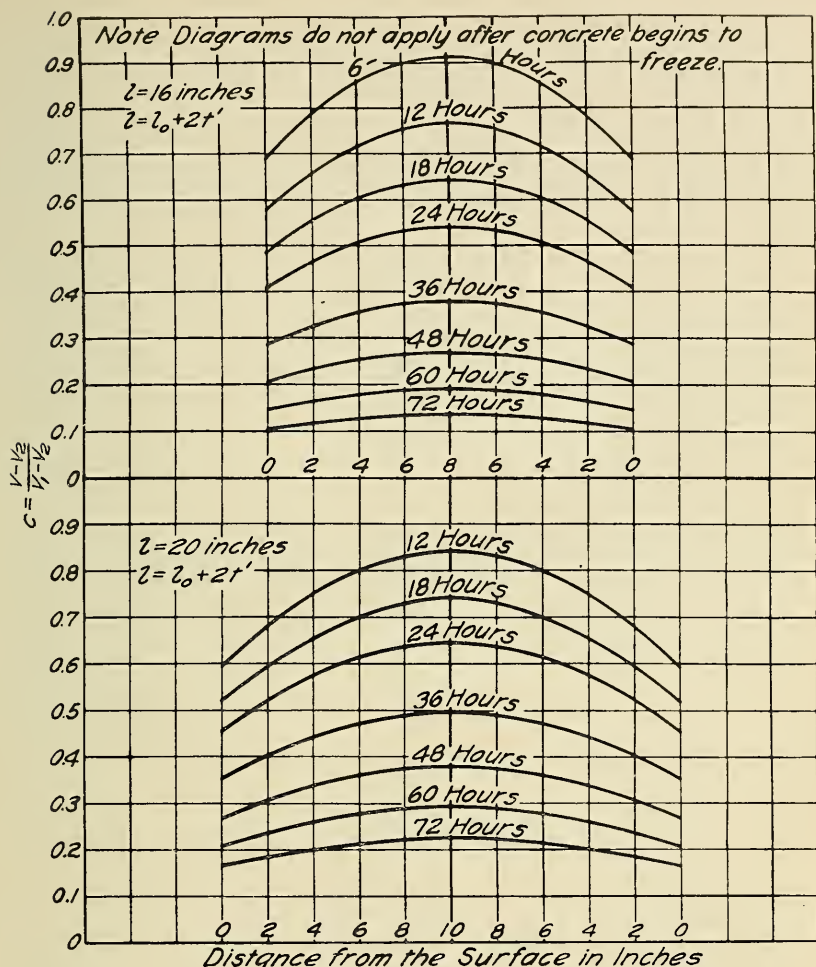


FIG. 17. DIAGRAMS FOR COOLING OF WALLS PROTECTED BY WOOD FORMS

the boards of the forms are equivalent in effect to an additional thickness of concrete, which may for the purpose be taken as five times the thickness of the board, and calculate the total thickness of the wall. From the diagram corresponding to the total thickness, take the ratio $\frac{v - v_2}{v_1 - v_2}$ which corresponds to the total distance to the point in question, including in this distance the equivalent thickness of concrete corresponding to the thickness of the form.

If this total thickness lies between the values shown on the diagrams the method of interpolation may be used.

As an example, assume $v_1 = 60$ deg. F., $v_2 = 10$ deg. F., $l_0 =$ the thickness of the wall = 14 in., and $t' =$ the additional thickness of concrete having the same effect as the given thickness of form. With a 1-in. board t' will be taken as 5 in., which must be added on each side of the wall.

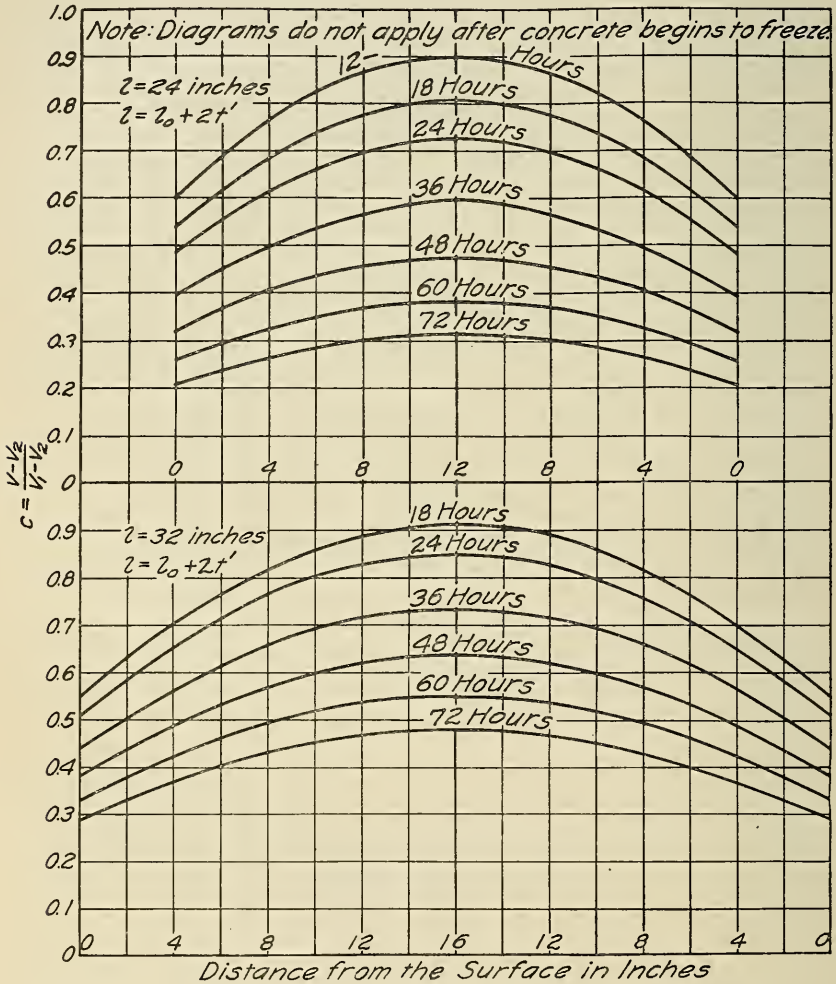


FIG. 18. DIAGRAMS FOR COOLING OF WALLS PROTECTED BY WOOD FORMS

Then the total thickness of the wall is

$$l = l_0 + 2t' = 14 + 10 = 24 \text{ in.}$$

From Fig. 18 the ratio $\frac{v-v_2}{v_1-v_2}$ after 36 hours, at a depth of 5 in., corresponding to the actual surface of the concrete, is 0.52. Then $v = 10 + 0.52(60 - 10) = 36 \text{ deg. F.}$ Similarly, the time at which the freezing temperature will reach the concrete is estimated to be 45 hours.

23. *Column of Circular Cross-section.*—Consider that concrete of temperature v_1 is deposited into wood forms of circular cross-section, the temperature of the surrounding air being v_2 . Assuming the equivalent thickness of concrete for the board of the form, let the total radius of the equivalent column be

$$l = r_0 + t'$$

where r_0 is the radius of the column and t' is the additional thickness of concrete having the same effect as the given thickness of form, which may for the purpose be taken as five times the thickness of the board. If both ends of the column are well protected from cooling, equation (10) applies to this case.

For the values of radius, l , of 10 in., 14 in., and 20 in., respectively, the ratio $c = \frac{v-v_2}{v_1-v_2}$ has been calculated by equation (10) in the manner described in Article 19 and plotted in Fig. 19.

The use of this figure is similar to that of Figs. 17 and 18.

24. *Heating the Materials.*—A good method to use in concreting in freezing weather is to heat the materials. Frozen aggregate is frequently considered to be the cause of failure of concrete laid in freezing weather. The water, the sand and the water, or the entire aggregate and the water should be heated, as called for by circumstances.

It is clear that heating the materials lengthens the time before the mixture becomes cold enough to freeze. In order that the freezing temperature may not penetrate into the surface of fresh concrete it is necessary that, using Fahrenheit units,

$$v_1 > \frac{32}{c} - \frac{(1-c)}{c} v_2 \dots \dots \dots (11)$$

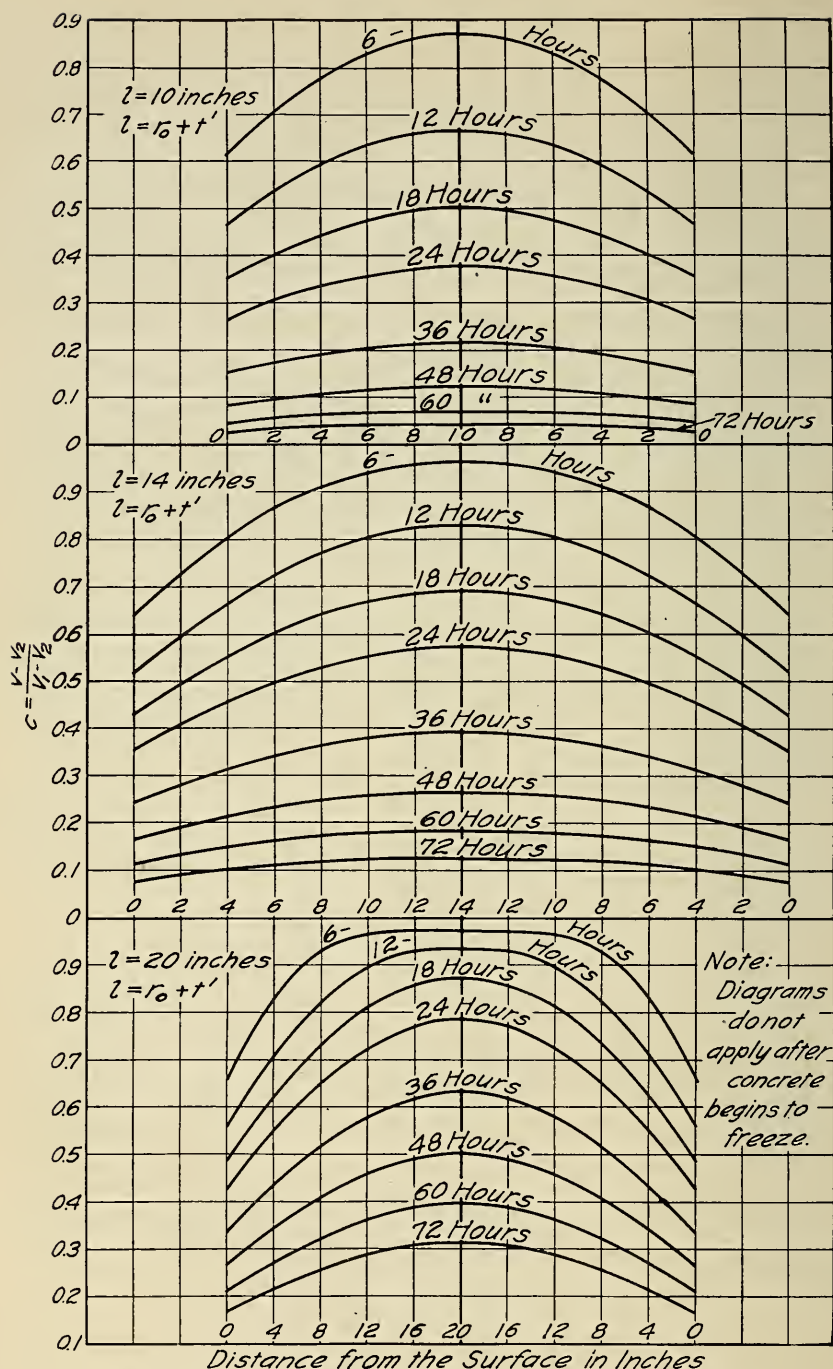


FIG. 19. DIAGRAMS FOR COOLING OF CIRCULAR COLUMNS PROTECTED BY WOOD FORMS

where c is the ratio $\frac{v - v_2}{v_1 - v_2}$ at the surface of the concrete at a given time after pouring.

This expression gives the temperature at which the concrete must be finished to prevent its freezing after it has been exposed to the low temperature for the given length of time. The values of c for foundations, walls, etc., may be obtained from the diagrams for the several cases already explained.

As an example, in order that the freezing temperature may not penetrate below the surface of the concrete foundation, used as an example in Article 21, after 48 hours with the temperature of the air 10 deg. F., the temperature at which the concrete must be placed is found from Fig. 13 and formula (11) to be 79 deg. F.

If, in the same example, the fresh concrete is deposited at 100 deg. F. instead of at 60 deg. F., the temperature used in the example in Article 21, the time required for the surface of the concrete to cool to 32 deg. F. will be found by extrapolation to be about 80 hours instead of 18 hours.

There is a practical limit to the value of v_1 , as excessive heating may accelerate too much the setting of the cement. The safe maximum temperature of the fresh concrete may be taken as about 100 deg. to 120 deg. F. Putting these values in formula (11), it becomes

$$v_2 > \left(\frac{32 - c(100 \text{ to } 120)}{(1 - c)} \right) \dots\dots\dots (12)$$

For a given length of time after pouring, the lowest expected temperature, when using this method of heating the materials, can be obtained from the above expression.

It is to be noted here, that, as heating the materials accelerates the rate of hardening, this also helps to insure the setting of the concrete before it can be damaged by frost.

In using this method it should be kept in mind that every effort must be taken to keep the fresh concrete at a temperature not below the specified one. The rapidity with which the work can be done after the heating is stopped, needs much consideration. As a practical precaution special care must be taken to remove all frozen lumps of aggregate and all ice and snow crystals.

25. *Lowering the Freezing Point of the Mixing Water by Using Chemicals.*—One way of preventing the freezing of fresh concrete in cold weather is to lower the freezing point of the water in the con-

crete by employing chemicals, usually common salt, or sodium chloride, and to a lesser extent calcium chloride.

Though this method is effective only for temperatures a little below freezing, and should not be used where there is danger of electrolysis, nor in concrete reinforced with steel, it is sometimes used because of the simplicity of the process.

Approximately, by Professor Tetmajer's rule reduced to Fahrenheit units, each per cent by weight of salt added to the water reduces the freezing point by one degree Fahrenheit, no more than 10 per cent by weight being considered safe practice.

Formula (11) is applicable to this case. Using 10 per cent salt brine, the relation between v_1 and v_2 is

$$v_1 > \frac{22}{c} - \frac{(1-c)}{c} v_2 \dots \dots \dots (13)$$

As may be seen from the curves of cooling for the several cases, in ordinary cold weather the freezing temperature will not penetrate many inches into the concrete in two or three days after pouring. Consequently, it is not necessary to use brine with the whole mass of concrete. Only the surface layer of the concrete need be made with brine.

If the temperature of the air is much below the freezing point of the mixture, equations (11) and (13) show that the advantage of the chemical is small. In fact, protection by some form of covering is generally much to be preferred, especially as the rate of hardening is reduced both by the presence of the salt and the lower temperature attained.

26. *General Conclusions.*—It is believed that the constants derived from the tests are applicable to the ordinary conditions of fresh concrete. The value of k , the diffusivity, is 0.0063; that of h , the ratio of the emissivity to the coefficient of conductivity, is 0.046.

From the cooling curves given for massive concrete, thin walls and slabs, and beams and columns, the time of cooling and the temperature attained by the concrete in any particular case may be estimated.

The protection afforded by board forms and by canvas is considerable, as was shown by the examples given.

If a canvas protection is used, care must be taken to prevent any circulation of cold air, and it is best to observe the temperature in the

air space by means of a thermometer. Knowing this temperature, the time when the freezing temperature will penetrate into the concrete may be estimated from the diagrams and the necessity of using artificial heat determined.

The heating of the materials is seen to furnish an excellent method of ensuring early hardening and delaying the fall in temperature. It is much to be preferred to the use of chemicals.

The effect of the wind should also be given consideration.

The diagrams are of some value in connection with the determination of the time for the removal of the forms, since they will enable the temperature attained by the concrete to be estimated. As the concrete gains strength much more slowly at a low temperature, and as frozen concrete is especially dangerous, information on the temperature attained is of value and the too early removal of forms may be avoided. If by this or other means the mean temperature of the concrete for a given time is known, the comparative strength of the concrete may be estimated from the diagrams given by Professor A. B. McDaniel in Bulletin No. 81 of the Engineering Experiment Station of the University of Illinois, which shows the influence of temperature on the strength of concrete, and thus the judgment may be aided in deciding on the safe time to remove the forms.

LIST OF
PUBLICATIONS OF THE ENGINEERING EXPERIMENT STATION

Bulletin No. 1. Tests of Reinforced Concrete Beams, by Arthur N. Talbot 1904. *None available.*

Circular No. 1. High-Speed Tool Steels, by L. P. Breckenridge. 1905. *None available.*

Bulletin No. 2. Tests of High-Speed Tool Steels on Cast Iron, by L. P. Breckenridge and Henry B. Dirks. 1905. *None available.*

Circular No. 2. Drainage of Earth Roads, by Ira O. Baker. 1906. *None available.*

Circular No. 3. Fuel Tests with Illinois Coal (Compiled from tests made by the Technological Branch of the U. S. G. S., at the St. Louis, Mo., Fuel Testing Plant, 1904-1907), by L. P. Breckenridge and Paul Diserens. 1908. *Thirty cents.*

Bulletin No. 3. The Engineering Experiment Station of the University of Illinois, by L. P. Breckenridge. 1906. *None available.*

Bulletin No. 4. Tests of Reinforced Concrete Beams, Series of 1905, by Arthur N. Talbot. 1906. *Forty-five cents.*

Bulletin No. 5. Resistance of Tubes to Collapse, by Albert P. Carman and M. L. Carr. 1906. *None available.*

Bulletin No. 6. Holding Power of Railroad Spikes, by Roy I. Webber. 1906 *None available.*

Bulletin No. 7. Fuel Tests with Illinois Coals, by L. P. Breckenridge, S. W. Parr, and Henry B. Dirks. 1906. *None available.*

Bulletin No. 8. Tests of Concrete: I, Shear; II, Bond, by Arthur N. Talbot. 1906. *None available.*

Bulletin No. 9. An Extension of the Dewey Decimal System of Classification Applied to the Engineering Industries, by L. P. Breckenridge and G. A. Goodenough. 1906. Revised Edition, 1912. *Fifty cents.*

Bulletin No. 10. Tests of Concrete and Reinforced Concrete Columns, Series of 1906, by Arthur N. Talbot. 1907. *None available.*

Bulletin No. 11. The Effect of Scale on the Transmission of Heat through Locomotive Boiler Tubes, by Edward C. Schmidt and John M. Snodgrass. 1907. *None available.*

Bulletin No. 12. Tests of Reinforced Concrete T-Beams, Series of 1906, by Arthur N. Talbot. 1907. *None available.*

Bulletin No. 13. An Extension of the Dewey Decimal System of Classification Applied to Architecture and Building, by N. Clifford Ricker. 1906. *None available.*

Bulletin No. 14. Tests of Reinforced Concrete Beams, Series of 1906, by Arthur N. Talbot. 1907. *None available.*

Bulletin No. 15. How to Burn Illinois Coal without Smoke, by L. P. Breckenridge. 1907. *None available.*

Bulletin No. 16. A Study of Roof Trusses, by N. Clifford Ricker. 1907. *None available.*

Bulletin No. 17. The Weathering of Coal, by S. W. Parr, N. D. Hamilton, and W. F. Wheeler. 1907. *None available.*

Bulletin No. 18. The Strength of Chain Links, by G. A. Goodenough and L. E. Moore. 1907. *Forty cents.*

Bulletin No. 19. Comparative Tests of Carbon, Metallized Carbon, and Tantalum Filament Lamps, by T. H. Amrine. 1907. *None available.*

Bulletin No. 20. Tests of Concrete and Reinforced Concrete Columns, Series of 1907, by Arthur N. Talbot. 1907. *None available.*

Bulletin No. 21. Tests of a Liquid Air Plant, by C. S. Hudson and C. M. Garland. 1908. *Fifteen cents.*

Bulletin No. 22. Tests of Cast-Iron and Reinforced Concrete Culvert Pipe, by Arthur N. Talbot. 1908. *None available.*

Bulletin No. 23. Voids, Settlement, and Weight of Crushed Stone, by Ira O. Baker. 1908. *Fifteen cents.*

**Bulletin No. 24.* The Modification of Illinois Coal by Low Temperature Distillation, by S. W. Parr and C. K. Francis. 1908. *Thirty cents.*

Bulletin No. 25. Lighting Country Homes by Private Electric Plants, by T. H. Amrine. 1908. *Twenty cents.*

Bulletin No. 26. High Steam-Pressure in Locomotive Service. A Review of a Report to the Carnegie Institution of Washington, by W. F. M. Goss. 1908. *Twenty-five cents.*

Bulletin No. 27. Tests of Brick Columns and Terra Cotta Block Columns, by Arthur N. Talbot and Duff A. Abrams. 1908. *Twenty-five cents.*

Bulletin No. 28. A Test of Three Large Reinforced Concrete Beams, by Arthur N. Talbot. 1908. *Fifteen cents.*

Bulletin No. 29. Tests of Reinforced Concrete Beams: Resistance to Web Stresses, Series of 1907 and 1908, by Arthur N. Talbot. 1909. *Forty-five cents.*

Bulletin No. 30. On the Rate of Formation of Carbon Monoxide in Gas Producers, by J. K. Clement, L. H. Adams, and C. N. Haskins. 1909. *Twenty-five cents.*

Bulletin No. 31. Tests with House-Heating Boilers, by J. M. Snodgrass. 1909. *Fifty-five cents.*

Bulletin No. 32. The Occluded Gases in Coal, by S. W. Parr and Perry Barker. 1909. *Fifteen cents.*

Bulletin No. 33. Tests of Tungsten Lamps, by T. H. Amrine and A. Guell. 1909. *Twenty cents.*

**Bulletin No. 34.* Tests of Two Types of Tile-Roof Furnaces under a Water-Tube Boiler, by J. M. Snodgrass. 1909. *Fifteen cents.*

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Bulletin No. 35. A Study of Base and Bearing Plates for Columns and Beams, by N. Clifford Ricker. 1909. *None available.*

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Bulletin No. 37. Unit Coal and the Composition of Coal Ash, by S. W. Parr and W. F. Wheeler. 1909. *None available.*

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**Bulletin No. 39.* Tests of Washed Grades of Illinois Coal, by C. S. McGovney. 1909. *Seventy-five cents.*

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**Bulletin No. 47.* Magnetic Properties of Heusler Alloys, by Edward B. Stephenson. 1910. *Twenty-five cents.*

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**Bulletin No. 56. Tests of Columns: An Investigation of the Value of Concrete as Reinforcement for Structural Steel Columns*, by Arthur N. Talbot and Arthur R. Lord. 1912. *Twenty-five cents.*

**Bulletin No. 57. Superheated Steam in Locomotive Service. A Review of Publication No. 127 of the Carnegie Institution of Washington*, by W. F. M. Goss. 1912. *Forty cents.*

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AN INVESTIGATION OF THE FATIGUE OF METALS

BY

H. F. MOORE

AND

J. B. KOMMERS



BULLETIN No. 124

ENGINEERING EXPERIMENT STATION

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UNIVERSITY OF ILLINOIS
ENGINEERING EXPERIMENT STATION

BULLETIN No. 124

Oct., 1921

AN INVESTIGATION OF THE FATIGUE OF METALS

A REPORT OF THE INVESTIGATION

CONDUCTED BY

THE ENGINEERING EXPERIMENT STATION
UNIVERSITY OF ILLINOIS

IN COÖPERATION WITH

THE NATIONAL RESEARCH COUNCIL
THE ENGINEERING FOUNDATION
THE GENERAL ELECTRIC COMPANY

BY

H. F. MOORE

RESEARCH PROFESSOR OF ENGINEERING MATERIALS
IN CHARGE, INVESTIGATION OF THE FATIGUE OF METALS

AND

J. B. KOMMERS

RESEARCH ASSOCIATE PROFESSOR OF ENGINEERING MATERIALS
ENGINEER OF TESTS, INVESTIGATION OF THE FATIGUE OF METALS

ENGINEERING EXPERIMENT STATION

PUBLISHED BY THE UNIVERSITY OF ILLINOIS, URBANA

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AN INVESTIGATION OF THE FATIGUE OF METALS

I. INTRODUCTION

1. *Inception of the Investigation.*—For three-quarters of a century the problem of the strength of metals under stresses repeated many times has engaged the attention of engineers. It has been recognized that loads which cause no apparent damage when applied a few times to a machine or structural part may cause failure if applied many times. Various investigations of the phenomenon of failure under repeated stress* have been made and the name “fatigue of metals” has been given to it. The earliest extensive investigation, and the best known, is that of Wöhler, whose results were published in 1870.

During the world war the question of strength of airplane parts under repeated stress became of prime importance. A special phase of this problem was a study of the strength of parts made of cold-drawn steel, and a short series of experiments was made in the laboratories of the University of Illinois at the request of the National Research Council.† This problem and other problems of material under repeated stress, notably repeated stresses in welded ships, brought the whole subject of fatigue phenomena of metals to the attention of the National Research Council. The result was the organization of an investigation by the coöperation of the National Research Council Division of Engineering, Engineering Foundation, and the Engineering Experiment Station of the University of Illinois. Later the General Electric Company joined this group.

The National Research Council furnished an Advisory Committee to formulate general policies for the investigation. The Engineering Experiment Station provided a laboratory, the time of one member of its staff, and the use of much apparatus. A previous progress report of this investigation gave a general summary of the knowledge of fatigue phenomena of metals current at the time of the beginning of the investigation. This was published in “Mechanical Engineer-

* See Appendix C, which gives a glossary of technical terms used in this bulletin.

† Moore and Putnam, Am. Inst. of Mining and Metall. Engrs., Bul. 146, p. 401, Feb., 1919.

ing'' (the organ of the American Society of Mechanical Engineers), for September, 1919, and is reprinted in substance as Appendix B of this bulletin.*

The investigation was financed, first, by Engineering Foundation from the fund given by Mr. Ambrose Swasey of Cleveland, Ohio. This grant was sufficient to permit the organization of a test party, the purchase of many pieces of apparatus, the preparation of a large number of specimens, and the maintenance of the laboratory for a term of two years. In 1920 the General Electric Company contributed an equal sum for investigations along certain lines of special interest to them, without any restrictions as to the free publication of the results. This contribution made possible the enlargement of the test party and the purchase of more equipment. The lines of investigation desired by the General Electric Company follow so closely the lines of the original investigation that, in compiling this report, data and results have been taken from both parts, but mainly from the original investigation financed by Engineering Foundation, as, owing to the fact that this part of the investigation had been in progress for a longer time, more material was available from this source.

2. *Personnel of the Advisory Committee of the Division of Engineering of the National Research Council.*—The personnel of the Advisory Committee is given below:

O. H. BASQUIN, Professor of Applied Mechanics, Northwestern University;

F. P. GILLIGAN, Secretary-Treasurer, Henry Souther Engineering Company;

HENRY M. HOWE, Metallurgist, Bedford Hills, New York;

ZAY JEFFRIES, Director of the Cleveland Section of the Research Bureau, Aluminum Company of America;

T. R. LAWSON, Professor of Rational and Applied Mechanics, Rensselaer Polytechnic Institute;

J. A. MATHEWS, President, Crucible Steel Company of America;

*For a more detailed account of previous investigations of the strength of materials under repeated stress see:

Unwin, W. C. "The Testing of the Materials of Construction," Chap. XVI, 1910.

Withey and Aston, "Johnson's Materials of Construction," Fifth Edition, Chap. XXVIII.

JOHN H. NELSON, Chief Metallurgist, Wyman-Gordon Company;

W. E. RUDER, Metallurgist, Research Laboratory, General Electric Company;

H. L. WHITTEMORE, Chief of Section of General Physical Testing, U. S. Bureau of Standards;

LEONARD WALDO, Consulting Engineer, New York City;

H. F. MOORE, Research Professor of Engineering Materials, University of Illinois, Chairman.

In connection with the work of the Advisory Committee there have been organized two sub-committees: first, a sub-committee on Heat Treatment of Specimens, J. H. NELSON, Chairman (resigned October 22, 1920), W. E. RUDER, Chairman (since October 30, 1920), and F. P. GILLIGAN; and second, a sub-committee on Statistics, T. R. LAWSON, Chairman, LEONARD WALDO, and H. L. WHITTEMORE.

3. *Outline of Investigation.*—At a meeting held February 19, 1920, the Advisory Committee planned reconnaissance tests of materials well scattered over the field of ferrous metals, in most cases studying two or more distinct heat treatments for each metal. It was decided not to enter the field of non-ferrous metals at this time.

For each heat treatment of each steel tested it was planned to make a series of tests of specimens under reversed bending stress, using various stresses, until an "endurance" of 100 000 000 reversals was reached; to make corresponding static tests in tension, compression, and shear (torsion); and to make various auxiliary tests, including hardness tests and impact tests.

It was planned to use magnetic analysis for examining the homogeneity of the material tested, and to study various accelerated tests for resistance to repeated stress in order to determine their reliability.

The main purpose of this first stage of the investigation was to determine whether for ferrous metals there exists any clearly defined relation between the static properties (elastic limit, yield point, ultimate tensile strength, elongation, reduction of area, hardness, etc.) and ability to resist reversed stress.

When later the General Electric Company became a party to the investigation there was added to this program the study of the

effect of range of stress (stress partially reversed or repeated but not reversed, as well as stress completely reversed) upon the strength of ferrous metals under repeated stress. Work on this phase of the investigation has not passed the preliminary stage.

4. *Organization of Test Party.*—The test party was organized as follows. The chairman of the Advisory Committee, H. F. MOORE, who was also a member of the technical staff of the Engineering Experiment Station, was in general charge. The University allowed him to give nearly all his time to this work. An engineer of tests, J. B. KOMMERS, was in immediate charge of the work of the test party. Two test assistants carried on the routine work. Two mechanics prepared specimens and repaired and built auxiliary parts for apparatus, and a clerk gave half her time to the office work. The professor in charge of the metallographic laboratory in the Department of Chemistry gave some time to the metallographic features. Certain students of marked ability were allowed to take thesis work in connection with the investigation.

5. *Acknowledgments.*—The Investigation of Fatigue of Metals was made a part of the research work of the Department of Theoretical and Applied Mechanics, and was carried on under the general administrative direction of the head of the department, PROFESSOR A. N. TALBOT.

DR. HENRY M. HOWE, while serving as Chairman of the Engineering Division of the National Research Council, took an active interest in the fatigue phenomena in metals and it was largely owing to his suggestion, activities, and influence that this extensive investigation was made possible.

Acknowledgment is due the following individuals, firms, and institutions:

WYMAN-GORDON COMPANY, Worcester, Massachusetts, manufacturers of drop forgings, for assistance in the work of heat treatment of certain steels;

THE UNIVERSITY OF WISCONSIN, for the loan of a testing machine, and for permission to use certain test data in connection with Appendix A of this bulletin;

THE AMERICAN SOCIETY OF MECHANICAL ENGINEERS, for permission to reprint the Progress Report, Appendix B;

THE ILLINOIS STEEL COMPANY, Chicago, Illinois,

THE JOHN A. ROEBLINGS SONS COMPANY, Trenton, New Jersey,

THE HALCOMB STEEL COMPANY, Syracuse, New York,

THE STANDARD STEEL COMPANY, Philadelphia, Pennsylvania,

THE CARNEGIE STEEL COMPANY, Pittsburgh, Pennsylvania

THE GENERAL ELECTRIC COMPANY, Schenectady, New York, and THE MIDVALE STEEL AND ORDNANCE COMPANY, Philadelphia, Pennsylvania, for steel supplied for the investigation;

THE AMERICAN ROLLING MILLS COMPANY, for a supply of Armco iron (in this bulletin this material is designated as 0.02 carbon steel);

MR. W. J. FRANCKE of New Brunswick, New Jersey, for a number of special flexure tests upon various types of steel, and for depositing one of his special flexure testing machines;

DR. CHARLES W. BURROWS, Grasmere, Borough of Richmond, New York City, for permission to use his patented method of magnetic analysis for detecting flaws in steel;

The Physics Department and the Chemistry Department of THE UNIVERSITY OF ILLINOIS, for the loan of apparatus and for the use of rooms in which special tests were made.

The work of the following members of the test party is gratefully acknowledged:

PROFESSOR D. A. MCFARLAND and PROFESSOR W. S. PUTNAM of the Department of Chemistry, for assistance in metallographic work;

F. H. FISH, F. M. HOWELL, F. M. POST, J. W. HARSCH, and PROFESSOR W. J. PUTNAM.

The following students in mechanical engineering of the 1921 senior class have taken thesis work in connection with this investigation, and their work has been of no small value:

MESSRS. R. F. PACKARD and MARTIN FRISCH, subject, "The Effect of Over-Stress on Subsequent Resistance to Repeated Stress"; GEORGE R. CASKEY, subject, "The Effect of Surface Finish on Resistance to Repeated Stress"; JOHN A. GOFF, subject, "The Effect of Radius of Fillet on Resistance to Repeated Stress."

II. MATERIALS, TESTS, TESTING APPARATUS, AND TEST SPECIMENS

6. *Materials*.—It having been decided to confine this preliminary investigation to ferrous metals, a series widely distributed among the varieties of steels in common use was selected. Table 1 gives the chemical analyses and reference numbers for the different metals tested.

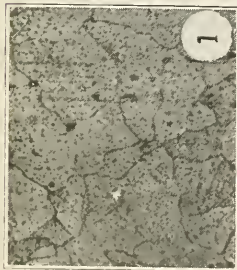
TABLE 1
CHEMICAL ANALYSES OF STEELS TESTED

Steel No.	Furnished by	Material Furnished in	CONTENT, PER CENT						
			Car- bon	Chro- mium	Nick- el	Sili- con	Man- gan- ese	Phos- phor- us	Sul- phur
1	Illinois Steel Co.....	2×1 flats.....	1.20			0.19	0.25	0.021	0.021
3	John A. Roeblings Sons Co.....	Billets 4-in. square .	0.52			0.24	0.56	0.037	0.029
4	John A. Roeblings Sons Co.....	Billets 4-in. square .	0.37			0.16	0.58	0.032	0.035
5	Halcomb Steel Co.....	2×1 flats.....	0.24	0.87	3.33	0.15	0.37	0.019	0.025
6	Carnegie Steel Co. through Standard Steel Co.....	2× $\frac{7}{8}$ flats.....	0.93			0.03	0.38	0.017	0.045
7	Midvale Steel & Ord. Co. through General Electric Co.....	1-in. squares	0.41	0.18	3.41	0.25	0.75	0.020	0.020
9	American Rolling Mill Co.....	1-in. rounds.....	0.02			0.02	0.03	0.005	0.042
10	Inland Steel Co.....	$\frac{1}{16}$ -in. squares.....	0.49			0.12	0.46	0.017	0.029
50	J. T. Ryerson & Son (Cold-drawn screw stock).....	$\frac{7}{16}$ -in. rounds.....	0.20			0.03	0.67	0.025	0.090
51	Univ. of Ill. stock (hot- rolled reinforcing rod).....	$\frac{1}{2}$ -in. rounds.....	0.18			0.06	0.37	0.013	0.039

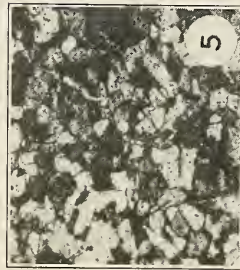
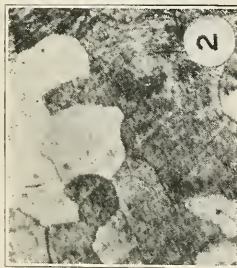
It was decided to use two or more heat treatments for most of the metals tested. The heat treatments used are shown in Table 2, and the resulting internal structures as shown by microscopic examination are given in Fig. 1.

Excepting materials 4, 5, 9, 50, and 51, all were first given a normalizing heat treatment to relieve any internal strain which might be present and to give a common basis for any other heat treatments.

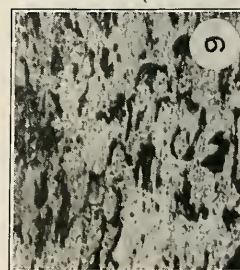
All the steels which were heat treated were treated in the form of rectangular bars after being cut up according to the diagram



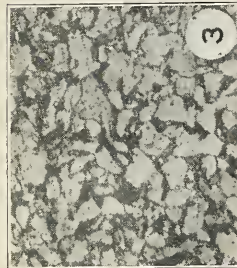
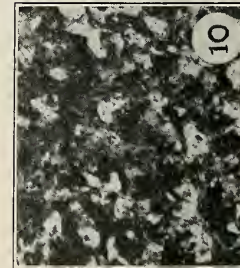
Steel No. 9, 0.02 Carbon, as Received



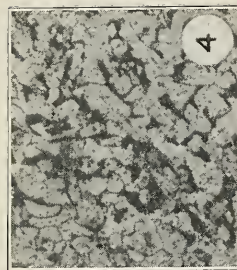
Steel No. 51, 0.18 Carbon, Hot-Rolled, Reduced in Tension
From 0.50 to 0.48 In. Diameter



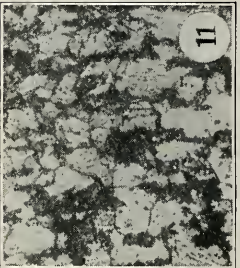
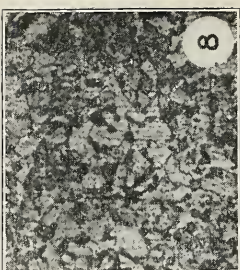
Steel No. 50, 0.20 Carbon, Cold-Drawn, as Received



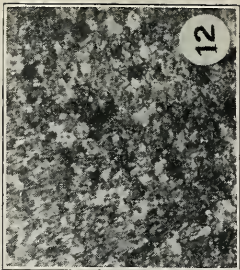
Steel No. 51, 0.18 Carbon, Hot-Rolled, as Received



Steel No. 51, 0.18 Carbon, Hot-Rolled, Reduced in Tension
From 0.50 to 0.44 In. Diameter



Steel No. 50, 0.20 Carbon, Cold-Drawn,
Annealed at 1300° F.



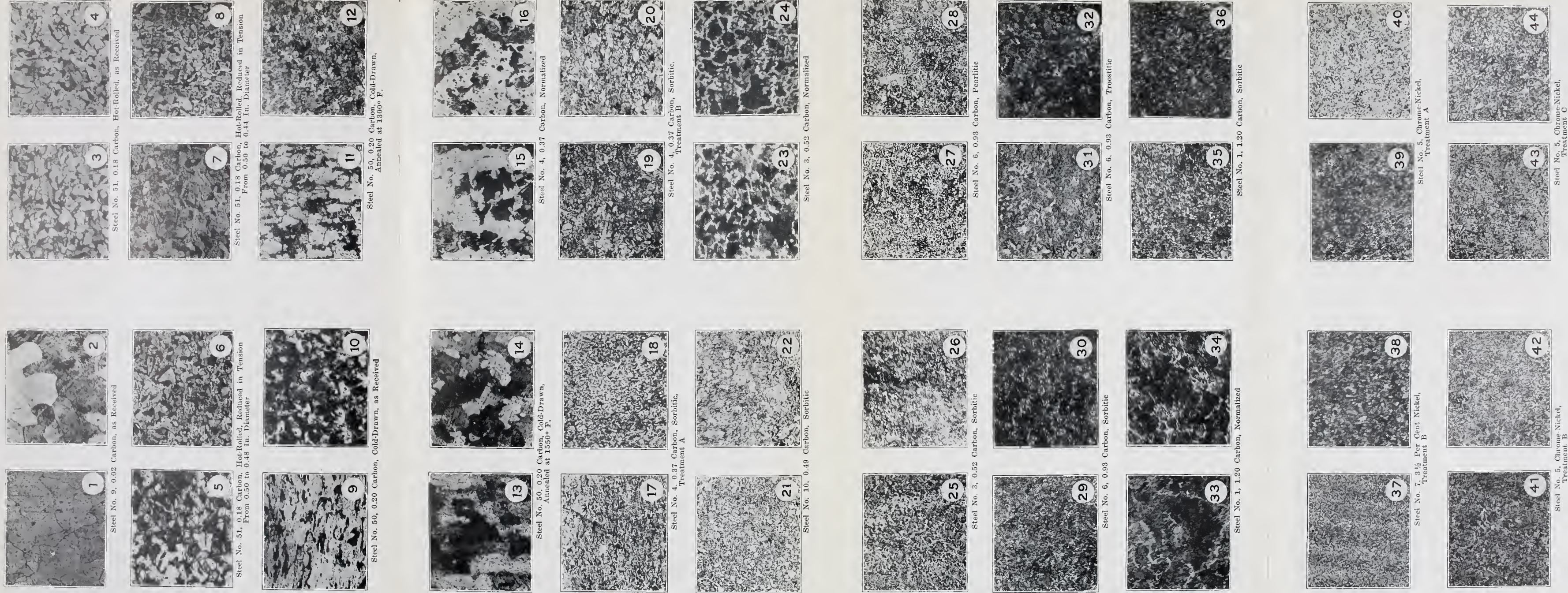


FIG. 1. MICROGRAPHS OF THE STEELS TESTED

Magnification of all micrographs is 100 diameters. Odd numbers refer to longitudinal sections, even numbers to transverse sections

shown in Fig. 2, except the following: 0.37 carbon, sorbitic, treatment A; 1.20 carbon, normalized; 1.20 carbon, sorbitic; and 0.93 carbon, troostitic. These steels were machined somewhat oversize, then heat treated, and later finished to size.

TABLE 2
HEAT TREATMENTS OF STEELS TESTED

No.	STEEL	HEAT TREATMENT
1	1.20 carbon, normalized sorbitic.....	Heat to 1460° F.; hold 15 min.; cool in furnace (this anneals the steel so that it can be machined); then heat to 1580° F.; hold 15 min.; cool in furnace with door open. First anneal as above; then heat to 1470° F.; quench in oil; reheat to 860° F.; hold 30 min.; cool in air.
3	0.52 carbon, normalized sorbitic.....	Heat to 1550° F.; hold 15 min.; cool in air. First normalize as above; then heat to 1450° F.; hold 15 min.; quench in water; reheat to 1200° F.; cool in air.
4	0.37 carbon, normalized sorbitic treatments A and B.....	Heat to 1495° F.; hold 15 min.; cool in furnace with door open. This steel was <i>not</i> first normalized. Heat to 1550° F.; hold 15 min.; quench in water; reheat to 1050° F.; cool in air.
5	Chrome-nickel, treatment A..... treatment B..... treatment C.....	Steel received annealed. Heat to 1525° F.; quench in oil; reheat to 700° F.; quench in oil. Steel received annealed. Heat to 1525° F.; hold for ½ hour; quench in oil. Reheat to 1450° F.; quench in oil. Reheat to 1200° F.; hold for 1 hour; cool in furnace. Steel received annealed. Heat to 1525° F.; hold for ½ hour; quench in oil. Reheat to 1450° F.; quench in oil. Reheat to 1200° F.; hold for 1 hour; quench in water.
6	0.93 carbon, normalized pearlitic..... sorbitic..... troostitic.....	Heat to 1600° F.; hold 15 min.; cool in air. First normalize as above, then heat to 1450° F.; hold 15 min.; cool in furnace. First normalize as above; then heat to 1450° F.; hold 15 min.; quench in oil; reheat to 1200° F.; hold 30 min.; cool in air. First normalize as above; then heat to 1450° F.; hold 15 min.; quench in oil; reheat to 850° F.; hold 30 min.; cool in air.
7	3.5 nickel, treatment B.....	Normalize by heating to 1525° F. and cooling in furnace; then heat to 1525° F.; quench in oil; reheat to 1210° F.; hold 2 hours; cool in furnace.
9	0.02 carbon.....	Tested as received.
10	0.49 carbon, normalized sorbitic.....	Heat to 1700° F.; hold 20 min.; cool in air. First normalize as above; then heat to 1425° F.; quench in water; reheat to 1200° F.; cool in furnace.
50	Cold-drawn as received..... annealed.....	Heat to 1300° F.; hold for 15 min.; cool in furnace. Heat to 1550° F.; hold for 15 min.; cool in furnace.
51	0.18 carbon as received..... cold stretched cold stretched..... cold bent.....	Reduced to diameter of 0.44 in.; then heat to 500° F.; cool in furnace. Reduced to diameter of 0.48 in.; then heat to 500° F.; cool in furnace. Bent to an angle of 45° at the middle, straightened cold.

4A13 4A32

4C13 4C26

A	B	C	D	E
F	G	H	I	J
K	L	M	N	O
P	Q	R	S	T
U	V	W	X	Y

Farmer
Impact Bending

4C21 4C28

A	B	C	D
E	F	G	H
I	J	K	L
M	N	O	P

Reverse Torsion
Torsion
Tension

4C33

A	B	C	D
E	F	G	H
I	J	K	L
M	N	O	P

Impact Tension
Compression
Reverse Bending

4A19

A	B	C	D	E	F	G	H
I	J	K	L	M	N	O	P
Q	R	S	T				

Reverse Bending
Impact Bending

No. 4, 0.37 Carbon Sorbitic

4B13 4B0

A	B	C	D	E
F	G	H	I	J
K	L	M	N	O
P	Q	R	S	

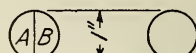
Farmer

4B26

A	B	C	D
E	F	G	H
I	J	K	L
M	N	O	P

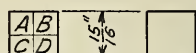
Reverse Torsion

No. 4, 0.37 Carbon Normalized



Farmer Other

No. 9, 0.02 Carbon



Farmer Other

No. 10, 0.49 Carbon

3B13 3B26 3C39

A	B	C	D	E
F	G	H	I	J
K	L	M	N	O
P	Q	R	S	
T	U	V	W	

3B39 3C26

A	B	C	D	E
F	G	H	I	J
K	L	M	N	
O	P	Q	R	

No. 3, 0.52 Carbon Normalized

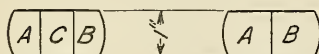
3A13 3A26

A	B	C	D
E	F	G	H
I	J	K	L
M	N	O	P

3A39 3C13

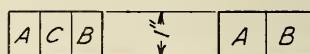
A	B	C	D	E
F	G	H	I	J
K	L	M	N	
O	P	Q	R	

No. 3, 0.52 Carbon Sorbitic



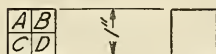
Farmer Other

No. 6, 0.93 Carbon



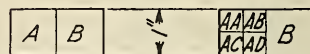
Farmer Other

No. 1, 1.20 Carbon



Farmer Other

No. 7, General Electric 3 1/2% Ni.



Treatment "A" Treatment "B-C"

No. 5, Chrome-Nickel

Fig. 2. MARKING DIAGRAMS FOR SPECIMENS

7. *Tests*.—Mechanical tests of metals may be divided into three groups: first, “static” tests, usually made on the ordinary tension-compression testing machine; second, repeated-stress tests, and third, impact tests. Static tests include torsion and flexure tests and Brinell hardness tests. Probably scleroscope hardness tests would also be classified with static tests.

The results of static tests indicate the resistance of a metal to the destructive action of a steady load or a load applied a few times. Static tests yield significant results concerning the suitability of material for buildings, tanks, and other structures, including most bridges.

The results of repeated-stress tests indicate the resistance of a metal to progressive failure under many repetitions of a given working load. Repeated-stress tests have not been thoroughly standardized, but would seem to yield significant results concerning the suitability of material for machine or structural parts which are to be subjected to many repetitions of loading when there is little danger of accidental heavy overload; springs, shafting, and car axles, for example.

The results of impact tests indicate the resisting power of material against shattering under sudden heavy overload. Impact tests would seem to yield significant results concerning the suitability of material for machine or structural parts which may be occasionally subjected to heavy overload, and which can still be used after some permanent distortion has taken place. Impact-resisting power seems to be a sort of insurance against complete collapse under sudden overload.

In this connection it may be noted that the strength-indicating results of static tests and the endurance limits given by repeated-stress tests are measured in the same units—pounds per square inch; the results of impact tests are measured in units of energy—foot-pounds or inch-pounds. It may be further noted that the results of an impact test are comparable with a measurement of the area under the complete stress-strain diagram for a static test, or, using a rough approximation, the results of an impact test are comparable with the product of ultimate strength and elongation given by a static test. The term *toughness* is sometimes used to indicate a combination of the static strength and the ductility, and the impact test may be said

to be the index of the toughness of a material. Again it may be noted that the strength-indicating results of static tests, and the results of repeated-stress tests indicate ability to withstand normal service conditions, while the results of impact tests—and the results of static tests which measure ductility—indicate ability to withstand occasional overload without complete shattering failure.

The principal series of tests in this investigation are the reversed-stress tests performed on the various materials. These tests are discussed in further detail in Section 8. As the problem of determining relations existing between strength under repeated stress and strength under ordinary static tests was the primary problem of the investigation, careful tests on ordinary testing machines formed another series of tests of prime importance. These tests included tension tests, compression tests, and torsion tests, the last named giving values for shearing strength.

In addition to the regular static tests, a limited number of special flexure tests were made, some in the laboratories of the University of Illinois and some at the laboratory of Mr. W. J. Franke of New Brunswick, New Jersey.

Brinell and scleroscope hardness tests were made. Charpy impact tests were made, both on notched bars in bending and on tension specimens.

Magnetic tests for homogeneity were made on the bars used for repeated-stress specimens for the rotating-beam machine before they were reduced at the center.

8. *Testing Machines and Apparatus.*—The rotating-beam type of testing machine was chosen for the basic series of repeated-stress tests. This machine has been used by many previous investigators, beginning with Wöhler in his historic series of tests. Fig. 3 is from a photograph of the machine used, and Fig. 4 shows a diagram of the machine.

In Fig. 3 and Fig. 4 the specimen, *A*, is in the form of a bar reduced in diameter at the middle of its length. It is held in ball bearings, *B*, *C*, *D*, and *E*, by means of draw-in collets (one collet is shown in detail at *F*). The specimen is driven by the shaft, *G*, operating through the flexible leather disc, *H*. Pulley *K* is driven by an electric motor. Load is applied by means of weights hung at *W*.

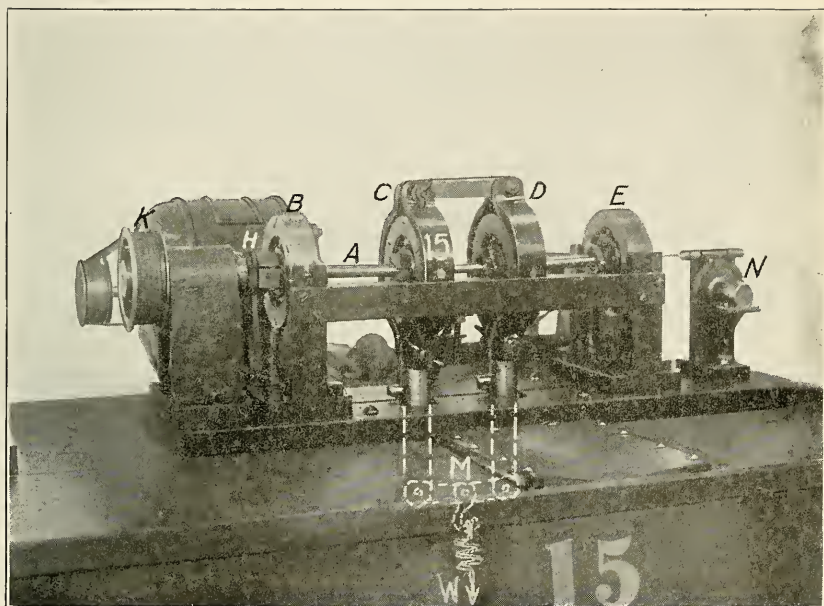


FIG. 3. ROTATING-BEAM TESTING MACHINE (FARMER)

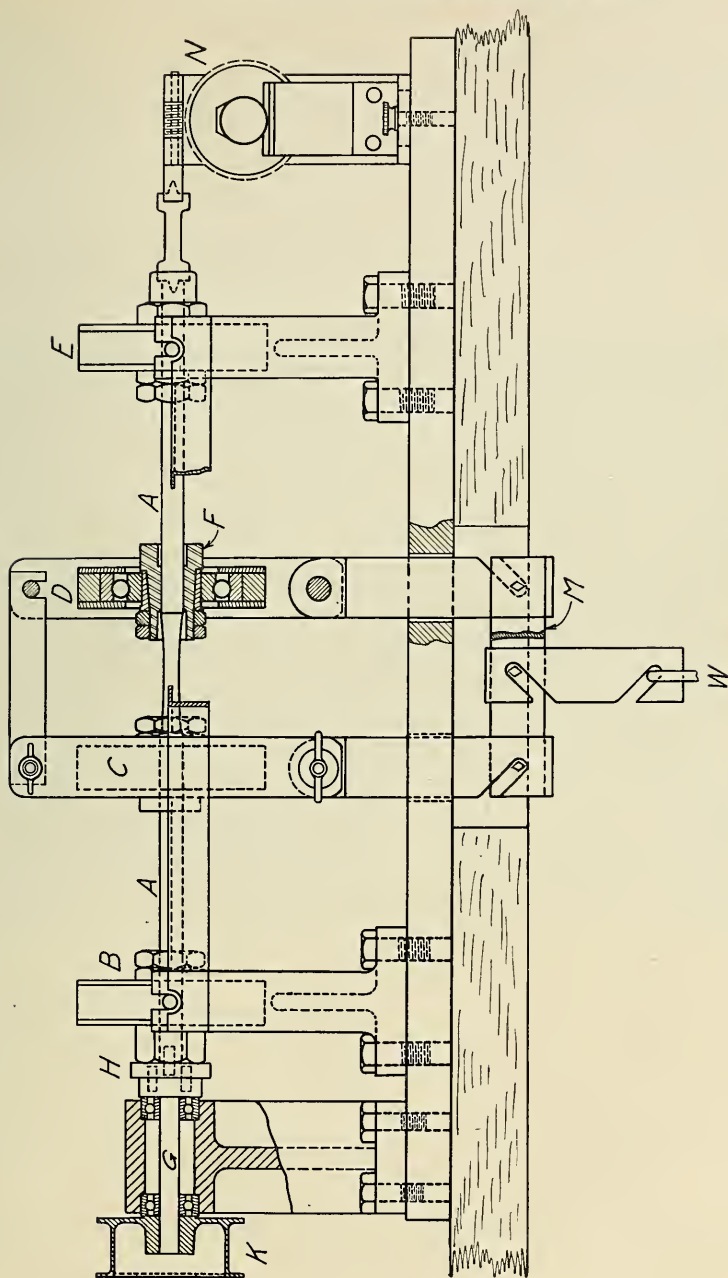


FIG. 4. DIAGRAM OF ROTATING-BEAM TESTING MACHINE (FARMER)

The weight, W , is distributed to the bearings, C and D , by means of the equalizing bar, M . In the position shown the specimen is under flexure and the fibers along the top of the bar are in compression while those on the under side of the bar are in tension. If the pulley is given half a revolution the stress in the fibers is completely reversed. The number of cycles of completely reversed stress given to the specimen is equal then to the number of revolutions of the pulley, K , which is indicated by means of the revolution counter, N . When the specimen breaks, the center bearings, C and D , drop together with their housings and strike a trigger (not shown in Fig. 4) which releases the switch controlling the motor.

The machine produces a uniform bending moment on the specimen between bearings C and D . This form of machine with two symmetrical loads has been used by many investigators, the first, so far as is known, being Professor Sondericker,* of the Massachusetts Institute of Technology in 1892. The machine used in the present investigation follows in its details quite closely that described by Mr. F. M. Farmer before the American Society for Testing Materials at its 1919 meeting; hence, this machine is sometimes referred to as the Farmer machine.

Fig. 5 is from a photograph and Fig. 6 shows a diagram of another type of rotating-beam machine which was used for some tests. This machine had been designed by Professor Kommers at the University of Wisconsin and is spoken of as the Wisconsin machine. The specimen is rotated and is loaded as a cantilever beam.

The rotating-beam type of machine was chosen for making the basic series of reversed-stress tests for a number of reasons. In this machine the magnitude of the stresses can be computed with a high degree of precision, and a prime requisite for the basic series of tests was that there should be little uncertainty as to the values of the stresses used. In its operation this machine is very free from vibration. The slight vibration of the specimen was observed to be in synchronism with the rotation of the machine, and probably was caused by the minute deviation from straightness of the axis of the specimen. A slight vibration in synchronism with the rotation of the

* Sondericker, J. "A Description of Some Repeated-Stress Experiments," Tech. Quar. Boston, April, 1892.

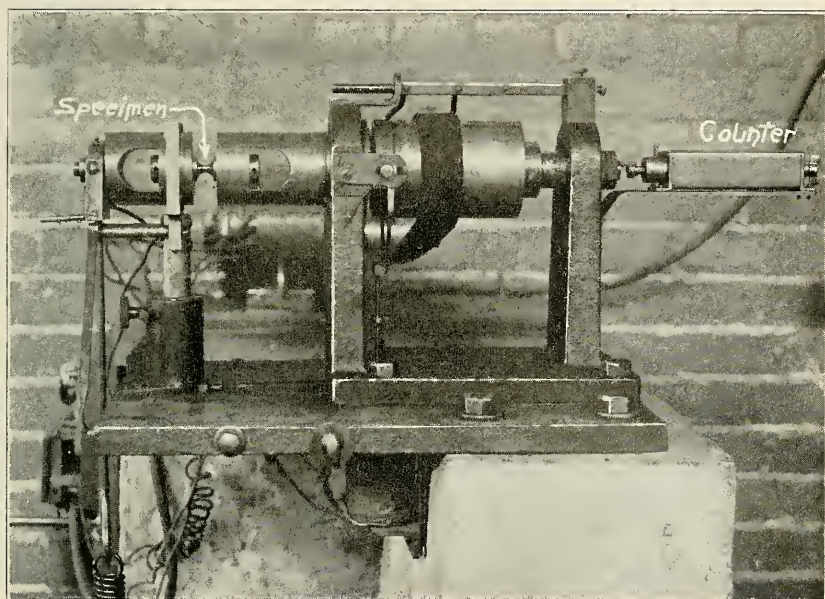


FIG. 5. ROTATING-BEAM TESTING MACHINE (WISCONSIN)

specimen does not alter the range of stress to which the specimen is subjected. The results obtained by this type of machine are not seriously affected by slight changes in speed. The writers made a

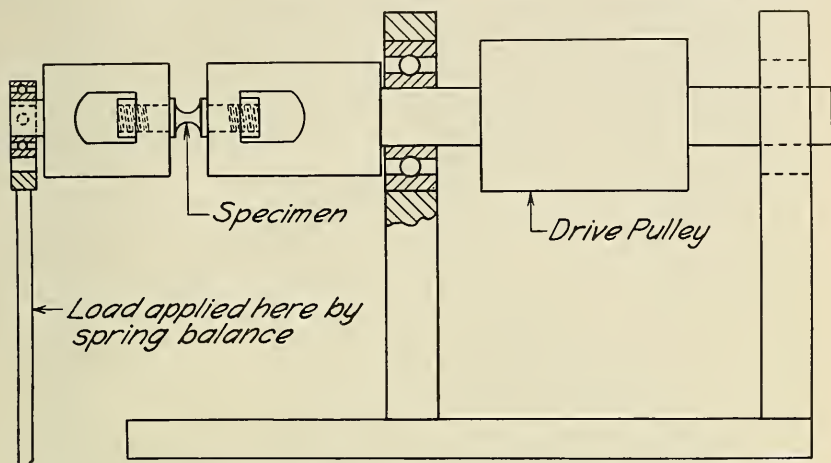


FIG. 6. DIAGRAM OF ROTATING-BEAM TESTING MACHINE (WISCONSIN)

somewhat careful search for test results at various speeds and found considerable evidence* that speeds below 2000 revolutions per minute have very little effect on the result of repeated-stress tests.

In another type of repeated-stress testing machine calibrated springs are used to resist and to measure the bending moment or the twisting moment applied to the specimen. Two machines of this type were used for a secondary series of reversed-stress tests. Fig. 7 is from a photograph of the Upton-Lewis machine used for flexural reversed-stress tests. Fig. 8 shows the scheme of the machine diagrammatically. In this machine the specimen is bent back and forth in one plane. The amount of deflection of the springs, and hence the magnitude of the bending moment applied to the specimen, is given by the width of the diagram drawn by the pencil point. The number of cycles of bending stress is shown by the counter. Fig. 9

*Stanton and Pannell, Inst. Civ. Engrs. (British), CLXXXVIII, p. 814, 1911.

Hopkinson, B. Proc. Royal Soc., A 86, January 31, 1912; Sci. Abs., 1912, No. 628.

Roos, J. O. Int. Assn. for Test. Materials, 1912, Art. V2B.

Kommers, J. B. Int. Assn. for Test. Materials, 1912, Art. V4B.

Stanton and Bairstow, Inst. Civ. Engrs. (British), CLXVI, p. 78, 1905-6.

Upton and Lewis, American Machinist, October 17, 1912, p. 633.

Eden, Rose, and Cunningham, Inst. Mech. Engrs. (British), 1911, Part 3-4, p. 839.

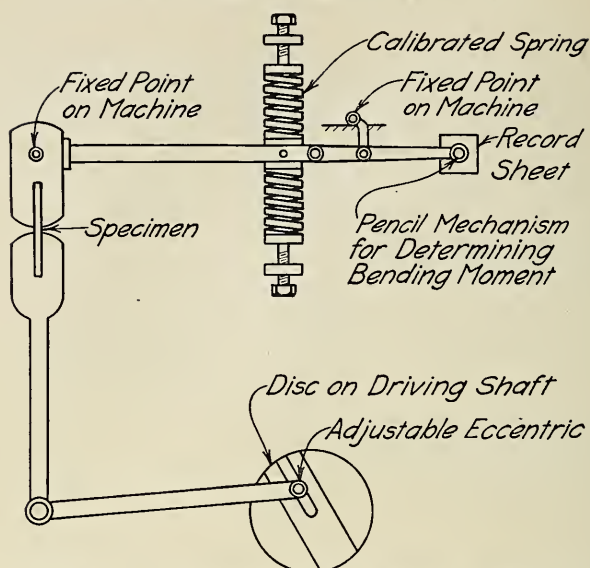


FIG. 8. DIAGRAM OF UPTON-LEWIS REVERSED-BENDING TESTING MACHINE

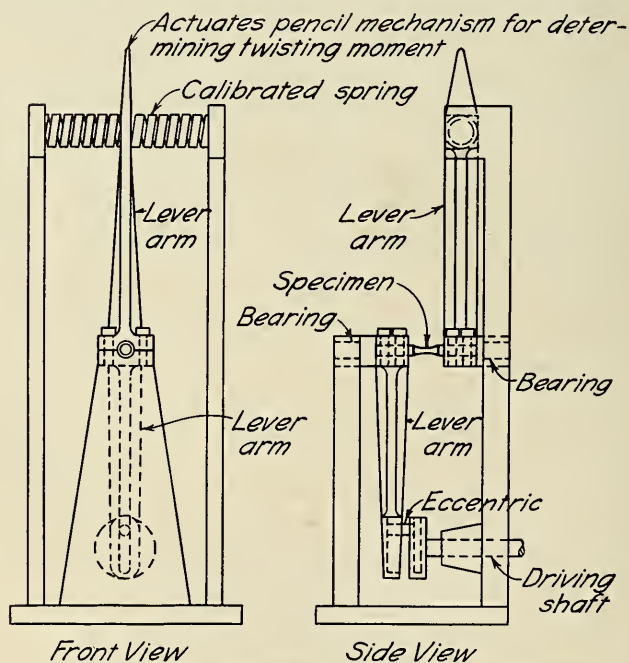


FIG. 10. DIAGRAM OF OLSEN-FOSTER REVERSED-TORSION TESTING MACHINE

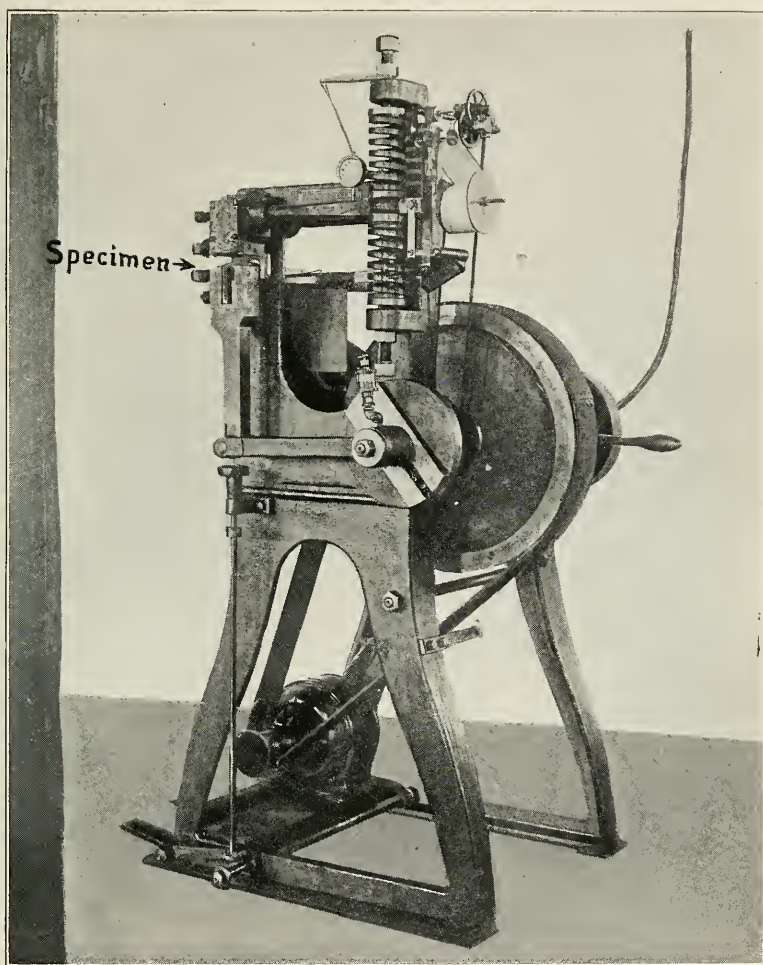


FIG. 7. UPTON-LEWIS REVERSED-BENDING TESTING MACHINE

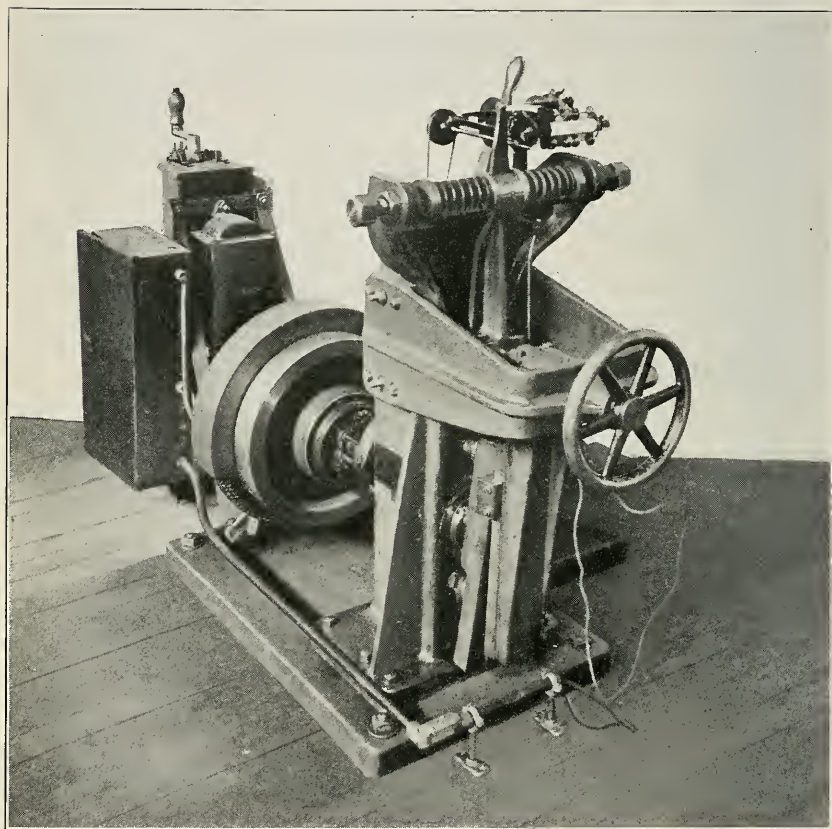


FIG. 9. OLSEN-FOSTER REVERSED-TORSION TESTING MACHINE

is from a photograph of the reversed-torsion machine, the Olsen-Foster, while Fig. 10 gives a diagram of the same machine. The specimen is subjected to an amount of twist in one direction from the neutral position, and then to the same amount of twist in the opposite direction. The scheme for measuring twisting moment and number of cycles of twisting stress is similar to that used in the Upton-Lewis machine.

These spring machines are, however, very much more expensive than the rotating-beam machine, and, moreover, the error inherent in spring calibration and the stresses caused by the inertia of reciprocating parts introduce some uncertainty as to the magnitude of some of the stresses. A further objection to these machines is that they cannot be run satisfactorily at speeds as high as 1500 revolutions per minute.

All the fatigue machines were provided with automatic devices which stopped the machines when the specimen failed. These devices made continuous operation of the machines possible.

Repeated-stress testing machines, which depend upon the inertia of reciprocating or rotating parts for producing stress, have sometimes been used. Such machines are markedly affected by variations in the speed, since the inertia effect is proportional to the square of the speed. In tests which last for days at a time the power circuit ordinarily available cannot be relied upon to give constant speed at the motors, unless there is a very elaborate and expensive installation of speed-regulating devices. This consideration caused the rejection of the inertia type of repeated-stress testing machine for this investigation.

Testing machines employing alternating current magnets to produce repeated stress were considered. These machines would be very much more expensive than the rotating-beam type of machine, and, moreover, slight variations in the frequency of the alternating current (such variations as would inevitably occur in the current supplied to the laboratory) might produce quite serious variations in the force set up by the magnets.

There would seem to be some advantage in using machines which produce repetitions of direct tension or compression in a specimen. The only available machines producing such stress were of the alternating current magnet type or of the inertia type, and were so ex-

pensive that the funds of the investigation would not have permitted purchasing a sufficient number of machines.

To sum up, the rotating-beam machine was chosen for the basic series of tests primarily because it seemed to be a machine in which, within the yield point, definite stresses could be set up with a high degree of precision, because it seemed least affected by slight variations of frequency in the electric circuits supplying power, and because it was the least expensive machine to construct.

As noted above, a careful study of the records of other tests seems to indicate that speed of testing has little effect on the results up to a speed of about 2000 revolutions per minute. The speed chosen for the Farmer machine was well within this limit, namely, 1500 revolutions per minute. With the Upton-Lewis machine a speed of 300 revolutions per minute was used; with the Olsen-Foster machine, a speed of 350 revolutions per minute was used, except for the 0.93 carbon steel, troostitic, the 0.49 carbon steel, sorbitic, and the chrome-nickel steel, treatment A, for which a speed of 260 revolutions per minute was used; with the Wisconsin machine a speed of 1200 revolutions per minute was used.

For the static compression tests and tension tests a 100 000-pound Riehle machine and a 10 000-pound Olsen machine were used. For the static torsion tests a Riehle 10 000-inch-pound machine of the pendulum type was used. In the last-named machine the twisting moment exerted on the specimen is read from a micrometer dial gage which indicates a motion of 0.001 inch, and which is actuated by the movement of the pendulum from its neutral position.

Fig. 11 is from a photograph of the Charpy impact machine which was used to make the impact-bending and impact-tension tests. In the single-blow impact testing machine, of which the Charpy machine is one type, a heavy pendulum is raised to a given angle from its normal position and then is allowed to fall against a specimen. Rupturing the specimen (a short beam with a notch in it for the impact-flexure test), the pendulum passes the neutral position and rises to an angle indicated by a "maximum" finger. The difference between angle of fall and angle of rise is a measure of the energy absorbed in rupturing the specimen.* A centering device

* Dix, E. H., Jr. Proc. Am. Soc. for Test. Materials, Vol. XIX, Part II, p. 720, 1919. This is a detailed discussion of the single-blow impact testing machine.

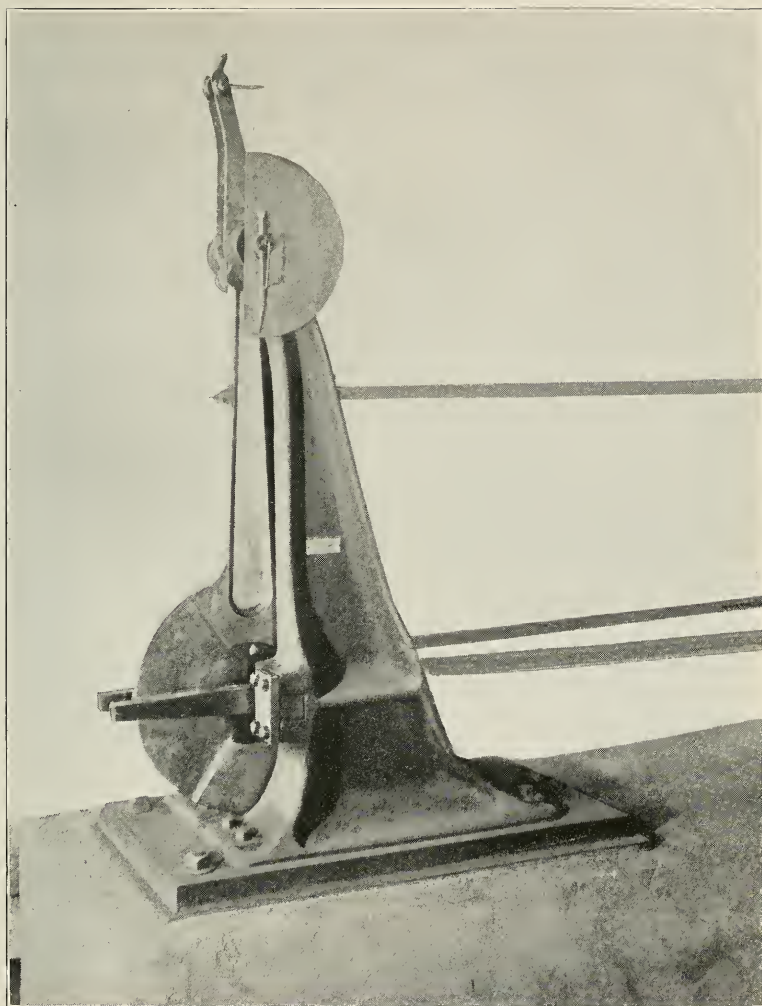


FIG. 11. CHARPY SINGLE-BLOW IMPACT TESTING MACHINE

was used for placing the impact-bending specimen in the machine, so that the pendulum would strike opposite the center of the notch.

Fig. 12 shows the apparatus which was designed for use with the Charpy machine in making the impact-tension test.* As is

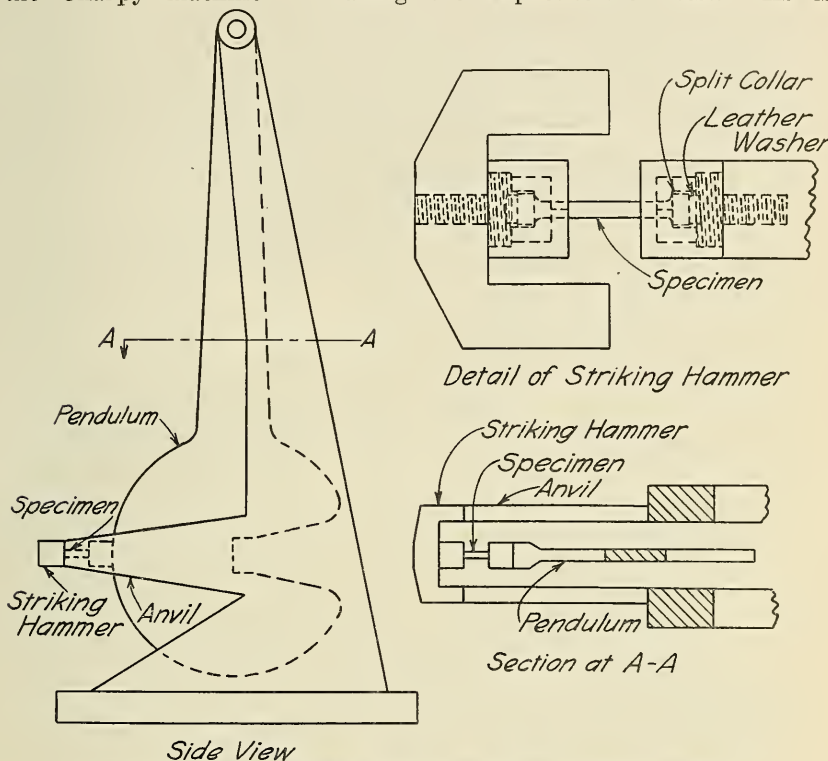


FIG. 12. TENSION TEST ATTACHMENT FOR CHARPY IMPACT TESTING MACHINE

*When making impact-tension tests there is one serious objection to the Charpy impact machine in its present form. The hammer strikes the anvil when the pendulum reaches its neutral position. The center of gravity would at that instant be moving in the horizontal direction. However, it will be seen that the point where the specimen is attached to the pendulum is still moving with a component in the downward direction. When a specimen is quite ductile so that its percentage of elongation is, say, 30 per cent, this allows the specimen to be bent downward while it is being stretched out. The result is that a certain amount of energy is absorbed by the specimen in this bending action, and this energy is credited to the steel as energy of rupture. With specimens that break without much elongation there is no appreciable error of this kind.

This error of the machine might be corrected in two ways. One would be to shorten the anvils or design the hammer in such a way that when the hammer strikes the anvil the specimen will be moving in a horizontal direction. This method would reduce the maximum capacity of the machine. A second method would be to design the anvil and hammer so that the striking surfaces would be at right angles to the direction in which the specimen is moving at the instant of striking.

shown in the sketch, the specimen is provided with spherical seats at the shoulders so that there may be slight adjustment of the specimen when the hammer strikes the anvil.

Fig. 13 is from a photograph of the double-blow machine used in making tests under repeated impact. Fig. 14 shows a diagram of the same machine. Two similar hammers actuated by cams strike the specimen alternately on opposite sides. The specimen shown in Fig. 14 is a grooved cantilever beam held in a vise, and

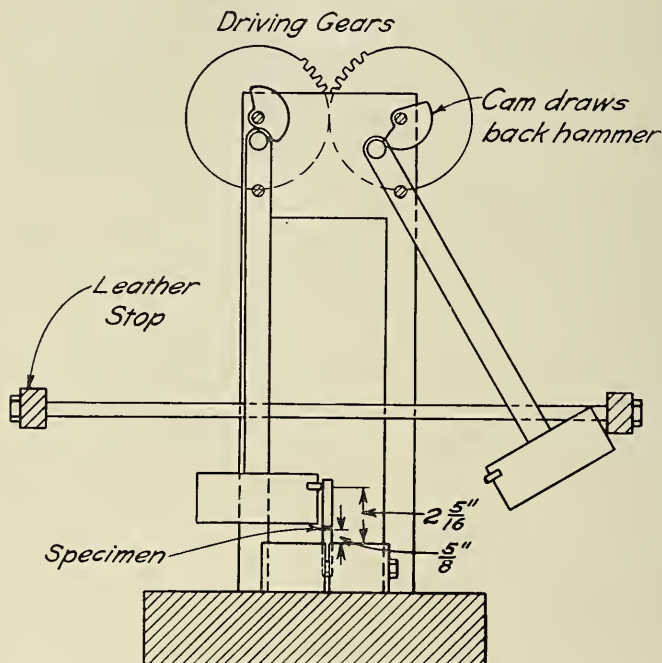


FIG. 14. DIAGRAM OF REPEATED-IMPACT TESTING MACHINE

is made from the ends of the broken Farmer rotating-beam specimens. The energy of each blow delivered is $\frac{1}{3}$ foot-pound, the velocity of the hammer as it strikes the specimen is 3.25 feet per second, and the machine delivers 65 double blows per minute. The number of blows withstood before failure is taken as the measure of the ability of the material to withstand this test. When a specimen breaks, the switch controlling the motor is automatically opened and the machine stopped.

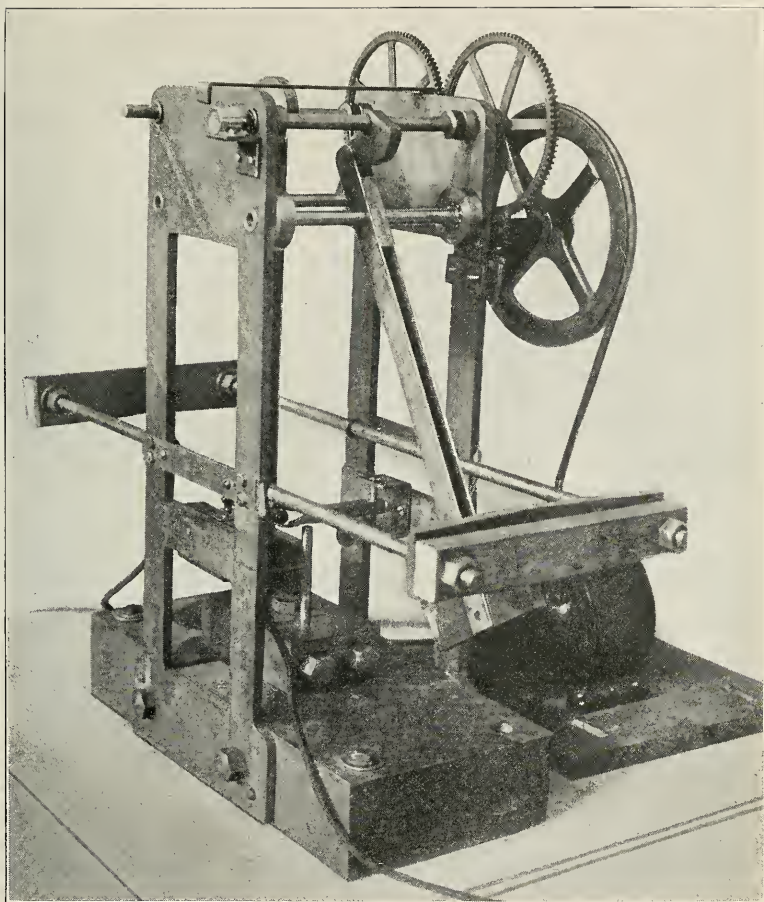


FIG. 13. REPEATED-IMPACT TESTING MACHINE

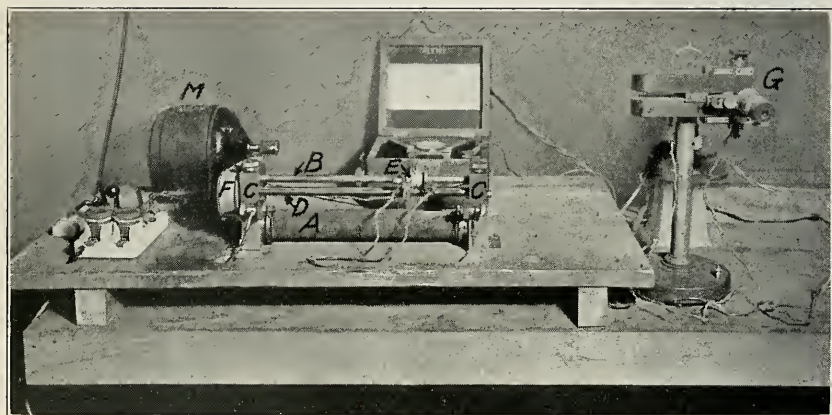


FIG. 15. APPARATUS FOR MAGNETIC TESTS

Fig. 15 shows a photograph of the magnetic testing machine which was used in making the test for homogeneity. This machine was constructed at the University of Illinois. It follows closely designs suggested by Dr. C. W. Burrows of New York, and by R. L. Sanford of the United States Bureau of Standards. The steel core at *A* was wound with wire which carried a current of 7.5 amperes. The magnetic circuit was completed through the specimen, *B*, and the uprights, *C*. At *D* is a screw threading through the small carriage at *E*. When the pulley, *F*, at the end of the screw is driven from the motor, *M*, the carriage, *E*, is moved over the specimen at a uniform speed. The carriage surrounds the specimen and upon the carriage are two coils, differentially wound, in which a small electromotive force is induced when the carriage moves through the magnetic lines of force. From the two coils terminals extend to a string galvanometer, *G*. If the specimen, *B*, is homogeneous in structure and free from both irregularities of form and internal stresses the electromotive force in one coil neutralizes that in the other and there is no deflection of the galvanometer. If there is a non-homogeneous section of the specimen, then, as each coil in turn passes over that section, the balance of the coils is disturbed and deflection of the galvanometer results.* Experiments showed that this device was particularly sensitive in pointing out places in the specimen which had been overstressed. The machine was used, therefore, to determine that the specimens to be tested were homogeneous. Readings were taken of the maximum variation of the galvanometer while the coils were moved across the middle four inches of the specimen. The uniformity of the readings was taken as a criterion of the uniformity of the various specimens.

Fig. 16 shows the detrusion indicator used in the torsion tests for determining the amount of twist in the specimen over a gage length of 2 inches. The arm is 10 inches long from the axis of the specimen to the point of contact of the Ames dial.

Hardness tests were made using both the Brinell machine and the scleroscope. The Brinell machine used was made by the Aktiebolaget Alpha of Stockholm, Sweden. The standard load used was

*For a more complete discussion of this type of apparatus see Scientific Papers of the U. S. Bureau of Standards, No. 343, and the 1917 Proc. of the Am. Soc. for Test. Materials, Part II, p. 87.

3000 kilograms (except for the 0.02 carbon steel, which was so soft that a load of 500 kilograms was used), and the standard diameter of ball was 10 millimeters. The machine indicates the standard load by "floating" a piston which works in its cylinder without packing, and which is loaded with standard weights. The diameter of the

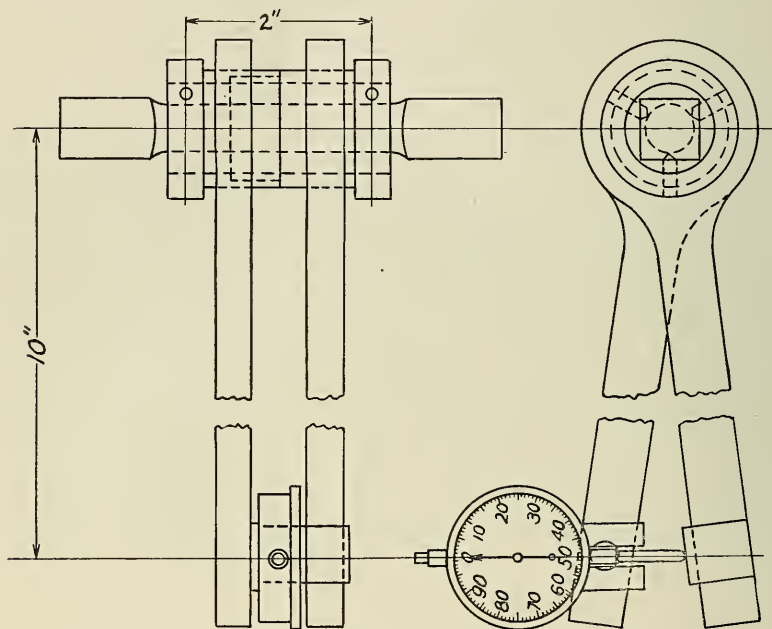


FIG. 16. DETRUSION INDICATOR FOR TORSION TESTS

ball used was measured by means of a micrometer, and found to vary less than 0.0025 millimeter from the standard diameter. The scleroscope used was a standard instrument manufactured by the Shore Instrument and Manufacturing Company of New York. The height of rebound of the hammer is read directly from a scale.

Fig. 17 is from a photograph of the heat-treating equipment used for heat treating most of the specimens. At A is shown a Hoskins electric furnace with inside dimensions 8 by 12 by 26 inches. This furnace was calibrated for uniformity of distribution of heat both empty and full, and it was found that, if specimens were kept in the rear half of the furnace, there was not more than

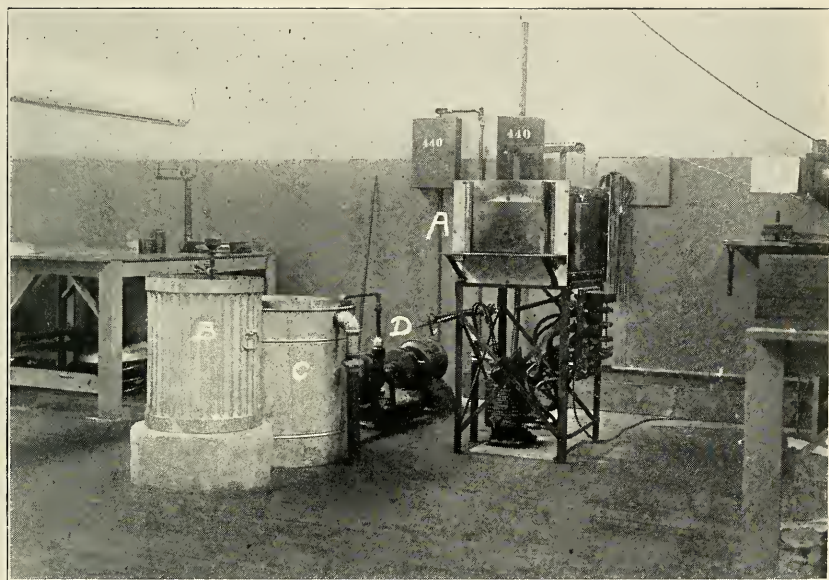


FIG. 17. HEAT TREATING EQUIPMENT

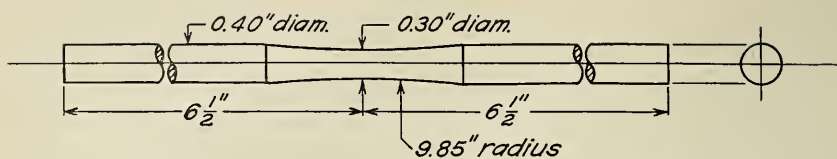
10 degrees centigrade variation of temperature for specimens in various locations in the furnace.

In measuring temperatures, two pyrometers were used, one of which had a platinum-rhodium thermocouple in the center of a dummy specimen, which was placed near the center of the pile of specimens in the furnace. This platinum-rhodium couple was connected to an Engelhard millivoltmeter. A constant-temperature cold junction was used with this pyrometer, and the cold junction was placed in a thermos bottle. The second pyrometer used a chromel-alumel couple connected to a Hoskins millivoltmeter. For this couple a correction was made for the variation of the cold-junction temperature, the cold junction being in the millivoltmeter case. Both pyrometers were calibrated at intervals by means of the freezing points of standard metals. In general, the chromel-alumel pyrometer was used merely as a check on the platinum-rhodium pyrometer.

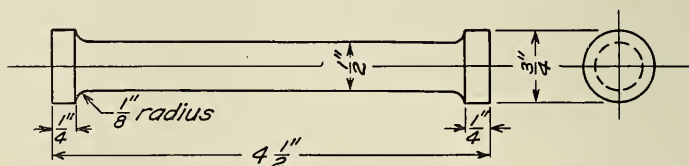
At *B* is shown the water-quenching tank, which during the operation of quenching was supplied with running water. At *C* is shown the oil-quenching tank. Houghton's soluble quenching oil was used, and during the operation of oil-quenching the oil was circulated through pipes which were surrounded by running cold water. At *D* is shown the circulating pump. By this circulation the maximum variation in the temperature of the quenching oil was kept within 17 degrees centigrade.

9. *Test Specimens*.—Figs. 18 and 19 show the various forms of specimens which were used for determining the static and endurance properties of the steels.

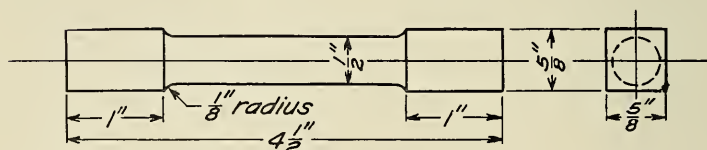
The shape of the specimen used for the rotating-beam reversed-stress machine, shown in Fig. 18(a), was the result of considerable study. It was found in the first place that, if the specimen was not reduced in cross-section at the middle of its length, the localized stresses at the collets where the load was hung would usually cause the specimen to fail at the collets (*F* in Fig. 4). Various shapes were tried for the reduced part of the specimen. For one design a part of the specimen was reduced in diameter, and the change of section from the smaller to the larger was attempted with a taper. For another design a straight reduced section with fillets at the ends was used. It was found that specimens broke at the root of



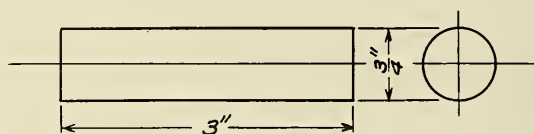
(a), Rotating Beam Specimen, Farmer Type



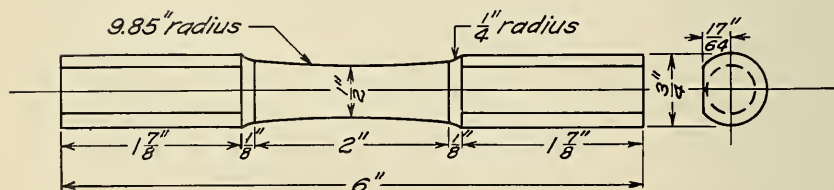
(b), Tension Specimen



(c), Torsion Specimen

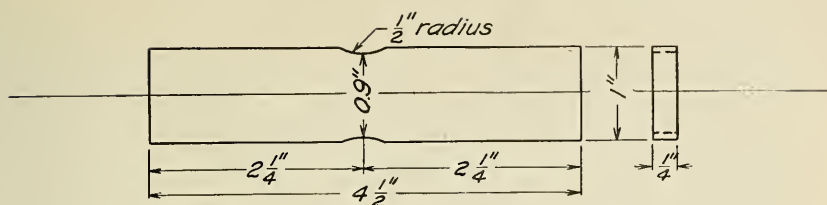


(d), Compression Specimen

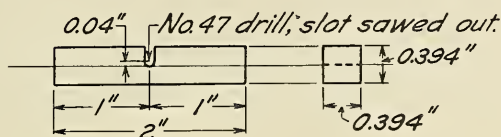


(e), Reversed Torsion Specimen, Olsen-Foster

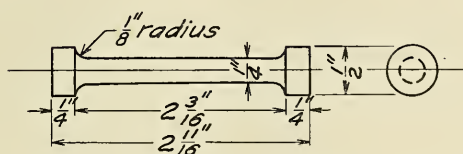
FIG. 18. TEST SPECIMENS



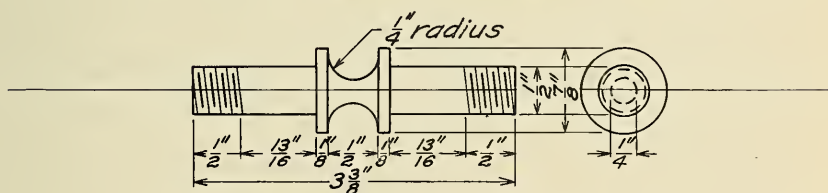
(a), Reversed Bending Specimen, Upton-Lewis



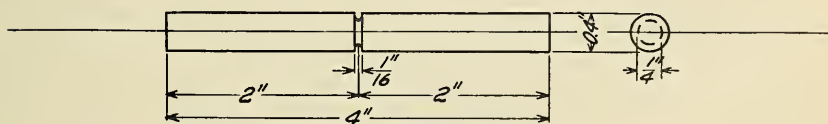
(b), Impact Bending Specimen, Charpy



(c), Impact Tension Specimen, Charpy



(d), Rotating-Beam Specimen, Wisconsin



(e), Repeated Impact Specimen, Double-Hammer

FIG. 19. TEST SPECIMENS

the taper, or at the root of the fillet, as the case might be. It seemed necessary, therefore, to have a specimen which changed gradually from one diameter to the other. In the final design the specimen was formed by a cutting tool swung on a radius of 9.85 inches. For the specimen shown in Fig. 18(a) the unit stress calculated by the usual beam formula gives very closely the actual stress existing at the minimum section, as was proved by some tests on celluloid models made at the laboratories of the General Electric Company at Schenectady, New York, for this investigation. Professor Coker's polarized light apparatus for determining stress was used. These tests showed that even when the radius was only 5 instead of 9.85 inches there was no measurable increase of stress on the outside fibers due to curvature of section. On the other hand, when this radius was only 0.5 inch, it was found that the increase of stress on the outside fiber was considerable. With a radius of 9.85 inches there is practically uniform stress over about 0.20 inch near the middle of the span (Fig. 20), the variation in computed stress, due to change of diameter, in this range of length being only 1 per cent.

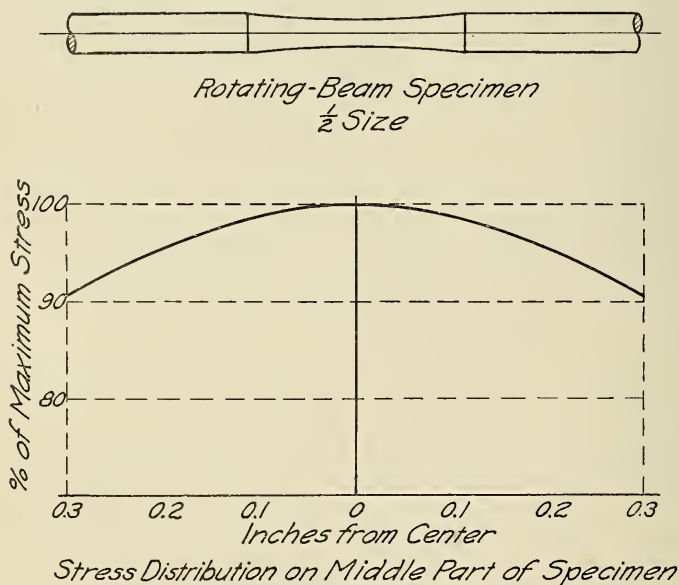


FIG. 20. VARIATION OF STRESS ALONG FARMER-TYPE
TEST SPECIMEN

Usually the diameter at the reduced section was made 0.30 inch, but in the case of the stronger steels it was found that with this diameter they would still break at the collets. It was necessary in some cases, therefore, to reduce the diameter to 0.27 inch.

In the case of the reversed-torsion specimen, shown in Fig. 18(e), exactly the same phenomena were exhibited as described for the rotating-beam specimen. When the ends of the specimens were joined by a fillet to the smaller diameter at the middle, the specimens always failed at the root of the fillet. The specimens were therefore cut down at the middle with the same radius as was used in the case of the rotating-beam specimen.

A similar problem was encountered with the reversed-bending specimens used in the Upton-Lewis machine. In this machine the distance between the jaws holding the specimen is 0.50 inch. The specimen was first cut down with a radius of 0.75 inch, but it continued to break at the edges of the jaws because of the local stresses existing there. It was, therefore, necessary to weaken the specimen further and a radius of 0.5 inch was finally adopted. With this radius the specimens break at the minimum width instead of at the jaw edges. Fig. 19(a) shows the specimen used for the Upton-Lewis machine. Of course with this radius the change of section is rapid enough to produce some localized stress, and this is shown by the fact that the endurance limits (unit stress which the steel will withstand indefinitely) determined by the Upton-Lewis machine are uniformly lower than those obtained from the rotating-beam machine.

Fig. 18 shows the specimens used for the static tests. Fig. 18(b) shows the tension specimen, Fig. 18(c) the torsion specimen, and Fig. 18(d) the compression specimen. In the compression tests a spherical-seated bearing block was used, and the specimen was centered in the machine by means of a template. In the tension tests spherical bearings were also used, and the shoulders of the specimen rested in split sockets.

In some cases tension specimens were made from the ends of rotating-beam specimens. The shape was similar to that of Fig. 18(b) except that the diameter of the reduced part of the specimen was only 0.25 inch. This form of specimen was used for check tests and for material supplied in the form of rods less than 0.75 inch in

diameter. In all tension tests the elongation was measured on a gage length which was four times the diameter of the specimen.

Fig. 19(b) shows the specimen used for the Charpy impact-bending test, while Fig. 19(c) shows the one used for the Charpy impact-tension test. Fig. 19(e) shows the specimen used for the repeated-impact tests. This specimen was made from the uninjured ends of Farmer rotating-beam specimens. The repeated-impact specimen is a grooved cantilever beam.

Brinell tests and scleroscope tests for hardness were made on the ends of the torsion specimens shown in Fig. 18(c).

The magnetic tests were made on the rotating-beam specimens before they were reduced at the center.

10. *Finish of Specimens.*—From previous investigations it was known that surface finish exerted a marked effect on the endurance of a specimen under repeated stress. It was planned, therefore, to use a standard finish which could be duplicated on successive specimens and which would at the same time be of sufficient fineness so that the endurance of the specimen would practically not be reduced because of the finish. The procedure on the rotating-beam specimen for the Farmer machine was as follows: first, to turn the reduced portion at the center with an ordinary lathe tool; next, to file the specimen to remove the deepest tool marks; and finally to polish first with No. 0 and then No. 00 emery cloth.

That this finish was satisfactory is indicated by the uniformity of results, and it is shown in the section on "Effect of Surface Finish" that the polish was sufficiently good to attain uniform and satisfactory results.

The fatigue specimens for the machines other than the Farmer type were finished in a similar manner.

11. *Numbering of Specimens.*—The system adopted for numbering the specimens was designed to make it possible to identify each specimen tested. The original billets or bars, as the case might be, were nearly all cut into 13-inch lengths, because the longest specimen, the rotating-beam fatigue specimen, was 13 inches long. The marking system used may be illustrated by an example. In the mark 5A39C, the numeral 5 showed that this specimen was made

of steel number 5 or chrome-nickel steel; the letter *A*, that the specimen was from bar or billet *A*; the numeral 39, that it was located 39 inches from the end of the bar; and the letter *C*, that the specimen occupied the position *C* in the cross-section of bar *A*, according to the diagram for cutting up the bars which had been adopted for steel number 5.

Fig. 2 shows the plan which was used in cutting up the various steels. Taking Steel No. 4, 0.37 carbon sorbitic, as an illustration, the numbers directly above the diagram show that one set of specimens was cut up from 0 to 13, or 13 inches long; that another set was cut up from 13 to 16, or 3 inches long; another from 16 to 21, or 5 inches long, etc. This was the first steel cut up; all succeeding steels were cut up into 13-inch lengths.

12. *Procedure in Tests—Accuracy and Sensitiveness.*—The regular static tension and compression tests were carried out on a 100 000-pound Riehle machine. This machine was calibrated by means of calibrating levers and dead loads before it was used for testing purposes. It was found that the machine was sensitive to 10 pounds up to 23 000, beyond which it was not tested. The machine was accurate to about 0.6 per cent at 5000 pounds and to 0.4 per cent at 20 000 pounds.

For the static tension and compression tests a Ewing extensometer was used to measure deformation over a gage length of 1.25 inches. This extensometer was calibrated with a Brown and Sharpe micrometer screw, and it was found that one division on the extensometer scale represented a unit deformation of 0.00006925 inch per inch of gage length, or the sensitiveness was 0.000006925 inch per inch by estimation.

The aim in the static tests being to get at least ten increments of strain (as shown by extensometer readings) up to the yield point of the material, the following procedure was adopted. An initial load, which in the tension test was 400 pounds, was put on the specimen and a reading taken on the extensometer. Load was then applied until the extensometer showed the desired increment of deformation, and load and extensometer readings were recorded. The load was then reduced to the initial reading of 400 pounds and the extensometer reading was again recorded. The load was then in-

creased until the extensometer showed the standard increment above the previous reading, and, after reading load and extensometer, the load was again reduced to 400 pounds. This procedure was continued up to the vicinity of the elastic limit, when smaller increments of extension were used—usually about half the initial values—in order to determine the form of the curve more exactly.

The purpose of this procedure was to make a determination of the elastic limit as based on permanent set, as well as the more commonly used limit based upon proportionality of unit stress and unit deformation.

Fig. 21(a) is a sample tension stress-deformation curve which will make clear the manner in which the two elastic limits were determined. Curve *A* is the usual curve showing unit stress as ordinate and unit deformation as abscissa. In determining the proportional elastic limit, an inclined line, *OB*, was drawn corresponding to a rate of deformation 25 per cent greater than that represented by the original curve. This is done by taking any point below the elastic limit and making the offset, *DE*, 25 per cent of the distance *CD*. The inclined line *OB* is then drawn through the point *E*. Next a line parallel to *OB* is drawn tangent to the curve. The unit stress at the point of tangency is taken as the proportional elastic limit.

The attempt is often made to determine the proportional elastic limit by determining the unit stress at which the stress-deformation curve first deviates from a straight line. It is difficult to determine this point accurately, and it seemed desirable to use some such method as was adopted in order to have a tangent line whose deviation from the curve could be detected at two different points. It is then quite easy to determine the unit stress half way between these two deviation points. The rate of deformation 25 per cent greater than the original curve was chosen because it gave a line only slightly less steep than the original curve. It is believed that this method of determining the proportional elastic limit is satisfactory, considering the use which is to be made of this property of the material. This is a slight modification of the method proposed by J. B. Johnson.*

Another consideration in favor of the method is the fact that it is independent of the scale to which a stress-deformation curve is

* Withey and Aston, "Johnson's Materials of Construction," Fifth Edition, p. 10.

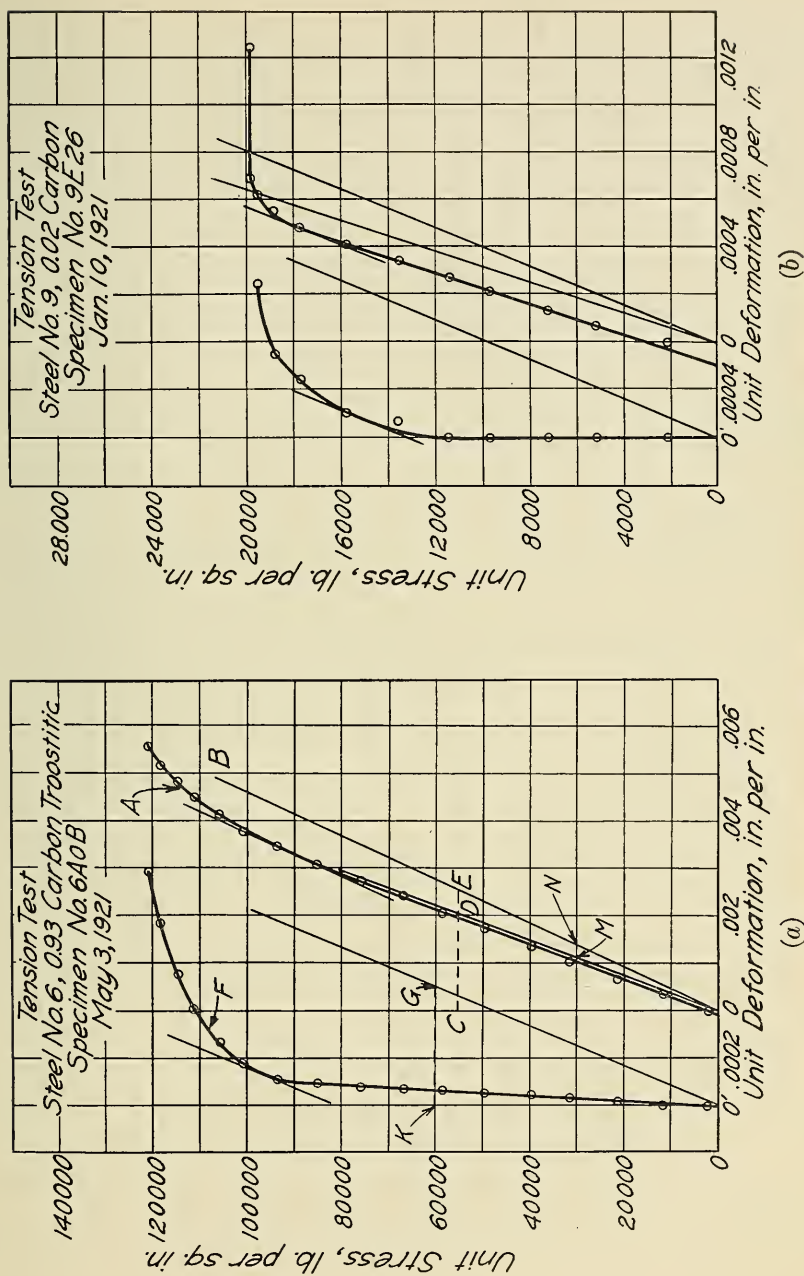


FIG. 21. REPRESENTATIVE STRESS-STRAIN DIAGRAMS FOR TENSION TESTS

drawn. The method will determine in all cases points where the proportional increase in rate of deformation is the same. This is not true for the method which makes use of the first deviation from a straight line; nor is it true of the method which employs a secant drawn parallel to the original curve, and which uses the unit stress at the intersection of the secant line and the curve as the elastic limit.

The elastic limit as based upon set may be called a set elastic limit. The set curve was plotted as shown at *F* in Fig. 21(a), the scale for set being usually about five or ten times that used for the curve *A*. The set elastic limit was taken as that unit stress at which the rate of permanent set increased appreciably. This method was chosen because it was assumed that too much reliance should not be placed on the set as determined at low stresses, but that a point in the curve should be chosen where there was no doubt that permanent set had commenced and was increasing.

The technique used in arriving at this value was based upon the idea of getting an offset for drawing a tangent line which would be about the same as the offset used in getting the inclined line *OB*, Fig. 21(a). Since the modulus of elasticity for steels is practically constant at 30 000 000 pounds per square inch, the inclined line *OB* used with curve *A* has at a unit stress of 30 000 pounds per square inch a horizontal offset from the original curve of 0.00025 inch per inch, *MN* in Fig. 21a, and at 60 000 pounds per square inch the offset would of course be 0.0005 inch per inch. The line for zero set would be a vertical line from *O'*. At a unit stress of 60 000 on the set curve, therefore, a point, *G*, was determined which had an offset, *KG*, from the line of zero set of 0.0005 inch per inch. A line through *O'* and *G* determined the inclined line, and one parallel to this and tangent to the curve determined the elastic limit as based upon set.

Tables 3 and 4 show that the proportional elastic limit and the set elastic limit as determined by the above methods are very nearly the same in magnitude in tension, compression, and torsion. In some cases the proportional limit is slightly the higher, and in other cases the set limit is the higher. Since there is so little difference in these two limits it seems obvious that the tedious method necessary to determine the set limit may be dispensed with in future tests of steels.

In the torsion tests readings of twisting moment and twist were taken in a manner similar to that already described for the tension and compression tests. The proportional elastic limits and the set elastic limits were also determined in a manner similar to that previously described. The micrometer dial of the detrusion indicator shown in Fig. 16 reads directly to 0.001 inch, and the reading can be estimated to 0.0001 inch. With this indicator, therefore, the twist of the specimen over two inches gage length can be read to 0.0001 radian directly, and to 0.00001 radian by estimation.

From the fatigue tests the fundamental information desired was the relation between the unit stress applied to the specimen and the number of cycles of stress necessary to cause rupture. The general practice with the Farmer machines was to stress the first specimen high enough so that it would break in a comparatively short time; then reduce the stress in the succeeding specimens, until finally a unit stress was arrived at which the steel could withstand for 100 000 000 cycles without failure. Wherever possible these long-time tests without failure were run on from three to five specimens for each of the steels investigated.

All values of stress calculated for the Farmer specimens are based upon the external load applied to the specimen, the weight of the specimen itself being neglected.

Only one Upton-Lewis and one Olsen-Foster machine were available. Consequently it was not practicable to make runs up to 100 000 000 cycles. In these machines, therefore, the stress was determined at which the specimen could withstand 2 000 000 cycles without failure. It will be found by consulting the curves which will be shown later that this determines the endurance limit fairly closely.

The Brinell and the scleroscope hardness tests were made on the rectangular surfaces of the uninjured ends of the static torsion specimens, the impressions being at right angles to the axis of the specimen. In each case impressions were taken on two adjacent sides, and in getting the diameter of the Brinell impression a microscope was used with a micrometer eyepiece which read to 0.1 millimeter directly, and to 0.01 millimeter by estimation.

It is not thought necessary to describe in detail the technique of impact tests or of magnetic tests.

III. TEST DATA AND RESULTS

13. *Summary of Test Data and Results.*—Tables 3, 4, and 5 give summaries of test results. Table 3 gives the results of the tension tests, and also the endurance limits as determined from the rotating-beam tests, so that comparisons may be conveniently made. Table 4 gives the results of the compression and the torsion tests. Table 5 gives the results of the hardness and the impact tests and also the endurance limits as determined by the rotating-beam, by the Upton-Lewis reversed-bending, and by the Olsen-Foster reversed-torsion machines. The last column in the table gives the “FR” point as determined by the Francke* test.

Table 6 shows a sample summary sheet, in this case for treatment A of the chrome-nickel steel. This table shows the number of results which were averaged in obtaining the results given in Tables 3, 4, and 5. Each Brinell and scleroscope result given in Table 6 is the average of several readings. Figs. 21-23 inclusive show representative stress-strain diagrams for tension, compression, and torsion tests.

Figs. 24-31 inclusive show the test results obtained on the Farmer machines for reversed-stress tests of the various steels. The method used in plotting these diagrams is discussed on page 90. Figs. 32 and 33 show similar diagrams obtained from the tests on the Upton-Lewis reversed-bending machine; while Figs. 34 and 35 show the diagrams for the Olsen-Foster reversed-torsion machine.

Fig. 28 shows S - N curves obtained in making a study of the effect of changes of shape on the endurance strength of steel. Fig. 29 shows similar curves obtained in making a study of the effect of surface finish on the endurance strength; while Figs. 30 and 31 show the S - N curves obtained in making a study of the effect of previous over-stressing on endurance strength.

Table 7 gives the results for all fatigue tests so far made in the course of the investigation.

The “rise of temperature” tests for endurance limit under reversed stress are treated in a special chapter, and the data for those tests are given in that chapter.

*Proc. Am. Soc. for Test. Materials, Vol. XX, Part II, p. 372, 1920.

TABLE 3
RESULTS OF TENSION TESTS

No.	STEEL	TENSION						Endurance Limit
		Set Elastic Limit	Proportional Elastic Limit	Yield Point	Ultimate	% of Elongation	% of Reduction of Area	
9	0.02 carbon, as received.....	14 400	16 100	19 000	42 400	48.3	76.2	26 000
51	Hot-rolled, as received.....	39 600	38 200	40 800	61 500	41.0	66.7	28 000
51	Hot-rolled, reduced to 0.48 in.....	60 100	60 000	63 800	67 600	22.3	63.3	35 000
51	Hot-rolled, reduced to 0.44 in.....	70 500	69 600	None	73 400	14.2	59.5	41 000
51	Hot-rolled, bent cold and straightened.....							30 000
50	Cold-drawn, as received.....	52 200	55 200	None	86 800	13.8	49.3	41 000
50	Cold-drawn, annealed at 1300° F.....	25 900	26 700	29 500	56 600	41.3	65.5	29 000
50	Cold-drawn, annealed at 1550° F.....	25 600	28 000	30 000	57 700	40.8	63.2	25 000
4	0.37 carbon, normalized.....	33 500	34 500	34 900	71 900	29.4	53.5	33 000
4	0.37 carbon, sorbitic, treatment A.....	82 000	80 600	87 800	102 600	23.3	65.1	57 000
4	0.37 carbon, sorbitic, treatment B.....	60 300	61 500	63 100	94 200	25.0	63.0	45 000
10	0.49 carbon, sorbitic.....	67 300	67 700	69 700	96 900	23.5	57.8	48 000

TABLE 3 (Continued)
RESULTS OF TENSION TESTS

No.	STEEL	TENSION						Endurance limit
		Set Elastic Limit	Proportional Elastic Limit	Yield Point	Ultimate	% of Elongation	% of Reduction of Area	
3	0.52 carbon, normalized.....	47 200	45 400	47 600	98 000	24.4	41.7	42 000
3	0.52 carbon, sorbitic.....	79 400	80 300	84 300	111 400	21.9	56.6	55 000
6	0.93 carbon, pearlitic.....	25 500	28 000	33 400	84 100	24.8	37.2	30 500
6	0.93 carbon, sorbitic.....	60 700	60 300	67 600	115 000	23.0	39.6	56 000
6	0.93 carbon, troostitic.....	102 200	97 200	None	188 300	9.9	29.3	98 000
1	1.20 carbon, normalized.....	59 100	58 600	60 700	116 900	7.9	11.6	50 000
1	1.20 carbon, sorbitic.....	119 600	120 400	130 100	179 900	9.0	15.2	92 000
7	3.5 nickel, treatment B.....	85 500	82 400	91 100	111 800	23.6	60.2	63 000
5	Chrome-nickel, treatment A.....	117 200	115 500	128 100	138 700	18.2	61.8	68 000
5	Chrome-nickel, treatment B.....	99 700	101 700	103 700	113 300	24.2	68.7	65 000
5	Chrome-nickel, treatment C.....	86 300	86 200	100 500	114 200	23.2	69.3	67 000

TABLE 4
RESULTS OF COMPRESSION TESTS AND TORSION TESTS

No.	STEEL	COMPRESSION				TORSION			
		Set Elastic Limit	Proportional Elastic Limit	Yield Point	Ultimate	Set Elastic Limit	Proportional Elastic Limit	Yield Point	Modulus of Elasticity
9	0.02 carbon, as received.....	19 400	19 200	20 600	31 200	12 400	12 500	13 600	11 770 000
4	0.37 carbon, normalized.....	36 800	36 300	38 100	59 200	19 800	20 300	22 500	11 790 000
4	0.37 carbon, sorbitic, treatment A.....	77 100	75 800	84 500	102 800	50 900	51 600	60 200	11 960 000
10	0.49 carbon, sorbitic.....	57 500	55 900	60 100	76 500	37 200	36 800	None	12 150 000
3	0.52 carbon, normalized.....	48 700	47 800	51 000	78 700	30 000	30 000	34 600	11 720 000
3	0.52 carbon, sorbitic.....	85 600	84 400	87 400	97 800	53 100	52 200	None	12 110 000
6	0.93 carbon, pearlitic.....	22 800	23 500	29 700	69 200	16 600	15 600	22 500	11 880 000
6	0.93 carbon, sorbitic.....	66 700	64 800	72 700	97 100	42 300	42 000	None	12 130 000
6	0.93 carbon, troostitic.....	105 300	106 500	None		77 500	75 200	None	11 960 000
1	1.20 carbon, normalized.....	55 400	55 300	57 900	96 500	36 400	36 500	39 700	11 700 000
1	1.20 carbon, sorbitic.....	104 400	102 700	111 500	149 200	82 700	80 600	None	11 940 000
7	3.5 nickel, treatment B.....	85 600	86 400	94 000	95 100	56 000	55 500	None	11 890 000
5	Chrome-nickel, treatment A.....	123 900	122 600	130 100	133 000	72 800	72 400	84 900	11 510 000
5	Chrome-nickel, treatment B.....	98 200	97 800	100 000	100 300	63 400	62 500	None	11 980 000
5	Chrome-nickel, treatment C.....	91 600	91 500	97 400	99 900	62 200	62 300	None	12 020 000

TABLE 5
RESULTS OF HARDNESS TESTS, IMPACT TESTS, FRANCKE TESTS, AND REVERSED-STRESS TESTS

No.	STEEL	Hardness		Impact Bend- ing	Impact Tension			Endurance Limits			FR Bend- ing	Re- peated Impact
		Brinell	Sclero- scope	Energy of Rupture ft. lb.	Energy of Rupture ft. lb.	% of Elonga- tion	% of Reduction of Area	Farmer Rotat- ing Beam	Upton- Lewis Re- versed Bend- ing	Olsen- Foster Re- versed Torsion	FR Point in Francke Test	Number of Double Blows
9	0.02 carbon, as received.....	69	18	19.3	84.9	27.0	72.7	26 000	23 000	12 500		260
51	Hot-rolled, as received.....							28 000				242
51	Hot-rolled, reduced to 0.48 in.....							35 000				280
51	Hot-rolled, reduced to 0.44 in.....							41 000				361
51	Hot-rolled, bent cold and straightened.....							30 000				
50	Cold-drawn, as received.....							41 000				316
50	Cold-drawn, annealed at 1300° F.....							29 000				202
50	Cold-drawn, annealed at 1550° F.....							25 000				223
4	0.37 carbon, normalized.....	132	18	15.8	168.4	32.4	52.7	33 000	30 000	16 000	33 000	183
4	0.37 carbon, sorbitic, treatment A.....	209	26	53.2	163.2	24.0	62.8	57 000		32 500		550
4	0.37 carbon, sorbitic, treatment B.....							45 000			44 000	503
10	0.49 carbon, sorbitic.....	197	23	22.5	152.3	24.0	55.1	48 000	39 000	26 000		319

TABLE 5 (Continued)
RESULTS OF HARDNESS TESTS, IMPACT TESTS, FRANCKE TESTS, AND REVERSED-STRESS TESTS

No.	STEEL	Hardness		Impact Bending ft. lb.	Impact Tension				Endurance Limits			Francke Bending in	Re- peated Impact
		Brinell	Sclero- scope		Energy of Rupture ft. lb.	% of Elonga- tion	% of Reduc- tion of Area	Farmer Rotat- ing Beam	Upton- Lewis Re- versed Bend- ing	Olsen- Foster Re- versed Torsion			
3	0.52 carbon, normalized.....	193	24	13.2	188.5	24.2	45.6		42 000	32 000		39 500	275
3	0.52 carbon, sorbitic.....	227	30	21.4	172.0	22.2	55.1		55 000	44 000		66 000	486
6	0.93 carbon, pearlitic.....	162	23	2.2	141.3	21.2	34.5		30 500	28 500	16 300	34 000	66
6	0.93 carbon, sorbitic.....	227	31	3.3	193.2	21.6	43.9		56 000	44 000			128
6	0.93 carbon, troostitic.....	380	51	4.4	186.8	15.0	41.3		98 000		52 000		227
1	1.20 carbon, normalized.....	224	31	1.9	88.8	8.8	14.6		50 000	45 000			39
1	1.20 carbon, sorbitic.....	369	45	2.0	109.1	6.3	11.0		92 000				172
7	3.5 nickel, treatment B.....	242	28	41.0	179.9	22.8	57.4		63 000				524
5	Chrome-nickel, treatment A.....	291	36	45.4	173.8	17.0	61.7		68 000	52 000	37 000	62 000	1073
5	Chrome-nickel, treatment B.....	247	28	53.8	175.3	22.2	63.2		65 000		31 500		714
5	Chrome-nickel, treatment C.....	246	29	56.0	187.6	23.4	67.0		67 000				722

TABLE 6
 REPRESENTATIVE SUMMARY SHEET
 Tests of Chrome-Nickel Steel, Treatment A

TENSION TESTS						COMPRESSION TESTS					
Specimen Number	Set Elastic Limit lb. per sq. in.	Proportional Elastic Limit lb. per sq. in.	Yield Point lb. per sq. in.	Ultimate lb. per sq. in.	Elongation per cent	Reduction of Area per cent	Specimen Number	Set Elastic Limit lb. per sq. in.	Proportional Elastic Limit lb. per sq. in.	Yield Point lb. per sq. in.	Ultimate lb. per sq. in.
5C26C				140 600	16.5	59.8	5C26C	126 500	125 000	132 700	136 700
5D13D	120 000	116 000	133 700	140 600	20.0	62.7	5D13D	128 000	126 500	138 000	140 000
5A91B	121 000	121 000	128 400	140 300	19.0	62.3	5A91B	127 000	124 000	130 300	133 900
5B65A	118 000	119 500	124 900	136 600	17.5	61.3	5B65A	121 000	120 000	127 200	131 100
5A39A	113 000	114 000	126 200	138 100	17.5	62.2	5A39A	122 500	120 000	126 700	131 000
5C78A	114 000	107 000	127 500	135 900	19.0	62.4	5C78A	118 300	120 000	125 700	125 700
Average	117 200	115 500	128 100	138 700	18.2	61.8	Average	123 900	122 600	130 100	133 000

TABLE 6 (*Continued*)
 REPRESENTATIVE SUMMARY SHEET
 Tests of Chrome-Nickel Steel, Treatment A

TORSION TESTS					HARDNESS	
Specimen Number	Set Elastic Limit lb. per sq. in.	Proportional Elastic Limit lb. per sq. in.	Yield Point lb. per sq. in.	Modulus of Elasticity lb. per sq. in.	Brinell	Scleroscope
5C26C	72 500	72 000	93 750	11 500 000	302	36
5D13D	77 500	78 000	86 100	11 380 000	302	36
5A91B	74 500	72 500	84 800	11 640 000	293	36
5B65A	72 500	70 500	83 200	11 630 000	289	36
5A39A	70 000	72 000	80 750	11 470 000	273	37
5C78A	69 500	69 500	80 500	11 420 000	286	35
Average	72 800	72 400	84 900	11 510 000	291	36

CHARPY IMPACT BENDING				CHARPY IMPACT TENSION				REPEATED IMPACT	
Specimen Number	Energy of Rupture ft. lb.	Specimen Number	Energy of Rupture ft. lb.	Specimen Number	Energy of Rupture ft. lb.	Elongation per cent	Reduction of Area per cent	Specimen Number	Number of Double Blows
5B78B	46.1	5B78B	47.8	5B78B	186.8	7.0	63.7	5D13A	1359
-5A104B	47.1	5A104B	45.4	5A104B	181.2	24.0	63.4	5A26A	1214
-5A26B	45.2	5A26B	46.9	5A26B	150.4	17.0	62.3	5B0B	1099
5C39C	44.9	5C39C	45.4	5C39C	180.4	22.0	61.7	5A117A	886
5C91C	41.8	5C91C	43.2	5C91C	170.3	15.0	57.2	5A130A	806
Average			45.4		173.8	17.0	61.7		1073

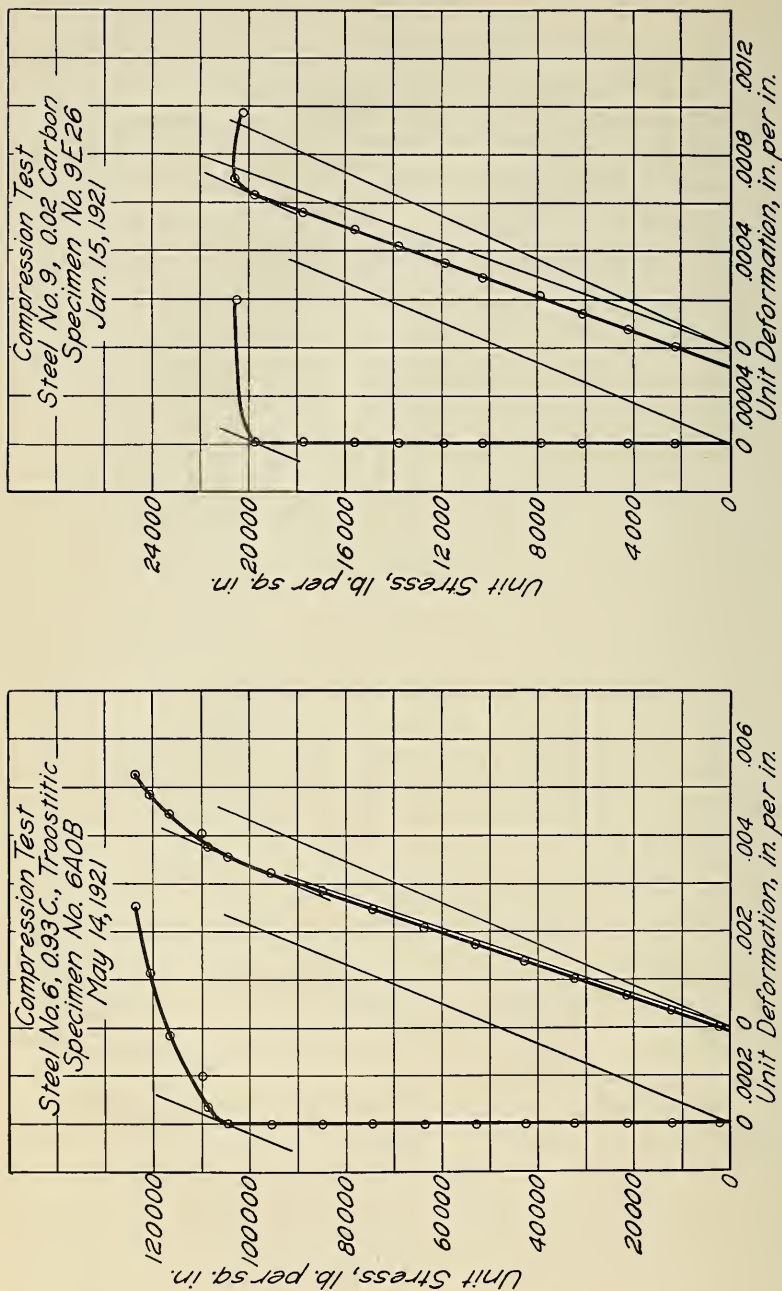


FIG. 22. REPRESENTATIVE STRESS-STRAIN DIAGRAMS FOR COMPRESSION TESTS

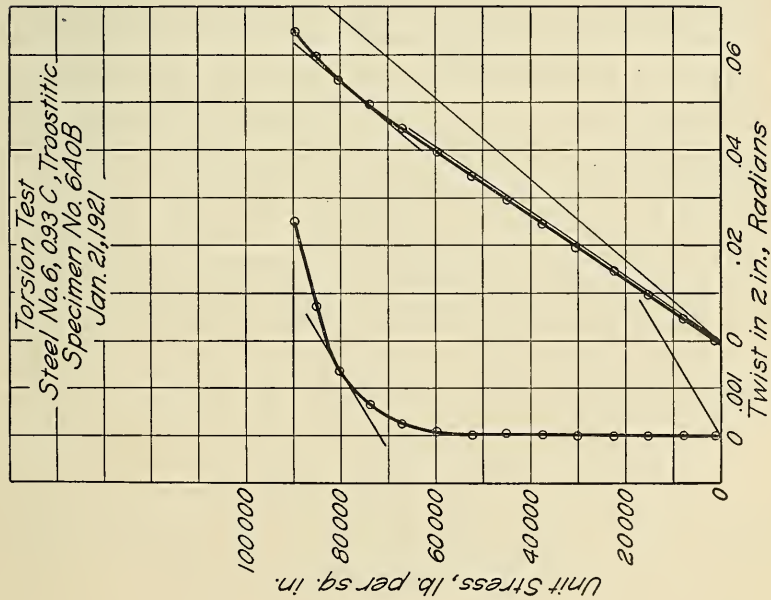
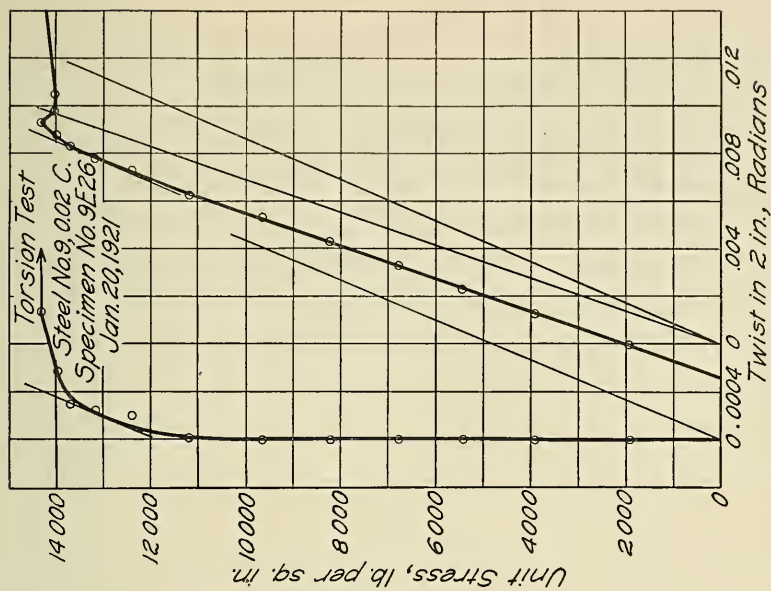


FIG. 23. REPRESENTATIVE STRESS-STRAIN DIAGRAMS FOR TORSION TESTS

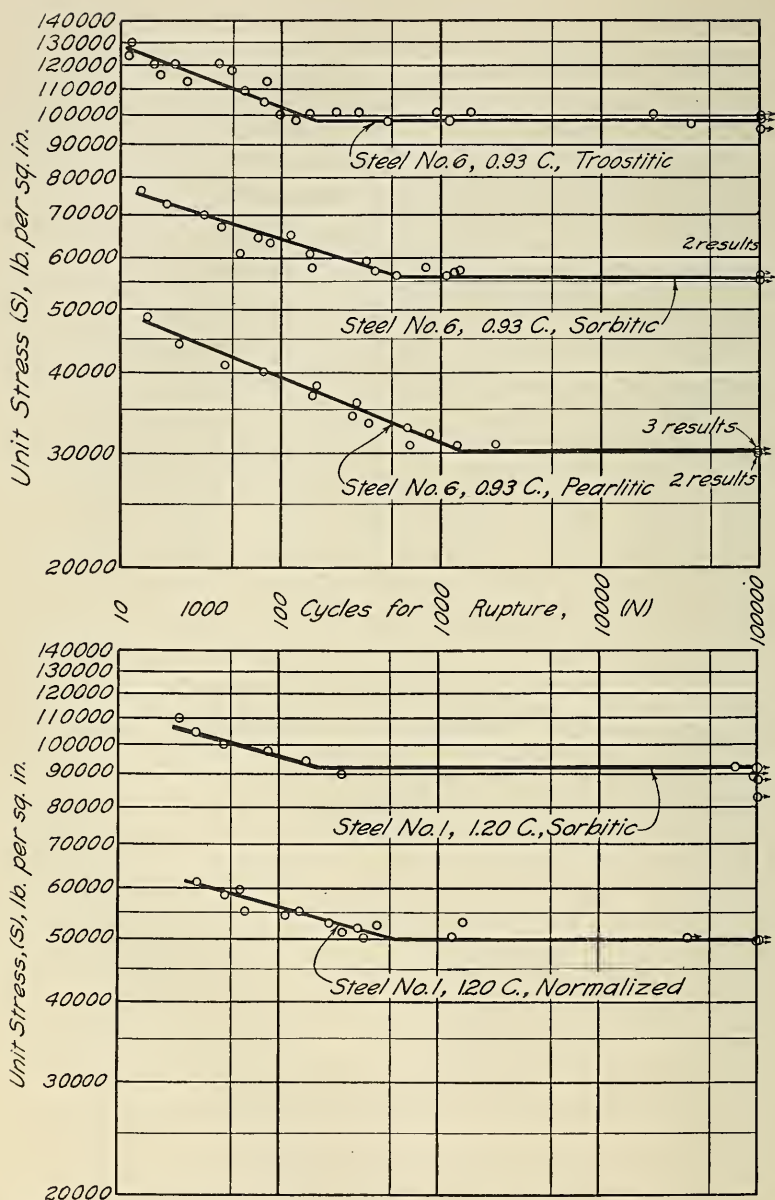


FIG. 24. *S-N* DIAGRAMS FOR 0.93 CARBON STEEL AND FOR 1.20 CARBON STEEL, ROTATING-BEAM TESTS

Logarithmic abscissas give thousands of cycles for rupture

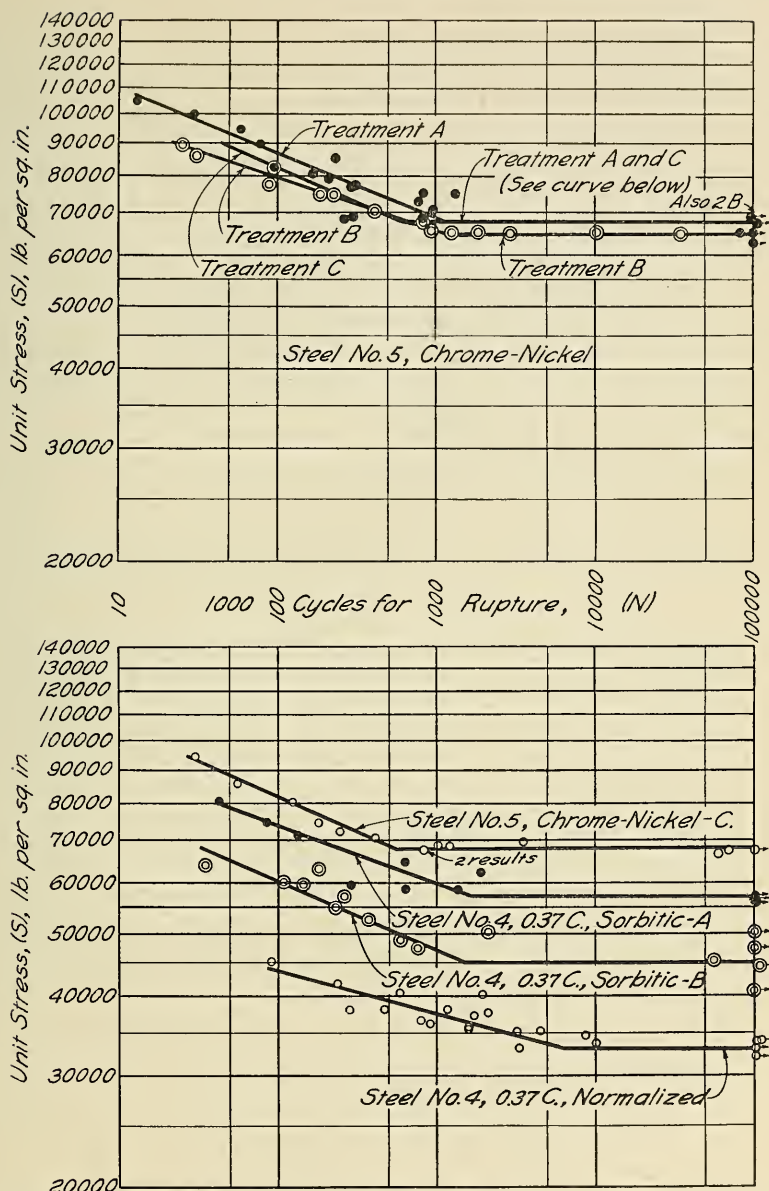


FIG. 25. *S-N* DIAGRAMS FOR CHROME-NICKEL STEEL AND FOR 0.37 CARBON STEEL, ROTATING-BEAM TESTS

Logarithmic abscissas give thousands of cycles for rupture

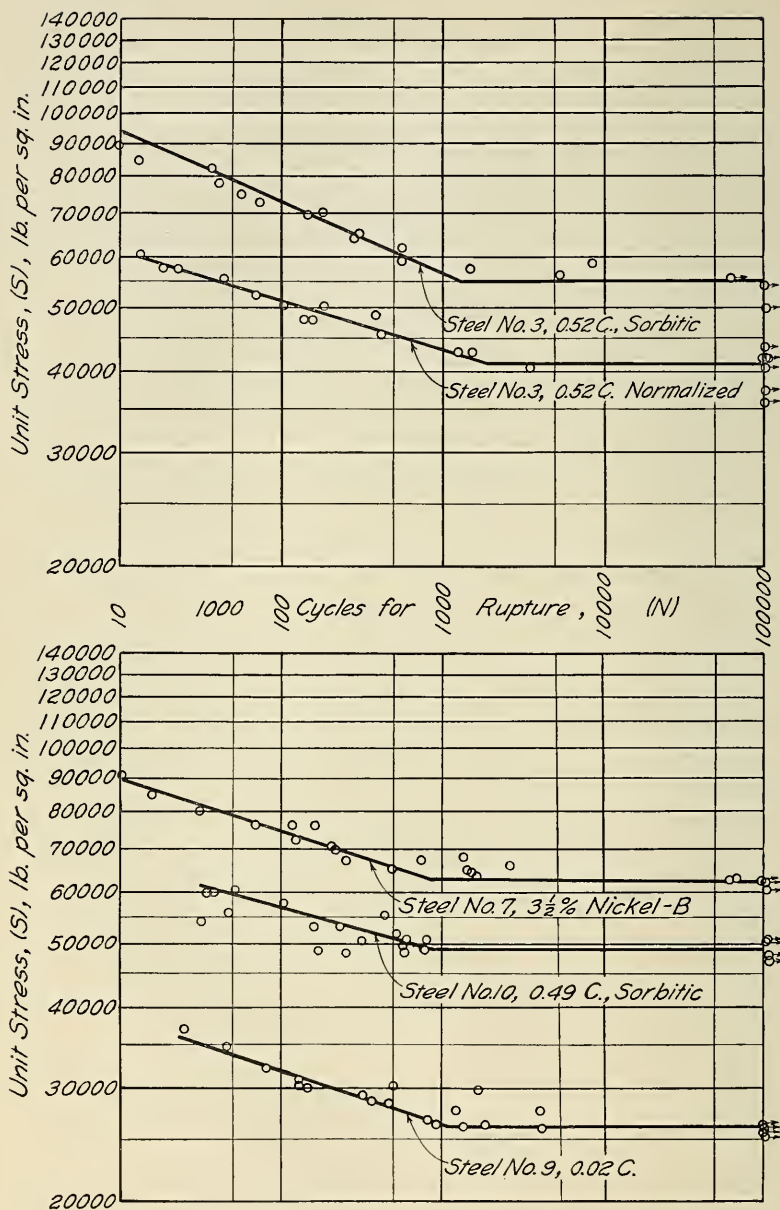


FIG. 26. *S-N* DIAGRAMS FOR 0.52 CARBON STEEL, 0.49 CARBON STEEL, 0.02 CARBON STEEL, AND 3.5 NICKEL STEEL, ROTATING-BEAM TESTS
Logarithmic abscissas give thousands of cycles for rupture

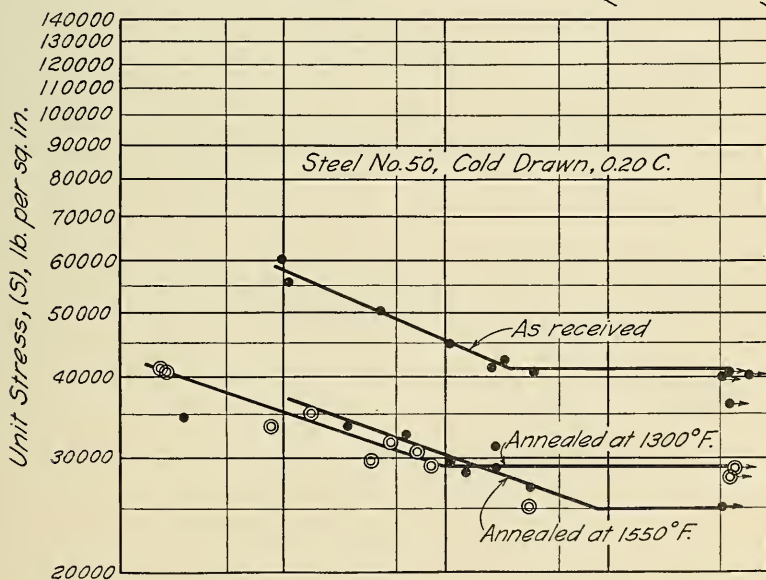
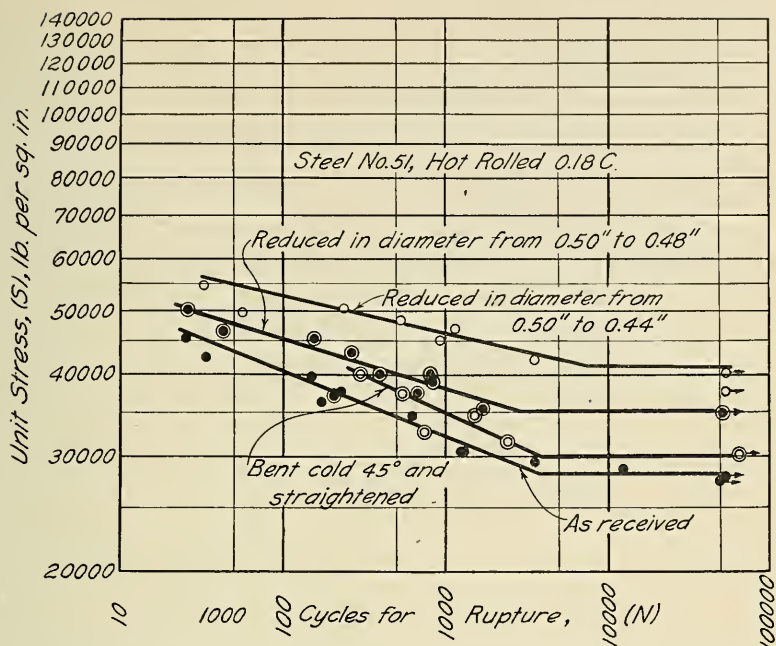


FIG. 27. *S-N* DIAGRAMS FOR HOT-ROLLED STEEL AND FOR COLD-DRAWN STEEL, ROTATING-BEAM TESTS
 Logarithmic abscissas give thousands of cycles for rupture

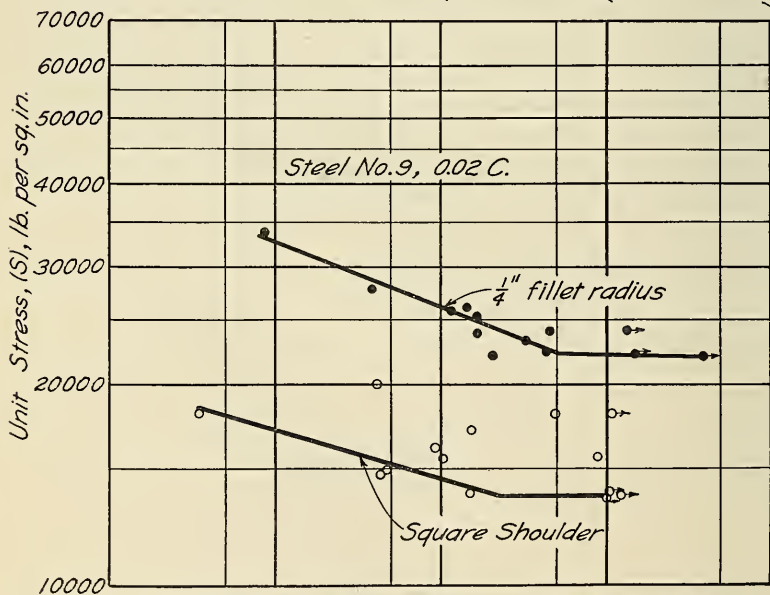
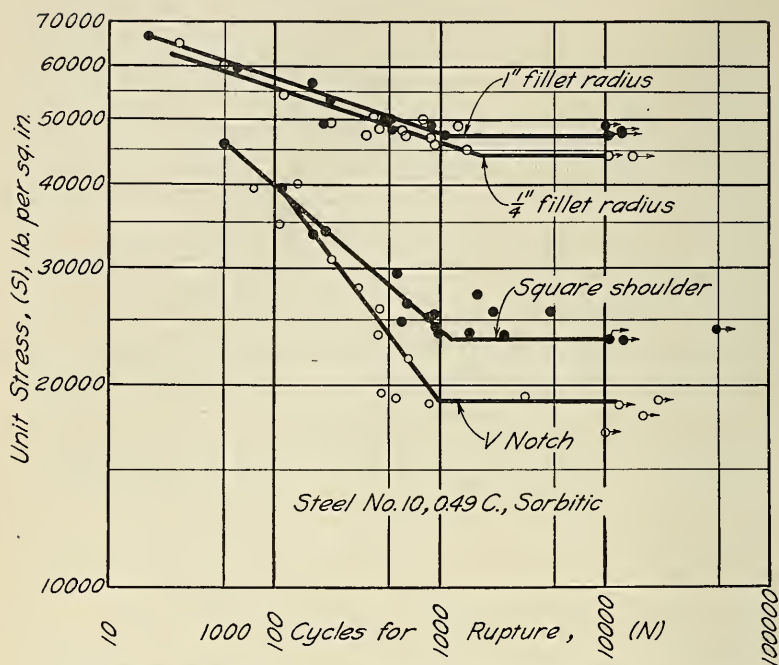


FIG. 28. S-N DIAGRAMS SHOWING EFFECT OF SHAPE OF SPECIMEN, ROTATING-BEAM TESTS

Logarithmic abscissas give thousands of cycles for rupture

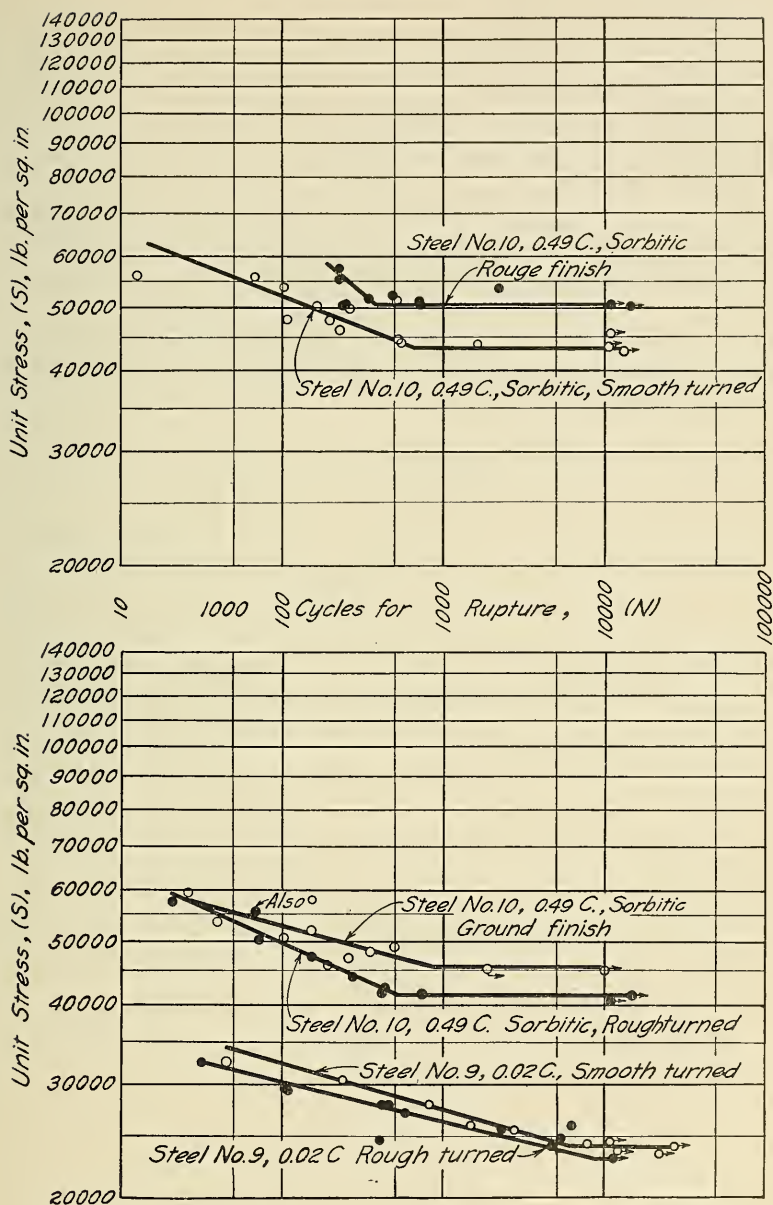


FIG. 29. S-N DIAGRAMS SHOWING EFFECT OF SURFACE FINISH, ROTATING-BEAM TESTS

Logarithmic abscissas give thousands of cycles for rupture

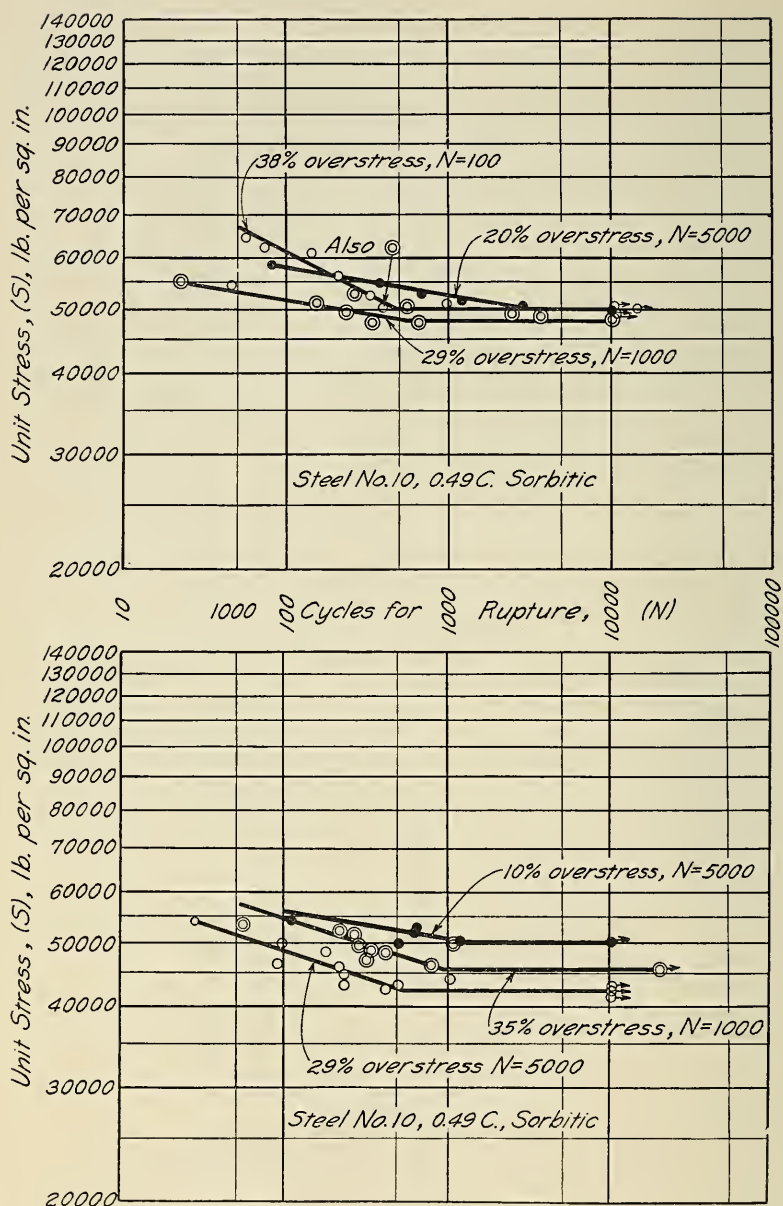


FIG. 30. S - N DIAGRAMS SHOWING EFFECT OF OVERSTRESS, 0.49 CARBON STEEL, ROTATING-BEAM TESTS
Logarithmic abscissas give thousands of cycles for rupture

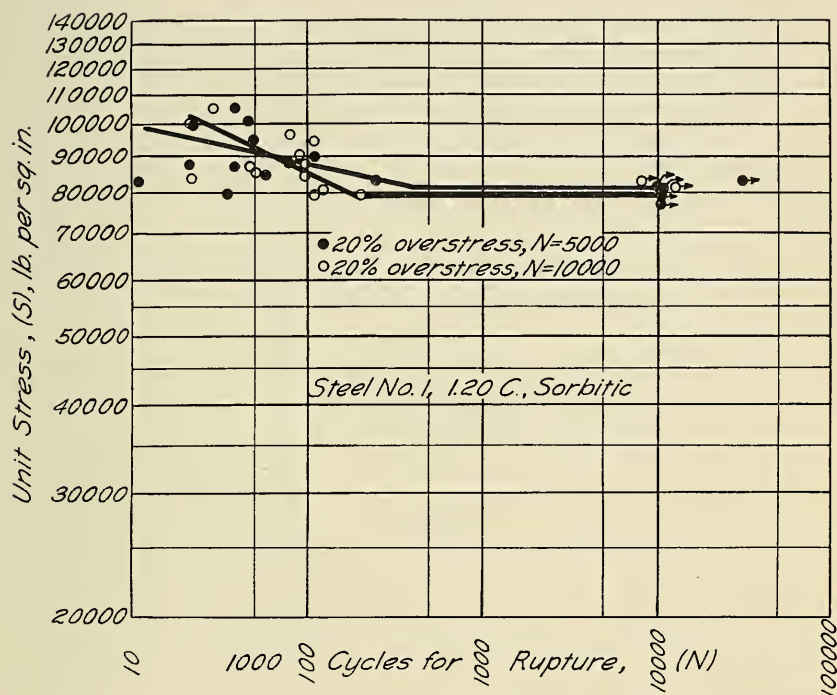


FIG. 31. S-N DIAGRAMS SHOWING EFFECT OF OVERSTRESS, 1.20 CARBON STEEL, ROTATING-BEAM TESTS

Logarithmic abscissas give thousands of cycles for rupture

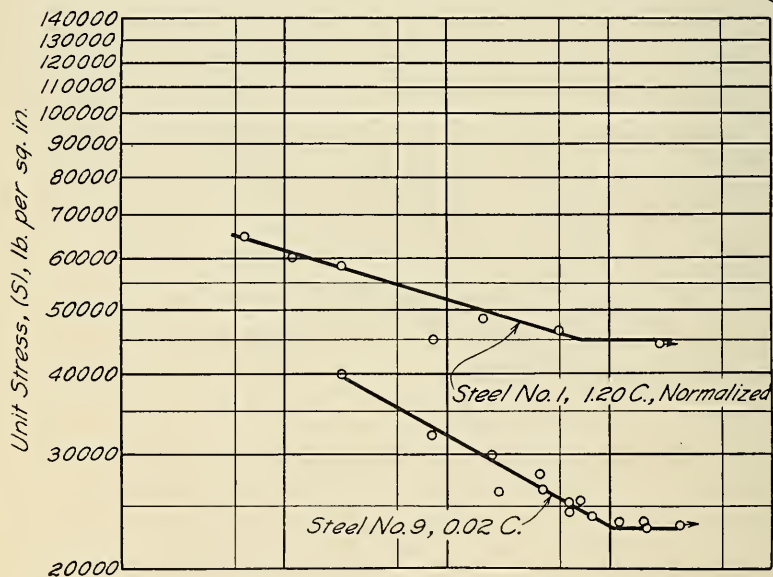
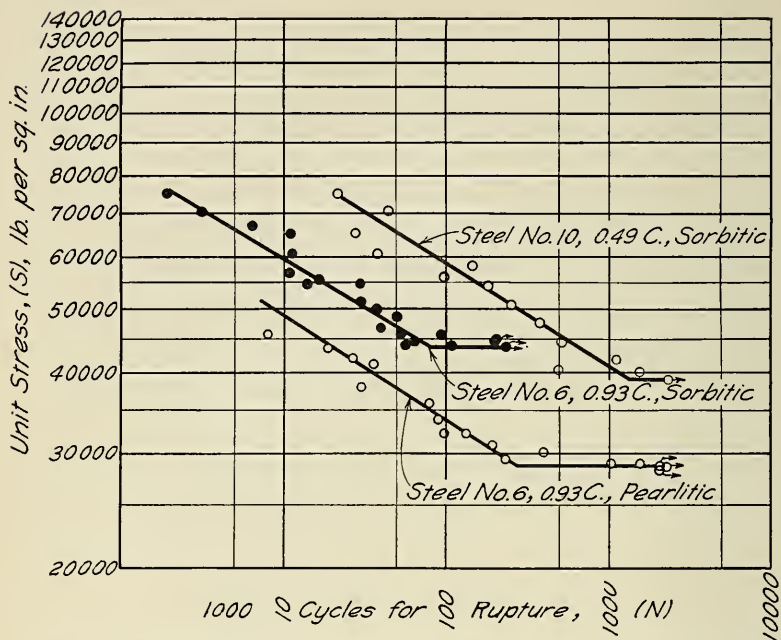


FIG. 32. S-N DIAGRAMS, UPTON-LEWIS TESTS, 0.93 CARBON STEEL, 1.20 CARBON STEEL, 0.49 CARBON STEEL, AND 0.02 CARBON STEEL
Logarithmic abscissas give thousands of cycles for rupture

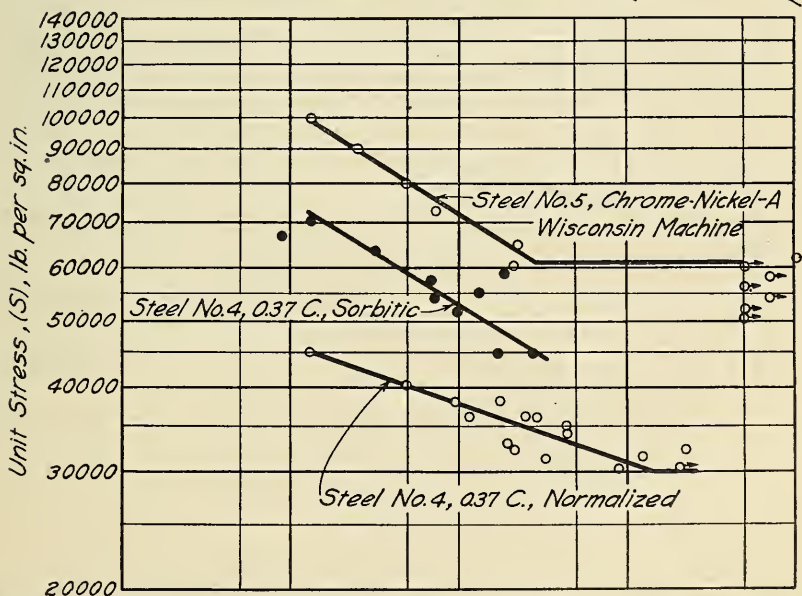
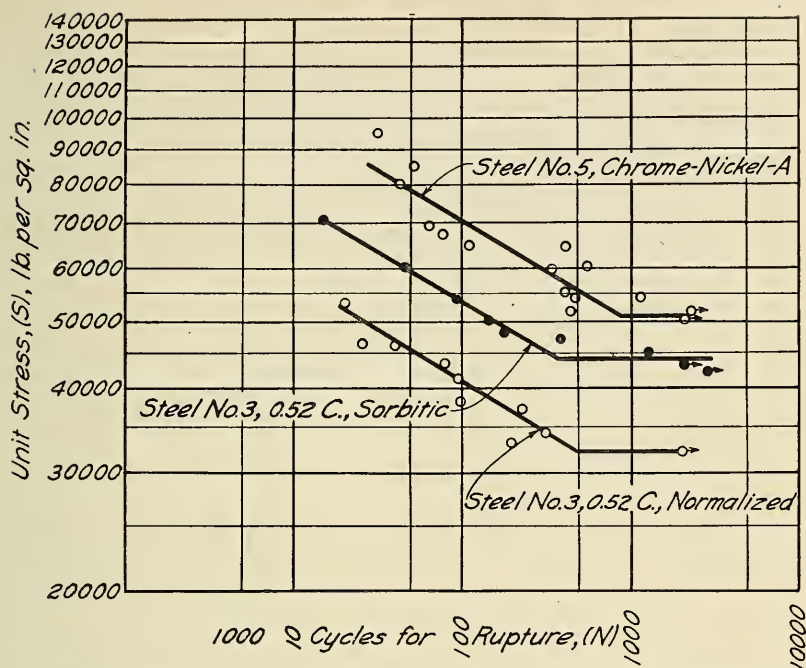


FIG. 33. *S-N* DIAGRAMS, UPTON-LEWIS TESTS, 0.52 CARBON STEEL, 0.37 CARBON STEEL, AND CHROME-NICKEL STEEL: TESTS ON WISCONSIN MACHINE, CHROME-NICKEL STEEL

Logarithmic abscissas give thousands of cycles for rupture

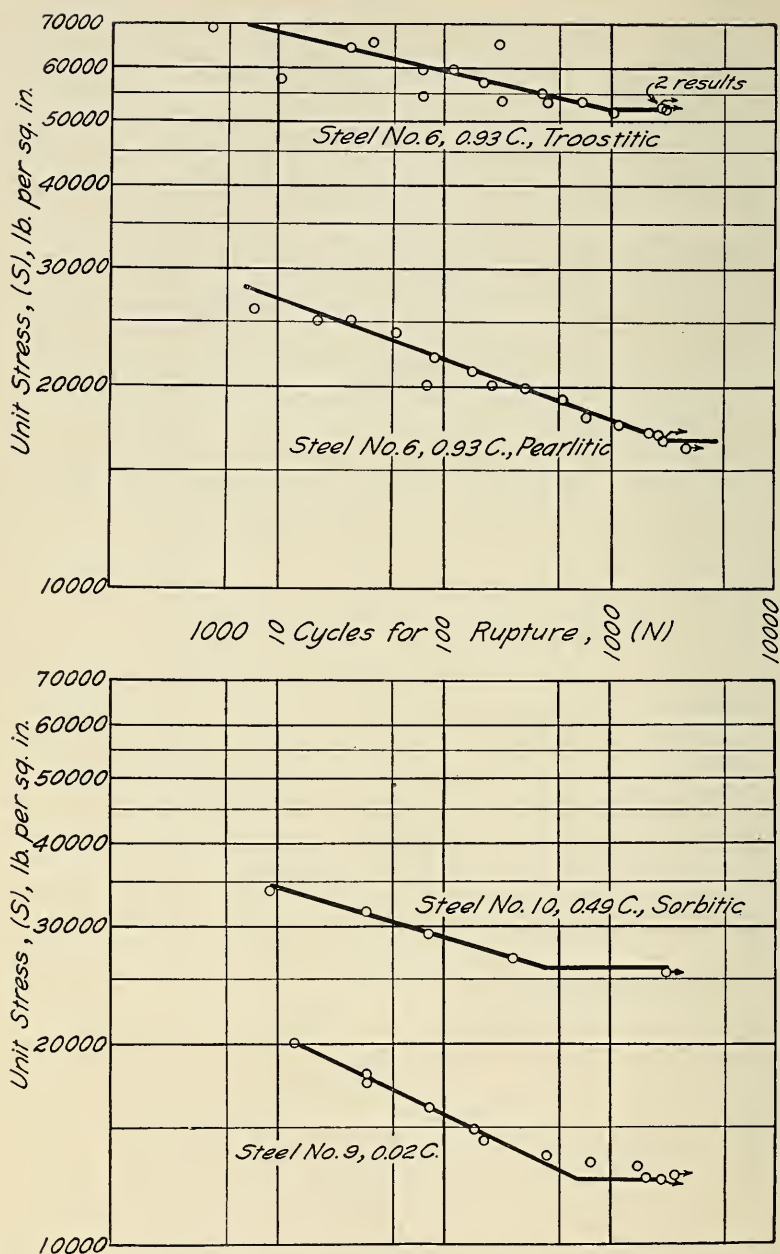


FIG. 34. *S-N* DIAGRAMS, OLSEN-FOSTER TESTS, 0.93 CARBON STEEL, 0.02 CARBON STEEL, AND 0.49 CARBON STEEL
Logarithmic abscissas give thousands of cycles for rupture

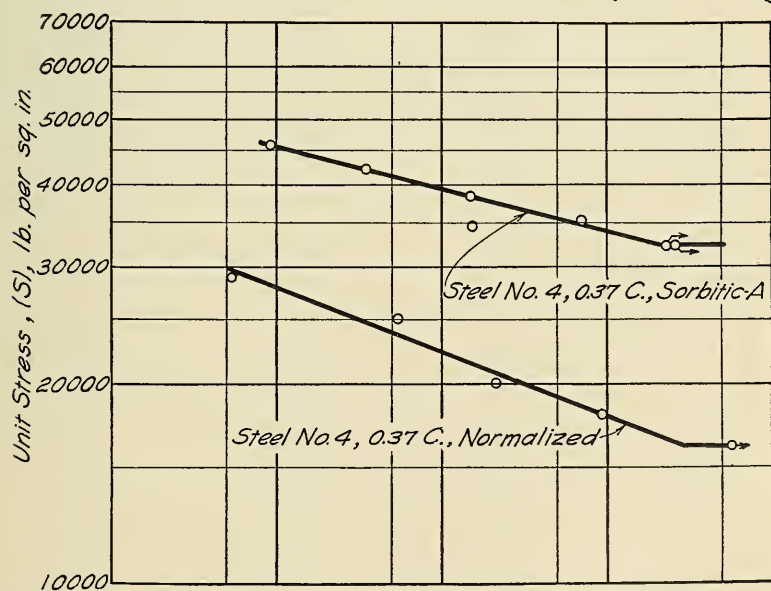
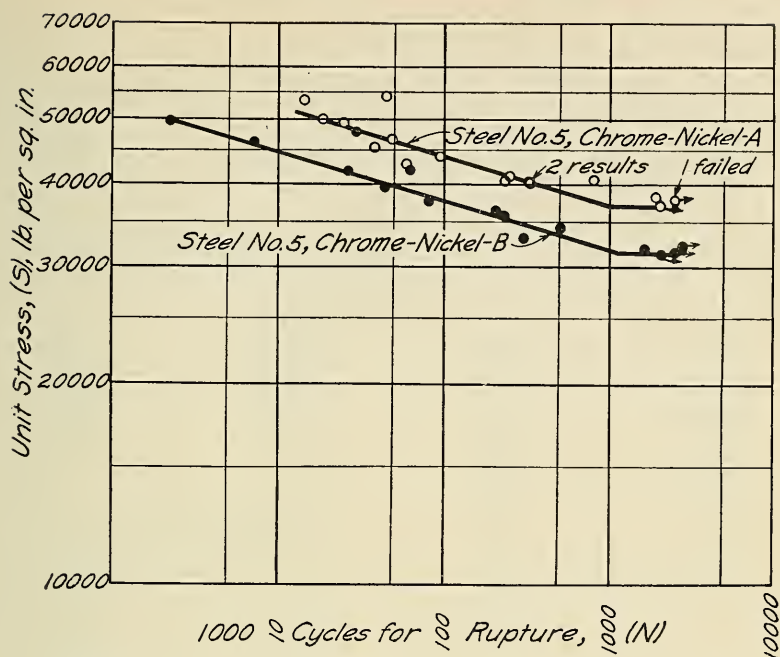


FIG. 35. S-N DIAGRAMS, OLSEN-FOSTER TESTS, 0.37 CARBON STEEL, AND CHROME-NICKEL STEEL

Logarithmic abscissas give thousands of cycles for rupture

TABLE 7

S-N RESULTS FOR REVERSED-STRESS TESTS

(1) Reversed-Bending Tests on Rotating-Beam Machine (Farmer Type)

Unit Stress lb. per sq. in. S	Cycles for Rupture in Thousands N	Unit Stress lb. per sq. in. S	Cycles for Rupture in Thousands N
Steel No. 1, 1.20 Carbon, Normalized		Steel No. 1, 1.20 Carbon, Sorbitic 20% Overstress, S=110 300 N = 5 000 (Cont'd)	
93 200	74 626†	83 200	10
63 500	1†	83 200	249
61 500	30	82 800	31 366*
59 600	58	81 600	10 139*
58 400	45	80 300	10 968*
55 000	62	79 900	35
55 000	139	79 100	10 855*
54 300	113†	76 600	10 720*
53 800	4†		
53 200	1 420	Steel No. 1, 1.20 Carbon, Sorbitic 20% Overstress, S=110 300 N = 10 000	
52 900	122		
52 800	27†		
52 600	400		
52 100	317		
51 100	252		
50 300	36 250*		
50 200	1 226		
50 100	345		
49 800	103 596*		
49 700	101 319*	99 900	22
Steel No. 1, 1.20 Carbon, Sorbitic		96 200	82
110 000	24	94 100	112
105 000	30	90 000	92
100 200	45	87 000	48
98 100	87	85 200	52
95 100	149	83 900	97
92 000	100 330*	82 800	8 336*
90 100	248	81 900	22
89 900	99 707*	80 900	12 917*
89 100	106 265*	80 100	126
81 800	102 005*	79 000	111
		79 000	206
Steel No. 1, 1.20 Carbon, Sorbitic 20% Overstress, S=110 300— N = 5 000		Steel No. 3, 0.52 Carbon, Normalized	
105 300	39	60 400	13
100 200	46	57 700	19
99 900	22	57 500	23
95 000	49	55 400	44
89 900	110	52 300	69
88 000	80	50 500	105
87 900	22	50 300	182
86 800	39	48 800	159
84 700	59	48 000	139
		47 500	380
		45 400	411
		44 000	749
		43 800	100 170*
		42 800	1 537
		42 700	1 251

* Specimen did not break.

† Specimen bent.

TABLE 7 (Continued)

S-N RESULTS FOR REVERSED-STRESS TESTS

(I) Reversed-Bending Tests of Rotating-Beam Machine (Farmer Type)

Unit Stress lb. per sq. in. <i>S</i>	Cycles for Rupture in Thousands <i>N</i>	Unit Stress lb. per sq. in. <i>S</i>	Cycles for Rupture in Thousands <i>N</i>
Steel No. 3, 0.52 Carbon, Normalized (Cont'd)		Steel No. 4, 0.37 Carbon, Normalized (Cont'd)	
42 200	4 124†	35 700	1 572
41 900	100 000*	35 400	1 536
41 600	100 168*	35 100	3 154
41 900	112 944*	35 100	4 463
40 400	3 544	34 700	8 478
40 400	102 332*	33 900	106 588*
37 300	102 738*	33 900	107 569*
35 800	100 614*	33 800	9 937
		33 200	103 602*
		33 100	3 248
		32 000	101 415*
Steel No. 3, 0.52 Carbon, Sorbitic		Steel No. 4, 0.37 Carbon, Sorbitic Treatment A	
89 800	10	81 400	44
85 000	13	74 700	85
82 200	37	71 600	136
78 100	40	64 700	631
75 400	57	62 100	1 926
73 000	73	59 600	292
70 200	183	58 800	641
69 500	148	58 700	1 364
65 500	303	57 200	100 965*
64 000	282	56 200	94 060*
62 000	562	55 600	90 179*
59 000	559		
58 400	8 790		
57 500	1 527		
56 200	5 540		
55 800	63 265*		
54 100	103 387*		
49 800	108 987*		
Steel No. 4, 0.37 Carbon, Normalized		Steel No. 4, 0.37 Carbon, Sorbitic Treatment B	
49 100	1†	80 000	3
48 800	0†	69 900	6
45 100	90	64 100	35
43 900	4†	63 000	180
42 200	4†	60 200	108
42 000	234	59 800	144
40 400	572	57 200	262
40 000	0†	55 100	237
40 000	4†	52 600	373
40 000	1 861	50 500	2 103
39 900	17†	50 000	101 403*
38 100	280	49 000	585
38 100	457	47 200	766
38 100	1 175	47 100	101 566*
37 700	2 027	45 300	56 550
37 300	1 667	44 200	108 518*
36 600	770	40 300	100 456*
36 200	884		

* Specimen did not break.

† Specimen bent.

TABLE 7 (Continued)

S-N RESULTS FOR REVERSED-STRESS TESTS

(I) Reversed-Bending Tests of Rotating-Beam Machine (Farmer Type)

Unit Stress lb. per sq. in. <i>S</i>	Cycles for Rupture in Thousands <i>N</i>	Unit Stress lb. per sq. in. <i>S</i>	Cycles for Rupture in Thousands <i>N</i>
Steel No. 5, Chrome-Nickel, Treatment A		Steel No. 5, Chrome-Nickel, Treatment C	
120 000	5	102 100	4
111 300	6	95 100	31
105 300	13	86 000	56
100 100	30	80 800	127
95 200	59	75 000	180
89 800	76	72 600	245
85 800	230	70 600	405
83 100	95	69 800	3 514
80 800	168	68 800	1 027
79 600	212	68 300	1 209
77 600	306	67 500	818
76 900	239	67 500	68 441
75 600	1 305	67 500	100 662*
75 300	807	67 300	817
73 100	765	66 500	58 281
70 200	945		
69 200	298		
68 800	99 908*		
68 600	272		
68 500	824		
68 200	829		
68 000	1 360		
67 800	100 735*		
67 200	105 669*		
65 200	101 602*		
65 200	83 364		
62 800	101 239*		
Steel No. 5, Chrome-Nickel, Treatment B		Steel No. 6, 0.93 Carbon, Pearlitic	
89 800	25	48 800	15
86 000	30	44 500	24
83 000	94	42 000	36
77 700	88	40 000	80
74 700	186	38 100	177
74 700	223	36 900	162
70 100	405	35 700	301
67 900	810	34 200	290
65 900	907	33 400	361
65 500	1 818	32 800	643
65 200	1 227	32 200	881
65 200	2 962	31 100	1 302
64 700	34 314	30 800	662
64 500	102 200*	30 800	2 270
64 500	103 979*	30 500	100 125*
		30 500	103 677*
		30 300	101 190*
		29 900	101 176*
		29 900	102 483*

* Specimen did not break.

† Specimen bent.

TABLE 7 (Continued)

S-N RESULTS FOR REVERSED-STRESS TESTS

(I) Reversed-Bending Tests of Rotating-Beam Machine (Farmer Type)

Unit Stress lb. per sq. in. S	Cycles for Rupture in Thousands N	Unit Stress lb. per sq. in. S	Cycles for Rupture in Thousands N
Steel No. 6, 0.93 Carbon, Sorbitic		Steel No. 6, 0.93 Carbon, Troostitic (Cont'd)	
80 000	8	98 000	470
76 600	14	98 000	1 136
73 000	20	97 000	36 840
70 000	34	95 200	100 853*
67 300	43		
65 400	118	Steel No. 7, 3.5 Nickel, Treatment B	
64 400	74	91 100	10
63 400	86	86 000	5
61 000	56	85 200	16
61 000	154	84 900	5
59 400	350	80 500	31
58 000	810	76 600	68
58 000	161	76 600	117
57 600	1 370	76 600	165
57 200	396	72 600	125
57 000	1 232	71 400	210
56 900	101 335*	69 900	219
56 800	100 869*	68 000	1 359
56 400	538	67 500	736
56 300	1 116	67 200	251
56 200	104 095*	66 200	2 616
56 000	100 504*	65 200	492
		64 900	1 411
		64 200	1 530
		63 800	1 616
		63 200	62 217
		63 200	69 506
		62 600	99 608*
		62 100	103 329*
		62 100	102 955*
		60 400	106 278*
		Steel No. 9, 0.02 Carbon, as Received	
		37 900	15
		34 700	46
		34 100	2†
		33 200	14†
		32 100	83
		30 900	131
		30 100	130
		30 000	502
		29 700	141
		29 700	1 758
		29 200	326
		28 700	369
		28 400	475
		27 600	4 022
		27 400	1 255
		26 700	313†
		26 700	814
Steel No. 6, 0.93 Carbon, Troostitic			
132 700	4†		
131 300	12		
131 000	4		
127 700	7		
127 700	8		
124 100	11		
120 500	22		
120 100	16		
120 100	41		
117 800	49		
115 100	18		
112 900	26		
112 800	81		
109 000	59		
106 400	0†		
105 300	78		
101 300	1 532		
101 100	302		
101 000	220		
101 000	936		
100 100	21 528		
100 000	149		
99 900	95		
99 400	104 325*		
99 000	100 900*		
98 000	122		

* Specimen did not break.

† Specimen bent.

TABLE 7 (Continued)

S-N RESULTS FOR REVERSED-STRESS TESTS

(I) Reversed-Bending Tests of Rotating-Beam Machine (Farmer Type)

Unit Stress lb. per sq. in. S	Cycles for Rupture in Thousands N	Unit Stress lb. per sq. in. S	Cycles for Rupture in Thousands N
Steel No. 9, 0.02 Carbon, as Received (Cont'd)		Steel No. 9, 0.02 Carbon, Square Shoulder	
26 200	2†	23 300	0†
26 200	944	22 000	0†
26 200	1 869	20 000	408
26 100	104 506*	18 600	416†
26 000	1 392	18 200	35†
25 900	4 251	18 100	4 912
25 900	100 576*	18 100	10 774*
25 400	104 871*	17 100	1 551
25 100	104 175*	16 000	914
		15 500	1 019
		15 500	8 891
		14 800	470
		14 700	434
		14 200	295†
		13 600	1 492
		13 600	12 828*
		13 600	12 234*
		13 400	10 000*
		13 700	10 077*
Steel No. 9, 0.02 Carbon, Rough Turned		Steel No. 9, 0.02 Carbon, 1/4" Radius	
32 300	31	34 000	87
29 700	104	32 300	4†
29 200	110	28 700	14†
27 900	406	27 800	393
27 900	453	27 000	22†
27 200	576	26 000	1 418
25 900	6 329	25 800	1 116
25 600	2 291	25 300	1 615
25 000	434†	24 500	46†
24 900	5 424	24 000	4 646
24 600	407	24 000	13 486*
24 100	578†	23 800	1 625
24 000	4 836	23 300	3 251
23 100	11 372*	22 800	4 378
		22 300	173†
		22 000	15 129*
		22 000	2 062
		22 000	39 129*
Steel No. 9, 0.02 Carbon, Smooth Turned		Steel No. 10, 0.49 Carbon, Sorbitic	
33 500	4†	64 500	2†
32 300	45	64 400	18†
30 300	236	62 000	5†
27 900	806	60 300	52†
25 900	1 480	60 000	38
25 400	2 756	59 900	35
24 900	2 042†		
24 400	7 781		
24 400	10 880*		
24 000	899†		
24 000	25 936*		
23 600	2 325†		
23 600	12 256*		
23 500	21 937*		

* Specimen did not break.

† Specimen bent.

TABLE 7 (Continued)

S-N RESULTS FOR REVERSED-STRESS TESTS

(I) Reversed-Bending Tests of Rotating-Beam Machine (Farmer Type)

Unit Stress lb. per sq. in. S	Cycles for Rupture in Thousands N	Unit Stress lb. per sq. in. S	Cycles for Rupture in Thousands N
Steel No. 10, 0.49 Carbon, Sorbitic (Cont'd)		Steel No. 10, 0.49 Carbon, Sorbitic, Ground Finish	
57 700	101	59 600	26
56 000	161	56 200	69
56 000	48†	53 800	40
55 300	432	52 100	155
54 100	32†	50 800	102
53 100	229	49 000	499
51 800	514	48 300	357
50 800	793	47 100	260
50 800	584	46 000	2 776
50 500	315	45 700	2 615
50 500	101 099*	45 900	192
50 300	101 934*	45 300	10 266*
49 500	568	42 200	8 691*
49 000	167		
49 000	766		
48 800	580		
48 500	252		
48 000	100 940*		
47 800	106 282*		
47 100	106 235*		
Steel No. 10, 0.49 Carbon, Sorbitic, Rough Turned		Steel No. 10, 0.49 Carbon, Sorbitic, Rouge Finish	
57 900	21	64 800	2†
55 400	68	57 600	230
50 500	72	55 100	230
47 400	154	53 500	2 203
44 600	274	52 100	494
42 200	430	51 800	354
41 800	417	51 100	721
41 400	744	50 700	243
41 300	15 156*	50 700	12 603*
40 800	11 167*	50 700	727
		50 400	246
		50 000	14 532*
Steel No. 10, 0.49 Carbon, Sorbitic, Smooth Turned		Steel No. 10, 0.49 Carbon, Sorbitic, Square Shoulder	
56 000	13	46 000	50
55 700	69	39 700	112
53 700	104	34 000	204
51 400	523	33 700	174
50 300	169	29 300	551
49 900	265	27 400	1 729
48 000	111	26 500	650
47 900	201	25 900	4 726
46 000	232	25 800	2 161
45 500	11 223*	25 400	908
44 600	534	25 300	862
44 100	554	25 000	530
43 900	1 666	24 600	950
43 600	10 647*	24 300	48 290*
42 700	13 534*	24 000	1 543
		24 000	987
		23 800	2 428
		23 500	10 734*
		23 300	13 131*

* Specimen did not break.

† Specimen bent.

† Specimen bent.

TABLE 7 (Continued)

S-N RESULTS FOR REVERSED-STRESS TESTS

(I) Reversed-Bending Tests of Rotating-Beam Machine (Farmer Type)

Unit Stress lb. per sq. in. <i>S</i>	Cycles for Rupture in Thousands <i>N</i>	Unit Stress lb. per sq. in. <i>S</i>	Cycles for Rupture in Thousands <i>N</i>
Steel No. 10, 0.49 Carbon, Sorbitic, 29% Overstress, <i>N</i> = 1000, <i>S</i> = 62 000 (Cont'd)		Steel No. 50, Cold-drawn, as Received	
50 300	392	70 000	6
49 500	236	60 300	97
49 300	2 430	55 600	108
49 000	3 279	50 300	397
48 400	10 140*	45 000	1 073
47 800	340	42 200	2 284
47 800	650	41 100	1 958
		40 700	56 976*
		40 400	3 480
		40 300	8 919
		40 000	74 956*
		40 000	51 376*
		36 200	57 606*
Steel No. 10, 0.49 Carbon, Sorbitic, 29% Overstress, <i>N</i> = 5000, <i>S</i> = 62 000		Steel No. 50, Cold-drawn, Annealed at 1300° F.	
54 000	28	41 200	18
50 000	100	40 800	19
48 800	183	35 000	147
47 200	29†	33 500	84
46 500	90	31 600	457
46 000	221	30 500	680
44 900	234	29 700	356
44 100	1 091	29 000	818
43 100	10 738*	28 900	61 754*
43 100	514	27 900	59 628*
42 900	238	25 600	5 289†
42 700	433	25 000	3 229
42 300	10 057*		
41 800	10 620*		
Steel No. 10, 0.49 Carbon, Sorbitic, 20% Overstress, <i>N</i> = 5000, <i>S</i> = 60 600		Steel No. 50, Cold-drawn, Annealed at 1550° F.	
58 500	84	34 700	25
55 000	383	33 400	248
52 900	684	32 500	571
51 300	1 220	31 200	2 042
50 500	2 883	29 500	1 099
49 800	10 022*	28 900	2 059
		28 200	1 380
		26 900	3 371
		25 100	52 244*
Steel No. 10, 0.49 Carbon, Sorbitic, 10% Overstress, <i>N</i> = 5000, <i>S</i> = 55 600			
54 300	116		
53 100	662		
52 100	643		
50 500	510		
50 500	1 220		
50 500	10 554*		

* Specimen did not break.

† Specimen bent.

TABLE 7. (Continued)

S-N RESULTS FOR REVERSED-STRESS TESTS

(I) Reversed-Bending Tests of Rotating-Beam Machine (Farmer Type)

Unit Stress lb. per sq. in. S	Cycles for Rupture in Thousands N	Unit Stress lb. per sq. in. S	Cycles for Rupture in Thousands N
Steel No. 51, Hot-rolled, 0.18 Carbon, as Received		Steel No. 51, Hot-rolled, 0.18 Carbon, Reduced from $\frac{1}{2}$ in. to 0.48 in.	
45 400	24	50 100	25
42 300	33	46 700	42
42 100	7†	45 200	154
39 700	148	43 100	260
37 700	221	40 000	385
36 200	172	40 000	795
34 600	602	39 000	841
30 300	1 244	37 700	671
30 300	1 292	37 000	203
29 200	4 631	35 800	1 712
28 600	12 452	34 800	52 383*
27 800	53 926*		
27 300	50 905*		
Steel No. 51, Hot-rolled, 0.18 Carbon, Reduced from $\frac{1}{2}$ in. to 0.44 in.		Steel No. 51, Hot-rolled, 0.18 Carbon, Bent Cold and Straightened	
54 700	32†		
50 500	230		
49 800	55	49 800	3†
48 300	524	45 300	1†
47 000	1 178	37 300	546
45 200	911	34 700	1 504
42 200	3 519	32 500	726
40 000	53 537*	31 400	2 384
40 000	53 582*	30 000	65 198*
37 600	53 826*		

* Specimen did not break.

† Specimen bent.

TABLE 7 (Continued)

S-N RESULTS FOR REVERSED-STRESS TESTS

(II) Reversed-Bending Tests on Upton-Lewis Machine

Unit Stress lb. per sq. in. S	Cycles for Rupture in Thousands N	Unit Stress lb. per sq. in. S	Cycles for Rupture in Thousands N
Steel No. 1, 1.20 Carbon, Normalized		Steel No. 4, 0.37 Carbon, Sorbitic	
65 000	6	70 700	13
60 000	11	66 700	9
58 400	23	64 300	32
48 600	167	58 900	186
46 700	495	57 800	69
45 000	81	54 800	131
44 200	2 047*	54 200	71
		51 600	98
		44 800	272
		44 700	168
Steel No. 3, 0.52 Carbon, Normalized		Steel No. 5, Chrome-nickel, Treatment A	
53 300	20		
46 500	26	95 200	31
46 000	40	85 100	52
43 300	79	80 000	42
41 200	95	69 300	64
38 000	98	67 800	78
37 000	225	64 800	111
34 200	315	64 400	413
33 000	195	60 100	558
32 000	2 017*	59 700	340
Steel No. 3, 0.52 Carbon, Sorbitic		55 100	406
70 700	15	54 200	1 182
60 000	46	54 100	469
54 000	93	53 200	444
50 000	143	51 600	2 307*
48 000	179	50 200	2 017*
47 000	392		
45 000	1 323	Steel No. 6, 0.93 Carbon, Pearlitic	
43 000	2 086*		
42 000	2 928*	45 600	8
Steel No. 4, 0.37 Carbon, Normalized		43 300	19
45 000	13	41 900	26
40 000	49	41 000	35
38 200	178	37 800	30
38 000	96	35 800	78
36 000	113	33 700	89
36 000	248	32 000	97
35 900	289	30 800	190
34 800	429	30 000	393
34 000	446	29 200	228
32 800	196	28 900	1 027
32 000	214	28 900	1 582
32 000	2 226	28 600	2 018*
31 200	1 224	28 000	2 011*
31 000	330	28 600	2 359*
30 200	2 007*		
30 000	895		

* Specimen did not break.

† Specimen bent.

TABLE 7 (Continued)

S-N RESULTS FOR REVERSED-STRESS TESTS

(II) Reversed-Bending Tests on Upton-Lewis Machine

Unit Stress lb. per sq. in. <i>S</i>	Cycles for Rupture in Thousands <i>N</i>	Unit Stress lb. per sq. in. <i>S</i>	Cycles for Rupture in Thousands <i>N</i>
Steel No. 6, 0.93 Carbon, Sorbitic		Steel No. 10, 0.49 Carbon, Sorbitic	
75 500	19	75 500	21
70 500	31	70 500	43
67 200	64	65 600	27
65 400	112	60 500	37
60 900	112	58 100	142
56 900	107	55 900	95
55 200	162	54 200	180
54 800	294	50 500	245
54 600	139	47 500	368
51 100	294	44 200	517
49 100	364	41 800	1 115
48 400	492	40 300	489
46 700	392	40 000	1 567
45 600	916	39 000	2 309*
45 400	524		
44 800	2 656*		
44 800	1 084		
44 300	625		
44 200	1 993*		
43 600	2 326*		
Steel No. 9, 0.02 Carbon, as Received			
39 800	22		
32 000	81		
29 900	187		
27 900	374		
26 400	394		
26 100	211		
25 400	661		
25 000	563		
24 400	563		
24 000	780		
23 600	1 614		
23 600	1 175		
23 300	2 723*		
23 000	1 694		

* Specimen did not break,

† Specimen bent.

TABLE 7 (Continued)

S-N RESULTS FOR REVERSED-STRESS TESTS

(III) Reversed-Bending Tests on Rotating-Beam Machine (Wisconsin)

Unit Stress lb. per sq. in. <i>S</i>	Cycles for Rupture in Thousands <i>N</i>	Unit Stress lb. per sq. in. <i>S</i>	Cycles for Rupture in Thousands <i>N</i>
Steel No. 5, Chrome-Nickel, Treatment A		Steel No. 2, Wisconsin Tests	
99 900	14	50 000	54
89 900	26	45 100	222
80 200	49	42 600	476
72 800	74	39 900	758
65 200	224	39 500	1 783
62 000	10 623*	39 100	450
60 100	216	39 000	903
60 000	5 231*	38 000	10 486*
58 100	7 215*	37 000	12 102*
56 100	5 222*	34 900	12 642*
54 000	7 250*		
52 000	5 122*		
50 300	5 193*		
Steel No. 1, Wisconsin Tests		Steel No. 3, Wisconsin Tests	
45 000	118	59 600	42
42 000	238	53 100	96
39 900	331	50 000	180
39 900	164	47 500	403
39 000	600	45 000	539
39 000	415	44 900	467
38 600	517	44 000	5 828
37 000	630	43 000	3 031
36 000	1 317	43 000	670
36 000	1 076	42 000	10 549*
35 200	12 446*	42 000	20 833*
34 900	806	42 100	15 716*
34 900	1 940		
34 000	1 916		
34 000	3 608		
34 000	4 857		
33 500	4 790		
33 500	10 662*		
33 100	10 759*		
32 500	14 173*		
30 000	14 984*		

* Specimen did not break.

† Specimen bent.

TABLE 7 (Continued)

S-N RESULTS FOR REVERSED-STRESS TESTS
(IV) Reversed-Torsion Tests on Olsen-Foster Machine

Unit Stress lb. per sq. in. <i>S</i>	Cycles for Rupture in Thousands <i>N</i>	Unit Stress lb. per sq. in. <i>S</i>	Cycles for Rupture in Thousands <i>N</i>
Steel No. 4, 0.37 Carbon, Normalized		Steel No. 5, Chrome-Nickel, Treatment B	
29 000	5	49 700	2
25 000	54	46 000	7
20 000	207	41 900	27
18 000	937	42 000	62
16 000	5 492*	39 500	44
		37 900	81
		36 400	208
		35 900	234
		34 200	512
		33 100	304
		32 200	2 804*
		32 000	1 660
		31 300	2 565*
		31 200	2 117*
Steel No. 4, 0.37 Carbon, Sorbitic			
45 800	10		
42 200	34		
38 500	146		
35 200	696		
34 700	153		
32 400	2 000*		
32 400	2 318*		
Steel No. 5, Chrome-Nickel, Treatment A			
54 100	44		
53 800	14		
50 200	18		
49 600	25		
46 800	49		
45 200	38		
43 800	94		
43 000	58		
41 000	250		
40 500	236		
40 000	337		
40 000	325		
38 100	1 906		
37 900	2 544*		
37 900	2 550*		
37 000	2 064*		

* Specimen did not break.

† Specimen bent.

TABLE 7 (Continued)

S-N RESULTS FOR REVERSED-STRESS TESTS

(IV) Reversed-Torsion Tests on Olsen-Foster Machine

Unit Stress lb. per sq. in. <i>S</i>	Cycles for Rupture in Thousands <i>N</i>	Unit Stress lb. per sq. in. <i>S</i>	Cycles for Rupture in Thousands <i>N</i>
Steel No. 6, 0.93 Carbon, Pearlitic		Steel No. 9, 0.02 Carbon, as Received	
26 000	7	20 000	13
25 000	18	18 000	35
24 000	52	17 300	34
22 000	87	16 000	82
21 000	147	14 900	157
20 000	79	14 300	178
20 000	196	13 500	420
19 800	305	13 200	780
19 000	509	13 000	1 505
18 000	703	12 700	2 446*
17 500	1 149	12 500	1 632
17 000	1 742	12 400	2 062*
16 800	1 926		
16 500	2 000*		
16 100	2 880*		
Steel No. 6, 0.93 Carbon, Troostitic		Steel No. 10, 0.49 Carbon, Sorbitic	
77 500	4	34 000	9
68 600	4	31 600	34
65 500	37	29 200	82
65 200	215	26 900	266
64 200	27	25 600	2 206*
59 500	74		
57 700	11		
57 200	173		
54 800	389		
53 500	685		
53 200	220		
53 100	418		
52 500	2 039*		
52 400	2 045*		
52 100	2 198*		
51 400	1 059		

* Specimen did not break.

† Specimen bent.

IV. DISCUSSION OF RESULTS

14. *Endurance Limit*.—The question whether any material can withstand an infinite number of repetitions of stress, however small, cannot of course be answered by direct experimentation. Earlier investigators determined a “limit” from plotted curves with values of unit stresses as ordinates, and numbers of repetitions of stress causing failure as abscissas. Denoting values of unit stress by S and number of cycles of stress necessary to cause failure by N , such diagrams may be conveniently designated as S - N diagrams, and are so designated in this bulletin. Having drawn such a diagram, the early experimenters then judged as best they could where the S - N diagram became horizontal, and took the S ordinate corresponding to this horizontal asymptote as the limiting stress which could be withstood an indefinite number of times. Fig. 36(A) shows some of the test data of this investigation plotted in this way. A careful reading of the writings of early investigators shows that they recognized the limitations of this method and the enormous extrapolation involved.

A modification of this method is to plot values of S as ordinates and values of the reciprocal of N as abscissas. (See Fig. 36 [B] in which some of the test data of this investigation are thus plotted). Then the limit for indefinite repetition of stress is found by extending the curve backward until it crosses the zero line of abscissas. This method involves an extrapolation which apparently is only slight, but which in reality is as great as that involved in the first method. In Fig. 36(B) the horizontal distance between A , corresponding to the smallest observed value of $1/N$, and the zero line for $1/N$ seems small, but in reality it corresponds to an infinite distance along the N -axis in the S - N diagram, Fig. 36(A). This method of plotting gives no assurance that the reciprocal S - N diagrams do not follow the broken lines in Fig. 36(B).

A modification of this second method, suggested by Mr. C. E. Stromeyer of Manchester, England, involves plotting values of S against values of $\frac{1}{\sqrt[4]{N}}$. This method gives a graph which is nearly

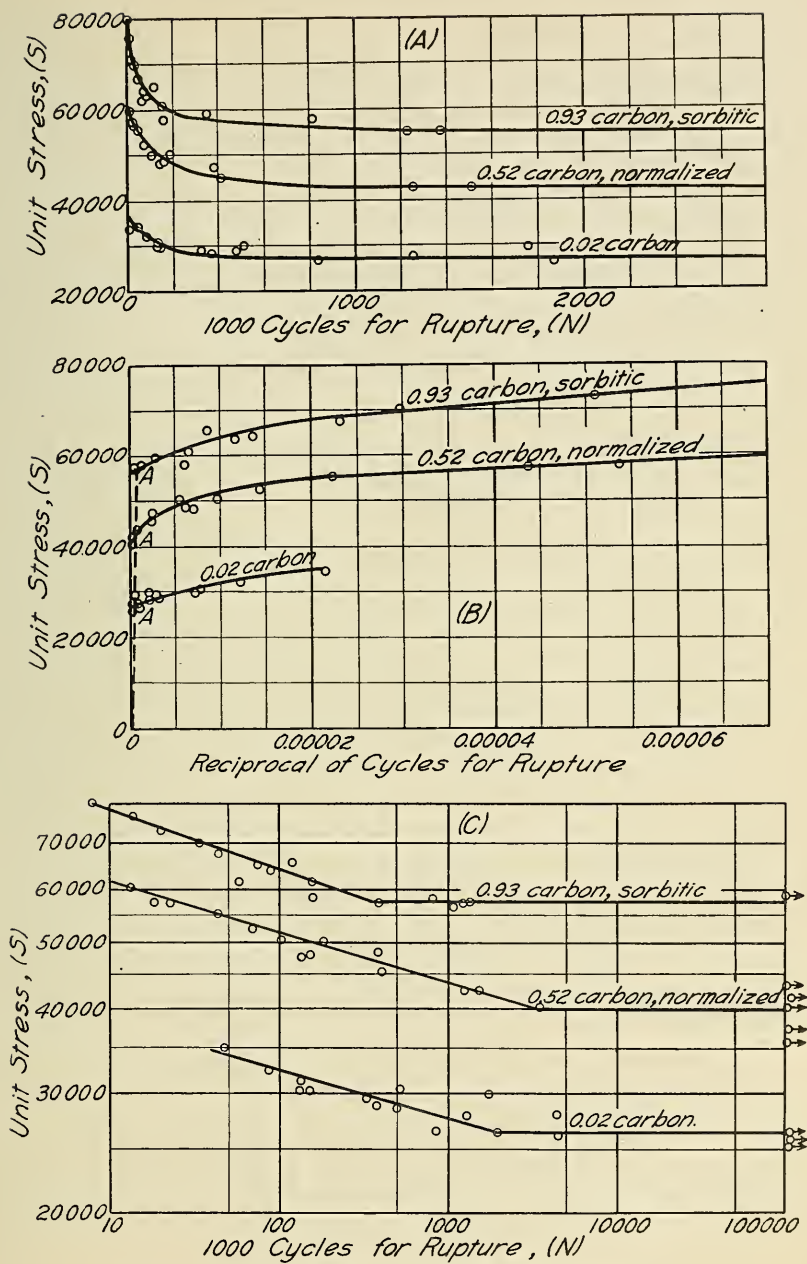


FIG. 36. VARIOUS METHODS OF PLOTTING S-N DIAGRAMS

a straight line, but involves the same extrapolation as does the method given above.

Some later authorities, including Professor Basquin of Northwestern University, and, later, Moore and Seely, have suggested that the relation between S and N may be expressed by an equation of

the form $S = \frac{B}{(N)^K}$, in which B and K are experimentally determined

constants. This viewpoint has been advocated as being a *safe* viewpoint for designers, rather than as an exact statement of relation, and it involves the assumption that any stress repeated often enough will eventually cause failure of a material. If, in place of plotting values of S and N , values of $\log S$ and $\log N$ are plotted, or values of S and N are plotted on logarithmic cross-section paper, the relation given in the above equation is represented by a straight line.

In Fig. 36(C) the results of some tests of this investigation are plotted to logarithmic coördinates. In this figure, as in all the S - N diagrams plotted to logarithmic coördinates, the plotted point for any specimen which was not broken is marked with an arrow. It may be noted that logarithmic plotting enables both small values and large values of N to be plotted with a reasonable degree of accuracy. It should be noted that Fig. 36(A), Fig. 36(B), and Fig. 36(C) show the same test data plotted in different ways. The diagrams of Fig. 36(C) indicate that up to a certain value of N a power equation is followed fairly closely, but then there comes a quite decided break in the graph, after which a horizontal line seems to represent the relation between S and N up to the value of N of 100 000 000, with no indication of a further break in the diagram. On a logarithmic S - N diagram, as on an ordinary S - N diagram, a horizontal line would seem to indicate that a limit for indefinite reversals of stress had been reached, and the ordinate of such a line gives a value of S which in this bulletin is designated as the "endurance limit."

It is of course possible that the relation between S and N is of such a nature that at some point beyond 100 000 000 reversals the diagram again slopes downward. However, it is believed that these experiments have been of sufficient extent and sufficiently numerous to justify regarding the unit stress corresponding to the horizontal line as a unit stress which can be completely reversed for an in-

definite number of times without danger of failure and in calling that unit stress the endurance limit. Additional confidence in this conclusion is given by the well-defined "break" shown in the logarithmic S - N diagrams, indicating as it does a decided change in the relation between S and N ; and still further confidence is given by the distinct rise of temperature noted at the endurance limit (this rise of temperature is more fully discussed in Chapter V).

Some of the S - N diagrams show considerable variation in values of N for a given value of S , especially at or near the endurance limit. The engineer usually wishes to determine a safe value of S , and it may be pointed out that for these same S - N diagrams there is little variation in S for a given value of N .

It is believed that the series of tests herein reported furnish a clearer demonstration of the existence and magnitude of an endurance limit for steel than has been furnished by previous tests. It is believed, moreover, that this investigation is a reconnaissance in the field of wrought ferrous metals sufficiently extensive to indicate the probability that there is an endurance limit for such metals in general.

In all of the curves the endurance limit is developed at a value of N less than 10 000 000. This fact can be used to shorten very greatly the time required for determining the endurance limit of ferrous metals. It is not at all certain that non-ferrous metals reach their endurance limits at such low values of N .

15. *Some Factors Influencing Endurance Limit.*—Fig. 24 illustrates the effect of heat treatment on the endurance limit. The very soft condition of the pearlitic structure of the 0.93 carbon steel results in an endurance limit of 30 500 pounds per square inch. This can be increased 84 per cent when the steel is given a sorbitic structure by heat treatment, and 221 per cent when it is given a troostitic structure. Figs. 25 and 26 illustrate the same fact and make clear the very great influence which heat treatment may exert upon the fatigue strength of steel.

Figs. 32 and 33 show the S - N curves obtained with the Upton-Lewis reversed-bending machine, and also one curve for chrome-nickel steel, treatment A, which is from tests made on the Wisconsin machine. The endurance limits from these tests on the Upton-Lewis

machine are given in Table 5, and it will be noted that they are uniformly lower than those obtained from the rotating-beam test. This difference is probably due in part to stresses set up by the inertia of vibrating parts in the Upton-Lewis machine, and in part to the fact that a $\frac{1}{2}$ -inch radius was used in reducing the section of the Upton-Lewis specimen, while a radius of 9.85 inches was used in the case of the rotating-beam specimen. It is well known that an abrupt change of section such as employed with the Upton-Lewis specimen causes localized stresses which tend to reduce the fatigue strength. This matter is discussed more fully in Section 20, "Effect of Shape of Specimen on Endurance."

For steel No. 5, chrome-nickel, treatment A, the endurance limit was determined with three different machines. The corresponding *S-N* diagrams are shown in Figs. 25 and 33. The Farmer machine gave an endurance limit of 68 000 pounds per square inch, the Wisconsin machine an endurance limit of 61 000 pounds per square inch, and the Upton-Lewis machine an endurance limit of 52 000 pounds per square inch. The two lower values in the Wisconsin and the Upton-Lewis machines, respectively, are believed to be due to the short-radius fillet of the Wisconsin specimen, and to the short-radius fillet of the Upton-Lewis specimen together with acceleration effects in the vibrating parts of the Upton-Lewis machine.

Figs. 34 and 35 show the *S-N* curves obtained with the Olsen-Foster reversed torsion machine. The endurance limits taken from these curves are given in Table 5. The results thus far obtained indicate that the ratio of the endurance limit in torsion to the endurance limit of the same steel in bending varies from 0.48 to 0.57. An average of eight results shows that the endurance limit in torsion is about 0.52 of the endurance limit in bending.

The effect of changes in the details of the heat treatment of material on the endurance strength is well brought out by the two curves in Fig. 25 for the 0.37 carbon, sorbitic steel. The specimens for treatment A were turned to about 0.02 inch oversize and then heat treated. The specimens for treatment B were about $\frac{5}{8}$ inch testing. Table 3 shows that the tensile results for treatment B were square, and were first heat treated and then turned down to size for considerably lower than for treatment A, and Fig. 25 shows that the endurance limit was also lower. These results indicate the neces-

sity of having the conditions of heat treating as nearly the same as possible in order to get uniform results.

In those cases in which specimens from the same steel were tested in a normalized and in a sorbitic condition, the *S-N* curves show that the endurance limit is higher for the sorbitic condition by 31 per cent for the 0.52 carbon, 73 per cent for the 0.37 carbon, and 84 per cent for the 1.20 carbon steel. For the 0.93 carbon, the sorbitic gives an endurance limit 84 per cent higher than the pearlitic steel.

An interesting fact, shown by Table 3, is that the 0.37 carbon sorbitic steel has a percentage of elongation almost as high as the 0.37 carbon normalized steel, and the percentage of reduction of area is even higher. For the 0.93 carbon steel the ductility is almost exactly the same for the sorbitic specimens as for the pearlitic; for the 0.52 carbon steel the elongation is slightly less for the sorbitic specimens than for the normalized, but the reduction of area is greater; while for the 1.20 carbon steel the sorbitic condition has actually a higher ductility than the normalized. These results indicate that heat treatment may greatly improve the static and fatigue strength properties of the material with no serious sacrifice of ductility.

The tests were not conducted with the intention of comparing carbon steels with nickel and chrome-nickel steels, but certain qualitative results may be mentioned. Table 3 shows that the sorbitic treatment for the carbon steels, with the one exception of the 1.20 carbon steel, develops about the same ductility represented by percentage of elongation as the heat treatments which were used for the nickel and chrome-nickel steels. The results indicate that the nickel and chrome-nickel steels have the advantage of somewhat higher endurance limits for the same ductility. This result is not unexpected, judging from the well-known fact that these alloy steels in comparison with carbon steels show high ductility for a given static strength.

16. *Relation of Static Strength to Endurance Limit.*—The tests described in this bulletin seem to indicate that the results of static tests made in an ordinary testing machine are not reliable as an index of the strength of the material under reversed stress as de-

terminated by the rotating-beam test. In confirmation of this, attention is called to Fig. 37, in which are plotted values of the endurance limit, the proportional limit, and the ultimate tensile strength for

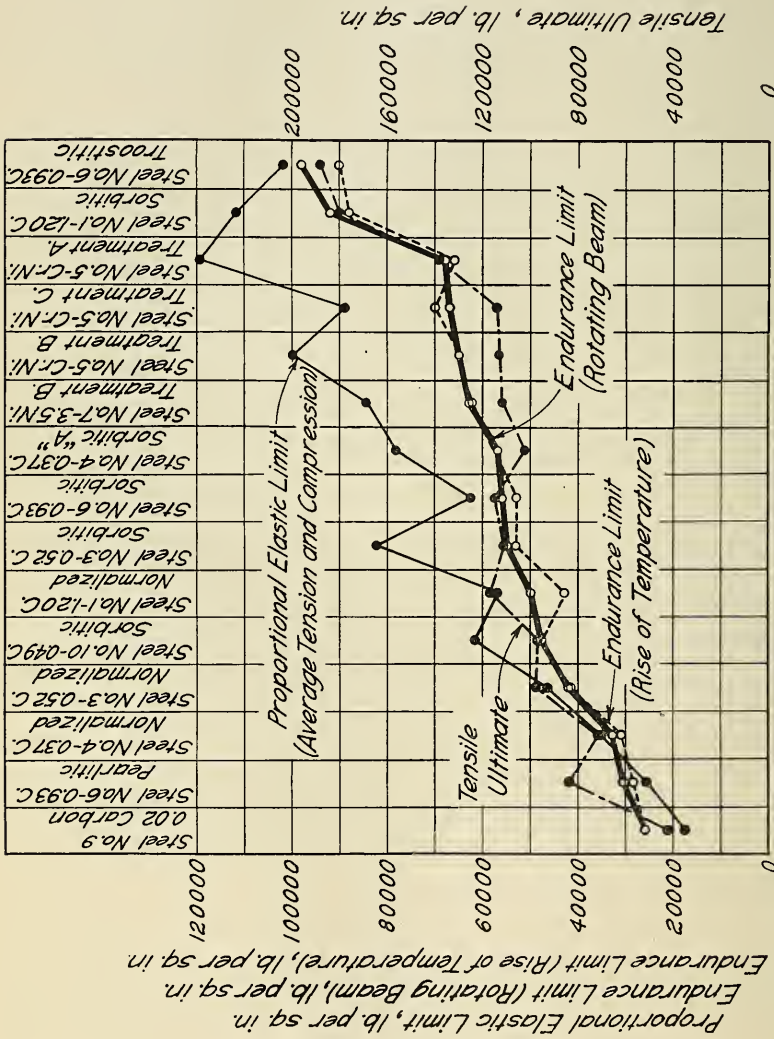


FIG. 37. COMPARISON OF ENDURANCE LIMITS WITH TEMPERATURE ENDURANCE LIMITS, PROPORTIONAL LIMITS, AND ULTIMATE TENSILE STRENGTHS

the various steels tested, arranged in such order that the endurance limits increase from left to right. The values for proportional limit do not follow at all closely the values for endurance limit. Computations from the values given in this figure show that the ratio of endurance limit to proportional limit varies from 1.5 to 0.5, making it apparent that the proportional limit cannot be used to predict fatigue strength in bending. The variation in the ratio of endurance limit to ultimate tensile strength is not as great as the variation just mentioned, but is considerable, the ratio ranging from 0.68 to 0.36. In general as the proportional limit and the ultimate tensile strength increase the endurance limit also increases.

Fig. 37 also shows values of the endurance limit as determined by a temperature test, which is discussed in detail in Chapter V.

A study of the endurance limits in torsion and the corresponding proportional limits shows that here also the proportional elastic limit is not a reliable index of fatigue strength. In this case, the ratio of endurance limit to the proportional elastic limit was found to range from a value of 1 for the 0.02 carbon steel and the 0.93 carbon pearlitic steel, to 0.51 for the chrome-nickel steel, treatment A.

The determination of the elastic limit and the ultimate strength (Fig. 37) depends upon the average properties of a considerable mass of material. A minute defect, such as a nick on the surface, an incipient crack in the structure, or a localized stress resulting from heat treatment, will not in general affect the determination to an appreciable extent. On the other hand, such a localized defect may readily act as a nucleus for structural damage which, under repeated stress, spreads in the form of minute cracks with continued localized stress present at the roots of these cracks, and finally causes failure.

It is readily conceivable that even for a material free from serious flaws non-homogeneity of structure might affect resistance to repeated stress to a much greater extent than it does static strength. Consider two materials, A, made up of strong and weak crystals with a large preponderance of strong crystals, and B, made up of crystals all alike, and all somewhat weaker than the strong crystals of A. It seems quite possible that material A will give higher elastic strength under static tests and lower strength under repeated stress than material B. Such might well be the case for the chrome-nickel

steel with heat treatment A as compared with the troostitic 0.93 carbon steel, the former corresponding to material A, and the latter to material B.

Fig. 25, which shows the *S-N* curves for the three heat treatments of steel No. 5, chrome-nickel, is especially interesting. By referring to Table 3 it will be seen that treatment A gives the highest ultimate strength and proportional elastic limit; treatment B is about 15 per cent lower, yet the endurance limit is almost as high as with treatment A. Treatment C gives a low proportional elastic limit compared with the other two treatments, yet the endurance limit is actually a little higher than that for treatment B and almost as high as that for treatment A. This is particularly interesting in view of the fact that the proportional elastic limit of material with treatment C is low compared with its yield point, and that the tension stress-deformation curve shows considerable permanent set below the yield point. On the basis of mechanical hysteresis it would ordinarily be expected that a material which showed considerable permanent set would probably be poor in resisting fatigue. This does not seem to be the case for treatment C of the chrome-nickel as compared with the other two treatments.

One rather remarkable result is indicated by Fig. 37. Computation from values given in the figure shows that the ratio of the computed stresses at the endurance limit to the proportional elastic limit of the 0.02 carbon steel was almost 1.5, and the endurance limit was actually 33 per cent higher than the average yield point in tension and compression. This result may at first appear impossible, and it is believed that it would be impossible in reversals of direct tensile and direct compressive stress, but it must be recalled that the endurance limit here mentioned was obtained from a rotating-beam test. Evidently the inner fibers of the beam, which are not stressed as highly as those on the outside surface, reinforce the outer fibers sufficiently so that the nominal stress calculated is actually higher than the yield point of the material. Of course the outer fibers are actually stressed only to the yield point, but evidently enough of the inner fibres are sufficiently stressed so that the resisting moment of the beam is increased to about 33 per cent above that based upon the yield point. While the above is an explanation of the possibility of the result it does not make the result less remark-

able, especially in view of the fact that only one other steel, the 0.93 carbon, gave an endurance limit even approximately equal to the proportional elastic limit of the material, as is clearly shown in Fig. 37. This result is called remarkable not because of the high endurance limit developed (the endurance limit of the 0.02 carbon steel was the lowest of all the steels tested) but merely because of the high ratio of endurance limit to proportional elastic limit and to yield point, and because it is believed that so high a ratio has not been observed before.

It seems clear that, in any case, the endurance limit for steel can be no higher than its yield point. That is to say, the actual unit stress existing in fatigue is limited to the yield point, although the nominal calculated stress, as in the case of the 0.02 carbon steel, may sometimes be higher than the yield point.

This matter has been dwelt upon at length because it may offer a clue to a quality necessary in a steel to produce a high ratio of endurance limit to proportional elastic limit. Apparently the only quality possessed by the 0.02 carbon steel which offers an adequate explanation of the above phenomenon was its great homogeneity. This steel was almost pure ferrite, while all the other steels were made up of several constituents. The only other steel which showed an endurance limit higher than the proportional elastic limit was the 0.93 carbon pearlitic steel. When the steel is made up all of pearlite or all of ferrite the ratio of endurance limit to proportional elastic limit is high. It seems to the writers that perhaps one of the qualities to be desired in order to get a high ratio of endurance limit to proportional elastic limit may be homogeneity of structure.

From the test data given in Appendix A it appears that annealing various grades of steel gives a high ratio of endurance limit to proportional limit. It would seem reasonable that either homogeneity of metallographic structure or freedom from internal strains (such freedom as would be brought about by thorough annealing) should tend to give a high ratio of endurance limit to proportional limit.

It does not seem likely that ductility, as shown by percentage of reduction of area and by percentage of elongation, will have much bearing on the strength in fatigue. These quantities are based upon the action of a bar as a whole and in ductile materials are largely

dependent upon the final necking down after the ultimate has been reached. In fatigue failures there is no necking down, but the material fails as though it were brittle. Furthermore, the action in fatigue is extremely local and does not involve a large portion of the bar. A study of Table 3 will show the basis for the foregoing statements. The 0.93 carbon troostitic steel, for instance, shows low elongation and reduction of area, but a high endurance limit and a high ratio of endurance limit to proportional elastic limit.

It has been suggested by various investigators that the use of very delicate instruments for measuring deformation might enable an experimenter to detect in a static test the minute fractures which are supposed to occur at the endurance limit. An apparatus of great delicacy has been developed by Mr. W. J. Francke of New Brunswick, New Jersey,* who has tested in his laboratory specimens of several of the steels studied in this investigation. The results in Table 5 showing the "FR" point determined in the Francke test indicate that the stress corresponding to this "FR" point coincides quite closely with the endurance limit in some cases, but not so well in others. More information will be necessary to determine the value of the Francke test as a criterion of fatigue strength.

17. *Correlation of Results of Hardness Tests and of Impact Tests With Results of Reversed-Stress Tests.*—The results of the Charpy impact-bending tests, the Charpy impact-tension tests, and the repeated-impact tests indicate that the results of the impact tests made cannot be used as a criterion of fatigue strength. Fig. 38, in which the plan of plotting results is similar to that used in Fig. 37, makes this clear. The results of these impact tests bear no consistent relation to the endurance limits for the various steels.

Fig. 38 shows on the other hand a quite consistent relation between Brinell hardness and endurance limit. *Within the range of the materials tested in this investigation* it would seem, therefore, that the Brinell hardness test may prove to be valuable in predicting the approximate value of the endurance limit. This approximate value could then be checked by fatigue bending tests on a number of specimens not exceeding a value of 10 000 000 cycles of stress.

* Proc. Am. Soc. for Test. Materials, Vol. XX, Part II, p. 372, 1920.

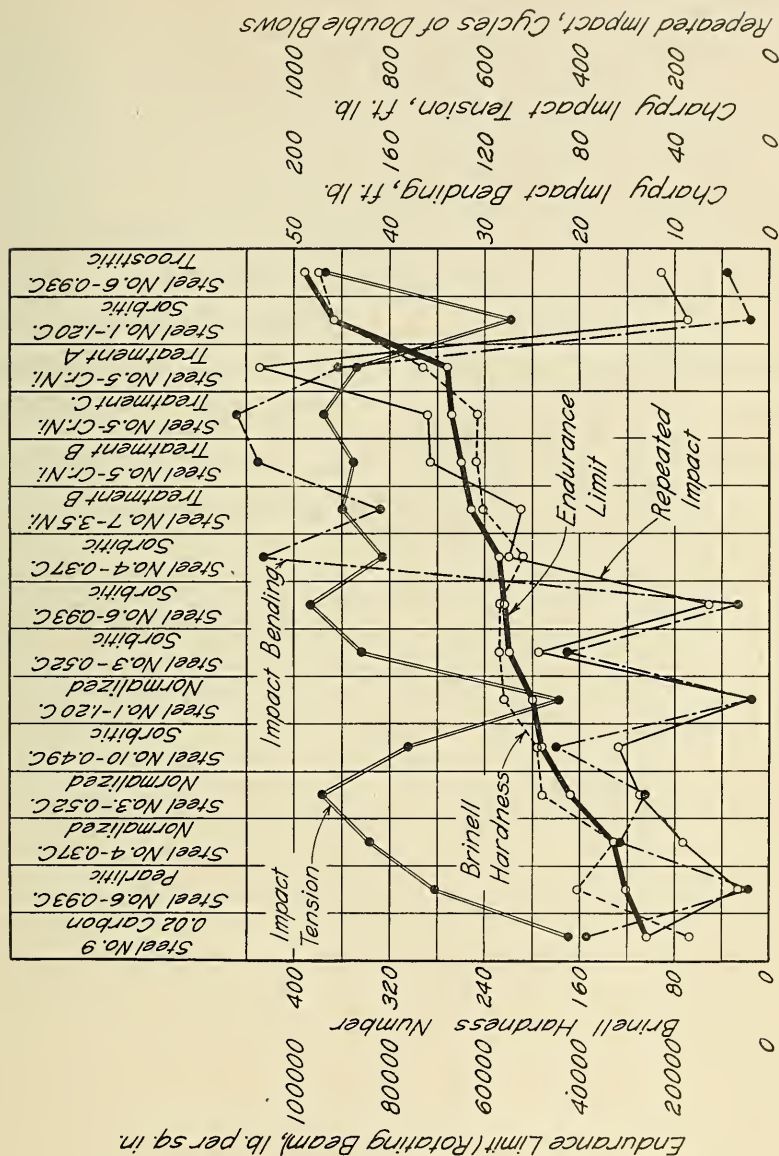


FIG. 38. COMPARISON OF ENDURANCE LIMITS WITH BRINELL HARDNESS, CHARPY IMPACT BENDING VALUES, CHARPY IMPACT-TENSION VALUES, AND REPEATED-IMPACT VALUES

By this procedure the endurance limit of a steel could be determined in a comparatively short time.

It seems significant that the elastic properties of a material should be so unreliable as criteria of fatigue strength, while the Brinell hardness is fairly satisfactory. The point to be emphasized is that the Brinell hardness test is an empirical test which stresses the material beyond its proportional elastic limit and gives it a permanent deformation. Perhaps the endurance limit in bending bears some relation to the properties which a material has when its elastic limit has been exceeded and the metal is in a plastic condition. The writers offer no explanation other than that of an empirical relation. Without further investigation the Brinell test should not be regarded as reliable for determining the endurance limit for non-ferrous metals, nor for determining the endurance limit for steel with "snowflakes" or other mechanical defects.

The "rise of temperature" test (Chapter V) is a short-time test for endurance limit which gave more accurate results than did the Brinell test, and which, moreover, seems to have a more logical basis.

Both the Brinell test and the "rise of temperature" test may be used as non-destructive tests on parts of machines and structures.

18. *Accelerated Methods of Determining Endurance Limit.*—The basic tests used in this investigation for determining the endurance limit are believed to involve the accurate measurement of number of reversals of definitely known stresses. These tests are, however, time consuming. While the endurance limit of steel may be determined by tests running no higher than 10 000 000 repetitions, several days must be consumed in making such a determination. Various accelerated tests have been proposed for determining the endurance limit. Among these may be mentioned impact tests, repeated-impact tests, short-time tests under repetitions of high stress, magnetic tests, and tests in which the rise of temperature caused by repeated stress is measured.

Impact tests under a single blow are frequently made, usually on flexure specimens which have been notched so that complete rupture will occur (Charpy, Izod, or other "notched-bar" tests). The action of a single impact is widely different from that of repeated

stress; both impact tests and repeated stress tests do, however, emphasize the influence of local defects. No correlation could be observed between the results of reversed-stress tests in this investigation and the results of single-blow impact tests.

Repeated-impact tests are sometimes made upon flexure specimens notched to localize failure, using a few hundred light blows in place of one heavy blow. In this class are tests on a "Stanton" machine or one similar in principle. These tests would seem to be a somewhat arbitrary compromise between a single-blow impact test and a repeated-stress test, and it is not possible to measure the stress set up in the notched bars used. The repeated-impact tests made in this investigation used a rather heavy blow for the size of specimen. The results of these repeated-impact tests failed to show any correlation with the results of the reversed-stress tests.

Short-time repeated-stress tests under high stresses have frequently been made and have given values for comparison, either on the basis of a constant stress and relative endurance of two steels, or on the basis of relative magnitude of stress corresponding to a given endurance. The results of the present investigation, and of some other investigations, seem to indicate that short-time, high-stress tests do not always give results corresponding to long-time, low-stress tests. This is illustrated by a comparison of the S - N diagrams for cold-drawn steel (Fig. 27) with the S - N diagram for 1.20 carbon steel normalized (Fig. 24). Comparing the values of N for a value of S of 60 000 pounds per square inch, the cold-drawn steel would seem the more resistant to repeated stress; comparing the values of N for a value of S of 45 000 pounds per square inch, the cold-drawn steel fails after about 1 000 000 cycles of stress while the 1.20 carbon normalized withstands 100 000 000 cycles without failure. Further discussion of such tests is given in Appendix A.

Magnetic tests to detect microscopic incipient fractures developed under repeated stress hold some promise of success, but are applicable only to ferrous metals. This test has not been thoroughly developed so far as the writers are aware, but promising preliminary results have been obtained by Dr. C. W. Burrows of New York, and by Mr. R. L. Sanford of the United States Bureau of Standards.

A study of the rise of temperature produced after a few hundred repetitions of stress has been proposed by various investigators,

and especially by Mr. C. E. Stromeier of Manchester, England, who made a preliminary experimental investigation of this test. This test has been given special study in the present investigation and the results of that study are given in detail in Chapter V by Messrs. Putnam and Harsch.

19. *Effect of Cold Work on Fatigue Strength.*—The upper half of Fig. 27 shows the *S-N* curves for Steel No. 51, which was received in the form of hot-rolled, half-inch, reinforcing bars. Part of this material was tested as received, part was stretched in tension until the diameter had been reduced to 0.48 inch, part had the diameter reduced in the same way to 0.44, and part was bent cold through 45 degrees and straightened. The cold stretching was such that the material reduced to 0.48 inch was stressed to a point between the yield point and ultimate. The material reduced to 0.44 inch was stressed up to the ultimate and was about to neck down. In each case after the material had been cold worked it was raised to a temperature of 500 degrees F. and held there for 15 minutes, so that it might recover its elasticity rapidly. The tension test results, which were determined from specimens $\frac{1}{4}$ inch in diameter taken from the ends of the rotating-beam specimen, are shown in Table 3. The specimens for the rotating-beam machine were prepared in the standard manner.

Fig. 27 shows that the cold bending raised the endurance limit slightly, about 7 per cent. The cold stretching raised the endurance limit in the first case about 25 per cent, and in the second case about 46 per cent. The cold stretching in the above two cases raised the proportional elastic limit about 57 and 82 per cent, respectively; and the ultimate about 10 and 19 per cent, respectively. The endurance limit is therefore influenced to a much smaller degree by cold stretching than is the proportional elastic limit.

The lower half of Fig. 27 shows the *S-N* curves for Steel No. 50, received in the form of 7/16 round bars. Some of this material was tested as received, some after annealing at 1300 degrees F., and some after annealing at 1550 degrees F. Taking the material annealed at 1550 degrees F. as a standard, the endurance limit for the 1300 degrees F. annealing is 16 per cent higher, although this value is somewhat in doubt, and that of the material as received is 64 per cent higher.

The tension test values in Table 3 show that the results after annealing at 1300 degrees F. are about the same as after annealing at 1550 degrees F.; but the results for the material as received are higher than those for the material annealed at 1550 degrees F. by 97 per cent and 49 per cent for the proportional elastic limit and ultimate strength, respectively. These last results again show that the endurance limit is not affected by cold work to the same degree as is the proportional elastic limit.

The results on Steel No. 50 bear out the results found by previous investigators. There is practical unanimity in the statement that annealing reduces the fatigue strength.

The results on Steel No. 51 contradict the results previously found by Moore and Putnam.* Although they did not continue the tests to a sufficiently high number of cycles of stress to determine the endurance limit, yet their curves show clearly that cold stretching reduces the strength in fatigue.

There is a possible explanation for the contradiction. Moore and Putnam used flat fatigue specimens prepared for the Upton-Lewis reversed-bending machine. The specimens were cold stretched and then tested without having the surface finished in any way. In the present tests the material was cold stretched, then the rotating-beam specimens had their diameter reduced by the standard radius of 9.85 inches from 0.48 and 0.44 inch to 0.30 inch, and the surface was given the standard polish. It is believed that the removal of the outside material and the polishing of the surface account for the difference. It is well known that a specimen stretched beyond its yield point has its surface roughened, and it is highly probable that this surface is a source of weakness in fatigue in that it is probably covered with minute cracks which are nuclei for the spreading of structural damage.

20. *Effect of Shape of Specimen on Endurance.*—One of the factors known to influence the endurance of a metal under repeated stress is a sudden change in cross-section. Stanton and Bairstow† have shown that specimens with Whitworth screw threads, and also those with square shoulders plus a small fillet, suffer a reduction in

*Am. Inst. of Mining Engrs., 1919, p. 401.

†Inst. of Civil Engrs. (British), Vol. CLXVI, IV, p. 78, 1905-6.

endurance strength of about 30 per cent for hard steel, for soft steel, and for wrought iron, while specimens with square shoulders suffer a reduction of about 50 per cent for hard steels and from 35 to 45 per cent for mild steels and for wrought iron. Eden, Rose, and Cunningham* found that a sharp V-notch reduced the endurance strength of bright drawn mild steel about 25 per cent. Square shoulders reduced the strength of both hard and soft steels by 40 per cent, while keyways at flange couplings reduced the strength of steel by 50 per cent, and the strength of wrought iron by 23 per cent. Wöhler† found in some tests on axle steel stressed from zero to a maximum in repeated tension that specimens with square shoulders as compared with specimens having well-rounded shoulders had their strength reduced about 37 per cent. On rotating-beam specimens of wrought iron the reduction of strength due to square shoulders ranged from 11 to 22 per cent.

In the present investigation some tests were run on the effect of shape of specimen on fatigue strength, especially on the effect of radius of fillet. Steel No. 10 (0.49 carbon) in the sorbitic condition was used in making the first series of these special tests. The shapes of the specimens are shown in Fig. 39, the cross-section being reduced by a 9.85-inch radius, by a 1-inch radius, by a $\frac{1}{4}$ -inch radius, by square shoulders, and by a 90 deg. V-notch, respectively. In each case enough specimens were tested to determine the endurance limit quite definitely.

Fig. 28 shows the *S-N* curves for these tests. Fig. 26 should also be consulted because it gives the *S-N* curve for the 0.49 carbon steel when finished with a standard radius of 9.85 inches. It should be explained that the material used for these tests was heat treated in two batches, and it was observed that the specimens from the first batch gave an endurance limit of about 50 500 pounds per square inch (for specimens with standard surface finish); while specimens from the second batch gave an endurance limit of about 48 000 pounds per square inch. The specimens with $\frac{1}{4}$ -inch radius and most of the specimens with square shoulders were from the first batch, while the specimens with 1-inch radius and those with the V-notch were from the second batch. The results shown for the standard specimen

* Inst. of Mech. Engrs. (British), 1911, III-IV, p. 839.

† Engineering (London), Vol. XI, 1871.

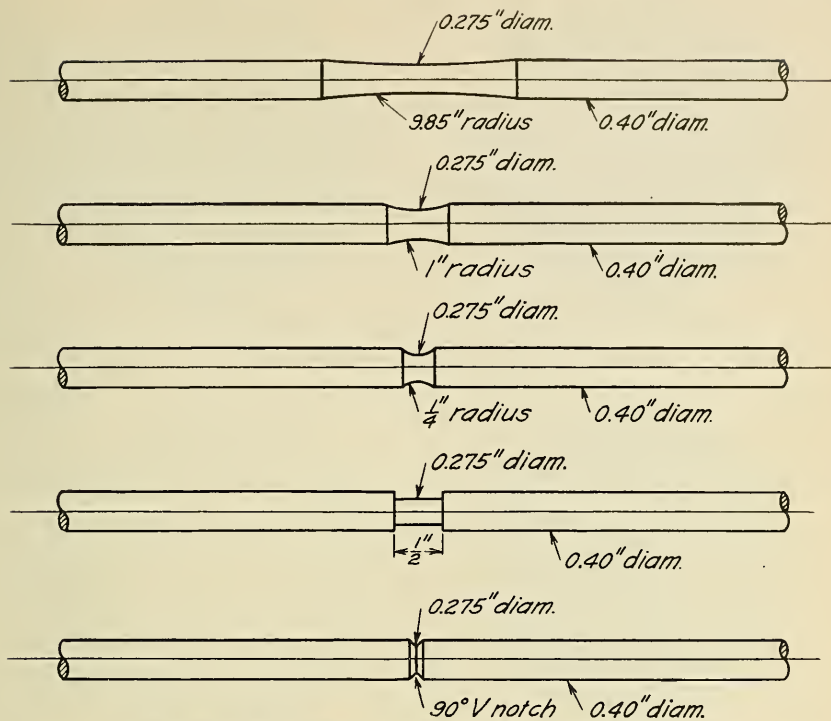


FIG. 39. SPECIMENS FOR STUDY OF EFFECT OF SHAPE ON ENDURANCE LIMIT

in Fig. 26 are from both batches. Specimens with a 1-inch radius are evidently almost as good in resisting fatigue as those with a 9.85-inch radius. The $\frac{1}{4}$ -inch radius shows a reduction in endurance limit as compared with the 9.85-inch radius of about 8 per cent; the square shoulders show a reduction in fatigue strength of about 51 per cent; and the V-notch a reduction of about 60 per cent.

Fig. 28 shows some effects of shape of specimen using Steel No. 9, 0.02 carbon, a very ductile steel. Fig. 26 shows the results for this steel with the standard specimen. For the specimens with $\frac{1}{4}$ -inch radius the endurance limit is reduced about 15 per cent, and for specimens with square shoulders, about 48 per cent.

The results on specimens of 0.02 carbon steel together with the results on specimens of 0.49 carbon steel indicate that the percentage of reduction of endurance limit due to the effect of abrupt change of section is about the same for the two steels.

These results indicate the importance of avoiding abrupt changes of section in members of machines which are to be subjected to repeated stresses. Whenever a change of section is necessary generous fillets should be provided at all shoulders.

A further factor in the effect of form of specimen which might well be the subject of investigation is the effect of a fillet with a given radius with various ratios of maximum diameter of shaft to reduced diameter.

21. *Effect of Surface Finish.*—The condition of the surface of a specimen has been shown to have a considerable influence on its fatigue strength. Eden, Rose, and Cunningham* found that polished specimens of mild steel which had their surfaces scratched with an ordinary sewing needle suffered an appreciable reduction in fatigue strength. Specimens of Bessemer steel with a turned surface showed a fatigue strength about 18 per cent lower than specimens of the same material which had been turned and polished. Sondericker† found that a rotating-beam specimen of soft steel with a groove 0.003 inch deep, cut with a diamond point, had its fatigue strength reduced by 40 per cent. In some tests in which annealed cold-rolled steel was stressed in reverse bending beyond the yield point, Komers‡ found that specimens which had been turned in a lathe and specimens which had been turned and then filed had their endurance reduced 30 and 18 per cent respectively, as compared with specimens which had been turned, filed, and polished. In the last named tests endurance was measured by the number of cycles of stress before rupture.

In order to study the effect of surface finish on the endurance strength of steel, five different series of specimens were prepared. Steel No. 10, 0.49 carbon sorbitic, was used for the first series.

The five degrees of finish were: first, the standard finish, made with No. 0 and No. 00 emery cloth; second, a high polish in which, after using No. 0 and No. 00 emery cloth, the specimens were polished with emery papers Nos. 1, 0, and 000, and finally with rouge and broadcloth, a microscope with a magnification of 100 diameters being

* Inst. of Mech. Engrs. (British), 1911, III-IV, p. 839.

† Tech. Quart. (Boston), March, 1899.

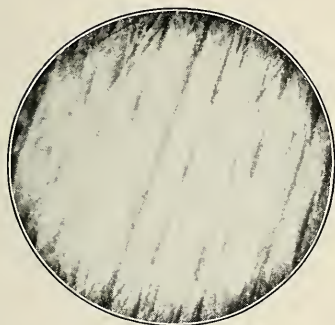
‡ Proc. Int. Assn. for Test. Materials, 1912, Art. V4a.



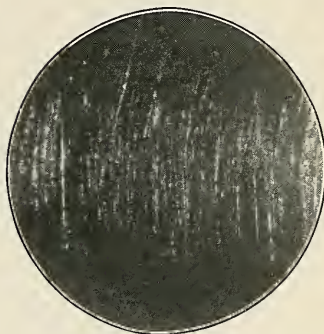
(a) Rough Turned Finish



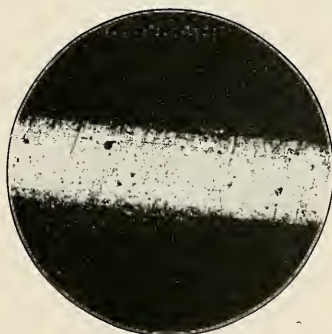
(b) Smooth Turned Finish



(c) Ground Finish



(d) Standard Finish



(e) Rouge Finish

FIG. 40. MICROGRAPHS OF SURFACE FINISH
Magnification 30 diameters

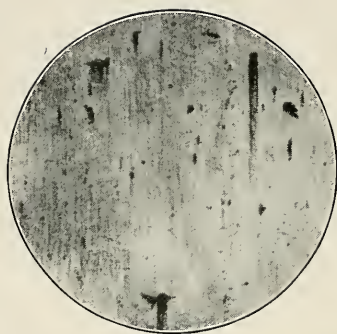


FIG. 41. MICROGRAPH OF SURFACE OF ROUGE-FINISHED SPECIMEN
Magnification 100 diameters

used to make sure that all scratches were removed; third, a ground finish obtained with a grinding wheel; fourth, a smooth-turned finish with a lathe tool; and fifth, a rough-turned finish with a lathe tool. Fig. 40 shows the photographs, under a magnification of 30 diameters, of the five degrees of finish. Fig. 41 shows the rouge finish under a magnification of 100 diameters, and indicates the degree to which the very fine scratches were removed.

Fig. 29 shows the *S-N* diagrams obtained with four of these series. Fig. 26 should be consulted also, since it shows the results obtained with the standard finish. As was the case with the tests discussed in Section 20, the material for the first series was taken from two batches of steel, heat treated at two different times. The first batch showed an endurance limit of about 50 500 pounds per square inch and the second batch an endurance limit of about 48 000 pounds per square inch. The rouge-finish specimens were taken from the first batch, the rough-turned were taken from the second batch, while the ground and the smooth-turned were taken from both batches, as were the specimens with standard finish.

The *S-N* curves make it clear that the rouge finish is probably slightly better than the standard finish in withstanding fatigue. The ground-finish endurance limit is slightly lower than the standard-finish, and the smooth-turned and rough-turned fall below the ground-finish in the order named. The rough-turned finish, the weakest, has an endurance limit about 18 per cent lower than the rouge finish.

There is so little difference between the rouge finish and the standard finish, especially in view of the fact that a number of the standard-finish specimens of the first batch ran out to 10 000 000 cycles of stress without failure at a unit stress of slightly over 50 000 pounds per square inch, that it is felt that the use of the standard finish is fully justified. The rouge finish was of course very expensive and time consuming, and is difficult to maintain free from corrosion; therefore it is not considered practical.

Fig. 29 shows some results of the effect of surface finish on specimens of Steel No. 9, 0.02 carbon, a very soft steel. The endurance limit for the rough-turned specimens seems to be slightly less than for the smooth-turned. The reduction in endurance limit of the turned specimens as compared with specimens with standard finish (see Fig. 26) ranges from 8 to 12 per cent.

Comparing the results of tests on specimens of 0.02 carbon steel with specimens of 0.49 carbon steel, sorbitic, it is seen that the percentage reduction in endurance limit due to smooth turning and to rough turning, as compared with the standard finish, is about the same for the two steels.

The results of this study make it clear that the finish of a machine member subjected to fatigue may have an appreciable effect on its endurance strength. It also appears that fine grinding would probably be satisfactory as a commercial finish.

22. *Effect of Overstress.*—From the standpoint of Bauschinger's theory regarding failure due to fatigue, and also from the fact that in practice many machine members subjected to fatigue are occasionally overstressed, it seemed desirable to determine whether stressing a material above its endurance limit for a certain number of cycles and subsequently testing it in fatigue in the usual manner would result in any change in the endurance limit.

The material used in this study for the first series of tests was Steel No. 10, 0.49 carbon, sorbitic.

Six different combinations of overstress and number of cycles were used. The endurance limit of the material having been determined in the standard manner (See Fig. 26), about 12 specimens were subjected to 5000 cycles of stress, the unit stress used being 10 per cent higher than the endurance limit previously determined. After this treatment the specimens were tested in the usual manner with varying stresses to determine whether the endurance limit would now be different. In the same manner other series of specimens were tested with 20, 29, 29, 35, and 38 per cent of overstress, the number of cycles of applied overstress being 5000, 5000, 1000, 1000, and 100 respectively.

The material for these tests, just as was the case in the previous section, was taken from two batches of steel heat treated at two different times. The first batch showed an endurance limit of about 50 000 pounds per square inch and the second batch one of about 48 000 pounds per square inch. The specimens subjected to 10, 20, and 38 per cent overstress were taken from the first batch of steel, and the calculation in per cent of their endurance limits is based upon an endurance limit of 50 500 pounds per square inch. Those

subjected to 29 and 35 per cent overstress were taken from the second batch, and the percentage calculation is based on 48 000 pounds per square inch. The specimens with 10, 20, and 29 per cent overstress were subjected to 5000 cycles of overstress at 1500 revolutions per minute, the specimens with 35 per cent overstress, and one set of the 29 per cent overstress specimens to 1000 cycles at 1500 revolutions per minute, and the specimens with 38 per cent overstress to 100 cycles applied slowly by hand.

Fig. 30 shows the $S-N$ curves obtained from these experiments. Fig. 26 should also be consulted since it shows the curve obtained for this material when there was no overstress. It is evident from the curves that 10 per cent and 20 per cent overstress applied 5000 times, 29 per cent overstress applied 1000 times, and 38 per cent overstress applied 100 times do not appreciably reduce the endurance limit. On the other hand, 35 per cent overstress applied 1000 times reduces the endurance limit about 4 per cent, while 29 per cent overstress applied 5000 times reduces the endurance limit about 11 per cent.

These last two results illustrate clearly the effect that the number of cycles of applied overstress has on the percentage reduction of endurance limit. Such a result is of course to be expected, because it is evident that if the higher stress were continued it would make the specimen fail at a stress on the sloping part of the diagram in Fig. 26. Since, however, this higher stress is not continued up to complete failure, it seems obvious that a smaller stress would take fewer cycles to cause failure than would normally be the case had there been no overstress. A few calculations indicate that the amount which the endurance limit is reduced in any case depends upon the ratio of the number of cycles of overstress to the total number of cycles of this stress which would cause failure.

The results for the specimens with 20 and 38 per cent overstress indicate that a machine member can withstand a considerable percentage of overstress without damage if that overstress is applied only a comparatively small number of times, and if the overload is not severe enough to make actual dents or nicks in the surface at points of localized pressure.

Fig. 31 shows some effects of overstress on specimens of steel No. 1, 1.20 carbon, sorbitic, subjected to a stress 20 per cent above

the normal endurance limit; in one case 5000 cycles of overstress were applied, and in the other case 10 000 cycles. The endurance limit seems to be slightly lower for the specimens subjected to 10 000 cycles of overstress than for the specimens subjected to 5000 cycles, and the reduction of endurance limit of the overstressed specimens as compared with material not subjected to overstress ranges from 12 to 14 per cent. Fig. 24, which gives the results for this material not subjected to overstress, should be consulted for comparison.

Comparing the results for 5000 cycles with 20 per cent overstress for the 0.49 carbon steel, sorbitic, and the 1.20 carbon steel, sorbitic, it is evident that the effect of overstress in reducing the endurance limit is more serious for the 1.20 carbon steel than for the 0.49 carbon steel.

The three studies of shape, surface finish, and overstress give a good idea of the relative seriousness of their effects in reducing the endurance limit. The maximum reduction noted in the overstress experiments was 14 per cent, that in the surface finish experiments was 18 per cent, while that in the experiments on shape was 60 per cent. While both overstress applied for a short time and poor surface finish reduce the endurance limit, the most serious reduction results from sudden changes in cross-section, especially in the case of machine members having square shoulders and sharp notches anywhere in their length.

Overloading a machine part may be accompanied by the formation of grooves and deep scratches due to wear, and these grooves and scratches may have a greater influence in reducing strength under repeated stress than the direct effect of the nominal overstress accompanying the overload.

23. *Theories of Nature of Fatigue of Metals.*—One theory of the nature of the fatigue of metals is the "crystallization" theory which had its origin in a study of the fractured surfaces of machine parts that had failed under repeated stress. These surfaces, even for soft material like wrought iron, appeared jagged and crystalline. This theory implied that under the action of repeated stress the internal structure of a metal changed from fibrous to crystalline, and that different metals varied in their resistance to this change.

By the use of the microscope all metals are seen to be of a crystalline structure under all service conditions, and no evidence of appreciable change of structure can be observed in a metal subjected to repeated stress. There does appear to be a breaking down of crystals as indicated by the formation of minute cracks or "slip bands" extending across the crystal. The crystallization theory has been practically abandoned.

A second theory was that advanced by the distinguished German investigator, Bauschinger,* that, under stress varying from zero to a maximum, it is possible for metals to acquire new elastic limits which for this range of stress may lie anywhere between the original proportional elastic limit and the yield point, and may in some cases even exceed the yield point. These new elastic limits, or "natural" elastic limits, as he called them, depend upon the number of cycles of stress to which the material is subjected, being higher for the greater number of cycles. For stresses within this natural elastic limit the metal is assumed to possess indefinite endurance, and he found experimentally that specimens under stresses varying from zero to a maximum would withstand several million repetitions of stress without failure. He did not, however, make any long-time tests under reversed stress. Bauschinger's theory does not imply a change of the crystalline structure but does imply some change in the inherent nature of the material.

The writers have seen very few published experimental results in which it was attempted to apply Bauschinger's theory to reversed stresses. Stanton and Bairstow† mention three specimens which had withstood 1 000 000 cycles of stress without failure and which were subsequently tested to determine their elastic limits. These specimens had been subjected to direct stress in which the ratio of tension to compression was 1.4 to 1. The results showed that in each case the new elastic limit was practically equal to the highest tension and compression which had previously been applied, although these new limits were very much lower than the original or "primitive" elastic limits.

* "Mittheilungen des Mechanisch-technischen Laboratoriums" der kgl. Techn. Hochschule, München, heft 13;

Dingler's Journal, Bd. 224;

Unwin, "The Testing of the Materials of Construction." 1910 edition, Chap. XVI.

† Inst. of Civil Engrs. (British), Vol. CLXVI, IV, p. 78, 1905-6.

Popplewell* determined the natural elastic limits by only 10 cycles of reversed direct stress and then attempted to check these values by rotating-beam tests. His natural elastic limits were lower than the primitive elastic limits, and the endurance limits from the rotating-beam tests were about equal to the primitive elastic limits. It seems to the writers that the check tests should have been made with a direct-stress machine instead of a rotating-beam, and it also seems questionable whether 10 cycles of stress would be enough to develop Bauschinger's natural elastic limits.

A series of tests to throw light on this matter is discussed in Section 36, "Effect of Repeated Understressing."

A third theory has been suggested by various investigators, and was developed by the writers, who afterwards found that it had been previously elaborated by Gilchrist.† This theory, while not directly contradictory to that of Bauschinger, presents a different picture of the mechanism of fatigue failure.

In discussing Wöhler's results Gilchrist has stated this theory as follows:

"(1) The average stress in the bars broken in Wöhler's machines did not reach the statical breaking load.

"(2) The fracture was caused by the statical breaking limit being exceeded at one point only, from which, when once started, rupture spread, at first rapidly, then more slowly, sometimes continuing to complete separation of the two parts of the bar, but occasionally stopping short of complete rupture.

"(3) The raising of the stress at the point where fracture commenced was due to an irregularity in the bar. This might be an irregularity or discontinuity in the metal, either on the surface or in the body of the bar.‡

"(4) A bar of uniform strength whose surface was perfectly smooth, with no sharp corners in the longitudinal configuration and the structure of which was perfectly homogeneous would endure, without breaking, an indefinite number of repetitions of a stress varying between zero and a value near to the breaking strength.

*Inst. of Civil Engrs. (British), Vol. CXC VII, III, p. 264, 1913-14.

†The Engineer (London), Vol. 90, p. 203.

‡The frequent occurrence of high localized stress which is usually overlooked in using the ordinary methods of computation is strikingly demonstrated by the mathematical work of Inglis and others, and by the experimental work of Coker and Preuss. See:

Trans. Inst. of Naval Architects (British), Vol. LV, Part I, p. 219, 1913;

Engineering, London, April 21, 1911, and March 8, 1912;

Zeit. des Ver. Deut. Ing., 1912, Part 1, p. 664;

"Johnson's Materials of Construction." Fifth Edition, p. 663

“(5) A bar similar to that under (4) could, under certain conditions, endure an indefinite number of repetitions of a load varying between tension and compression of equal values both beyond the ordinary primitive elastic limits.”

The writers agree in general with the foregoing statements, but, on account of the fact that a perfectly homogeneous material is not likely to be found for engineering purposes, they believe that the limitations of statement (4) as applied to ordinary metals should be set forth for the sake of clearness. It appears to them that the practical limit even for the case of stress varying from zero to a maximum will be the yield point of the material.

This theory may be called the theory of non-homogeneity or of localized stress. The effect of external non-homogeneity due to scratches, tool marks, square shoulders, and notches is well known and is discussed in another section of this bulletin. Internal non-homogeneity may be due to blow-holes, pipes, inclusion of slag, irregularity of crystalline structure on account of the presence of two or more constituents of varying strength, variation in orientation of crystals, or the presence of initial stresses caused by mechanical working or heat treatment. Owing to the minute area over which it exists, this localized stress produces no appreciable effect under a single load, but under load repeated many times there is started from this area a microscopic crack, at the root of which there exists high localized stress which under repetition of stress spreads until it finally causes failure.

The writers do not look upon these fatigue failures as being due necessarily to accidental flaws or irregularities. Such failures may, in practice, often be due to such causes, but the definiteness of the endurance limits found in the present tests points to the conclusion that the endurance limit is a property of the material just as much as the ultimate strength. If in these tests the failure is due to flaws, then it is believed that these flaws are an inherent part of the structure of the particular steel which is being tested.

As evidence supporting this theory the following facts are given:

(1) In the case of material of homogeneous structure, such as a specimen of 0.02 carbon steel, which is almost pure ferrite, or a specimen of 0.93 carbon steel, which is almost pure pearlite, it is found possible to develop an endurance strength nearly or quite

equal to the primitive elastic limit. This is a direct verification of (5) in Gilchrist's theory.

(2) In the case of steels composed of a mixture of ferrite and pearlite the endurance limit is found below the primitive elastic limit, indicating to the writers' minds a progressive failure, first of the weaker constituent, then of the stronger.*

(3) Mathematical analysis, special measurement of stress distribution, and repeated-stress tests of specimens with notches and specimens with rough surface finish all indicate how important mechanical imperfections in surface may be in increasing localized stress and hence in reducing fatigue strength. There would seem to be no reason for doubting that internal non-homogeneity produces a similar effect.

The actual mechanism of fatigue failure has not been studied for a great variety of steel structures. It is known that in the softer steels the failure seems to be due to the production of slip bands (see Appendix B) which finally develop into cracks. For a very homogeneous material like 0.02 carbon steel it seems probable that a certain unit stress will be sufficient to cause slipping in the gliding planes of the crystals. Thereafter, the adjacent material will begin to slip under succeeding repetitions of stress until all the other material has yielded the same amount. This action will continue until the first material has reached the ultimate strength and actually opened up a crack. From then on the action will be rapid and failure will occur soon.

Just what happens in a material like the chrome-nickel, treatment A, in which the endurance limit is very much below the proportional elastic limit is not altogether clear. It is likely that non-homogeneity and especially the presence of internal stresses play an important part.

*It should be borne in mind that localized weakness may be caused by strains due to heat treatment as well as by non-uniformity of metallographic structure. Tests reported verbally by D. J. McAdam, of the U. S. Naval Academy Eng. Exp. Sta., indicate that thorough annealing as well as homogeneity of structure seems to be effective in giving a high ratio of endurance limit to proportional elastic limit. This is further confirmed by test results on annealed steel of various carbon contents as shown by Fig. 44 and Table 10.

V. "RISE OF TEMPERATURE" METHOD OF DETERMINING ENDURANCE LIMIT

By W. J. PUTNAM and J. W. HARSCH*

24. *Thermal Phenomena in Metals Under Stress*.—The determination of endurance limit by the actual application of repeated stress, by a test on a rotating-beam testing machine, for example, requires so much time that it is not a practical commercial test and various other methods have been tried, such as the examination of the hysteresis loop, magnetic exploration, and calorimetric determination.

The fact was first mentioned by Lord Kelvin† in 1855 that a material subjected to stress within its ordinary static elastic limit is subject to a change of temperature, cooling if in tension, and heating if in compression. Since that time Turner,‡ Rasch§, and Capp and Lawson§ have made use of this thermal change as a method of determining the elastic limit in tension, defining it as the least stress at which the temperature of the specimen begins to rise.

In 1913 C. E. Stromeier** tried the method of determining the fatigue limit under alternating stress conditions by means of the heat generated due to inelastic action. He located the endurance limit at that minimum alternating stress which after a few hundred rapid repetitions will just generate heat in the test pieces. His determinations of heat were made by sensitive mercury thermometers immersed in a steady stream of water just before and just after it had flowed

*Professor W. J. Putnam of the Department of Theoretical and Applied Mechanics of the University of Illinois, and J. W. Harsch, Metallurgist with the Investigation of the Fatigue of Metals. Professor Putnam was responsible for the preliminary development of the apparatus and methods of testing, and Mr. Harsch developed the present form of apparatus used, and carried out the "rise of temperature" tests made in connection with this investigation.

†Quart. Math. Journal, 1855.

‡Trans. Am. Soc. of Civil Engrs., Jan., 1902.

§Proc. Int. Assn. for Test. Materials, Art. VII, 1909.

§Proc. Int. Assn. for Test. Materials, Art. IX, 1912.

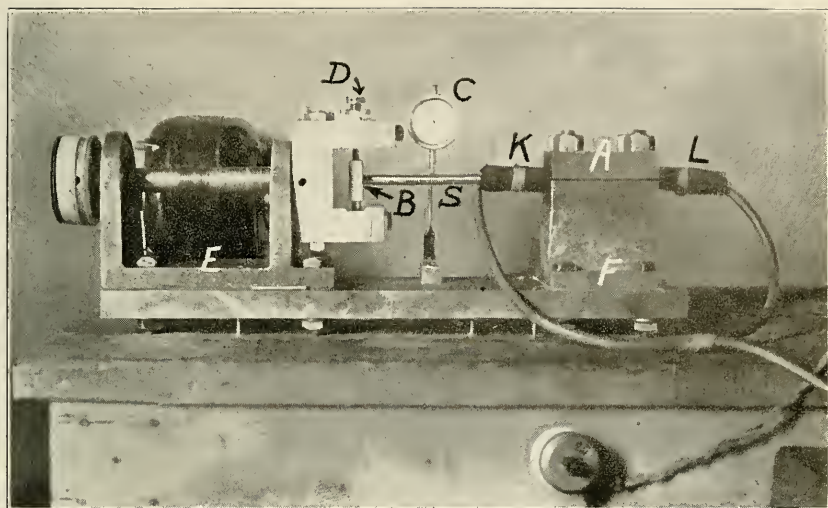
**Memorandum of the Chief Engineer of the Manchester (England) Steam Users Association, 1913. See also Engineering, London, June 19, 1914.

over the test piece, and he disregarded differences of temperature of less than 0.02 degree C. This method involves the determination of the quantity of heat generated, and the difference in temperature of the entering and leaving stream depends on the rate of flow. The work of Stromeyer was, however, incomplete, inasmuch as the actual fatigue tests carried out to check the calorimetrically determined endurance limits were for the most part carried to less than one million repetitions of stress.*

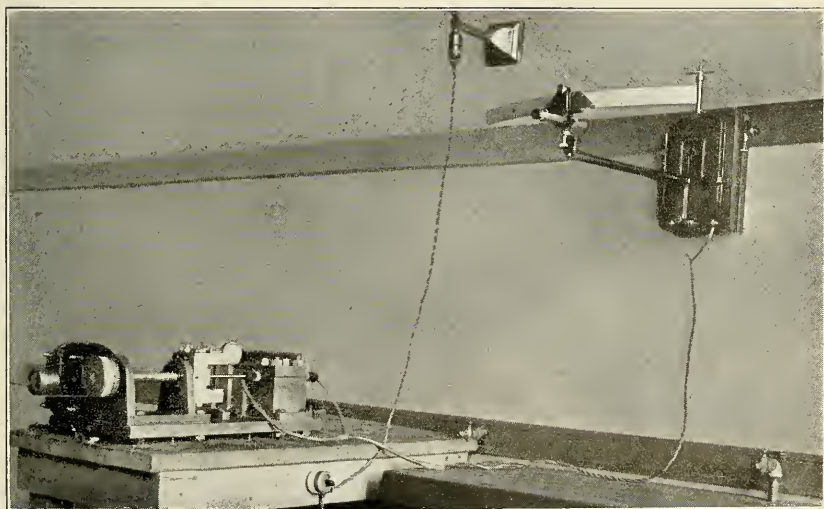
25. *Apparatus.*—The “rise of temperature” machine, Figs. 42(a) and 42(b), used in the present series of tests was designed to test the regular 13-inch, rotating-beam specimen, Fig. 18, and to produce the same type of stress as that produced in the rotating-beam machine used for the main series of tests described in this bulletin. As shown in Fig. 42(a), the specimen *S* is held horizontally in V-notch grips, *A*. The load is applied on a ball bearing, *B*, at the end of the specimen, the bearing being heat-insulated from the specimen by a fiber collar 1/16 inch thick. The machine is also heat-insulated from the base at points *E* and *F* to prevent heat traveling from the bearings through the base to the specimen. The load on the specimen is measured by reading the deflection of the Ames dial, *C*, a load-deflection curve for the specimen having been previously obtained by recording a set of loads and corresponding deflection readings. The load can be varied by an adjusting screw, *D*, in the head of the machine. A thermo-couple cannot be readily attached to a rotating specimen, and it will be noted that the machine is so designed that the specimen does not turn, but that the head of the machine rotates. The left-hand end of the specimen is thus moved in a circle concentric with its axis, the radius of the circle being that deflection corresponding to the desired stress.

After considering various methods of temperature measurement it was decided to measure the temperature change of the test piece itself, and it was first attempted to measure change of temperature by measuring the change in resistance of small coils of copper wire

* Since the preparation of the manuscript for this bulletin there has been reported a series of tests in the British National Physical Laboratory in which the “rise of temperature” test is shown to give a fairly accurate measure of the endurance limit. A special short-time deflection test is also recommended for determining the endurance limit. The tests were made by H. J. Gough and were reported in “The Engineer” (London), Aug. 12, 1921, and an abstract appears in “Mechanical Engineering” for October, 1921.



(a)



(b)

FIG. 42. MACHINE FOR "RISE OF TEMPERATURE" TESTS

held directly against the specimen. One coil was attached at the section of greatest stress and the other to the unstressed portion of the specimen just beyond the holding clamp, thus neutralizing the effect of any change in temperature of the machine. The coils having been balanced on a Wheatstone slide-wire bridge, any change in temperature would unbalance the system by an amount which, as indicated by the galvanometer, would denote the amount of change in temperature. The unbalancing, however, changed the amount of current flowing through each coil and therefore caused an added change in temperature of the coils. Then, too, the coils were so delicate that the repeated handling and the slight friction to which they were subjected soon destroyed the insulation on the wires.

Differential copper-constantan thermo-couples made from No. 30 *B* and *S* gage wire were then used. These were attached in locations corresponding to those used for the resistance coils, the one, *K*, at the section of greatest stress being tightly taped against the steel, and the other, *L*, separated from the steel by a single layer of thin paper. This arrangement proved very satisfactory and calibration showed it to be sensitive to 0.003 degree C. The couples were connected in series with a Leeds and Northrup, Type H, D'Arsonval galvanometer, shown in Fig. 42(b). When the couples were at the same temperature no deflection was shown, but when either varied in temperature from the other a corresponding deflection occurred.

26. *Materials*.—By the rise of temperature method several steels in the general series were tested, thus checking the reliability of this method since the endurance limit of each steel was definitely known from long-time tests, and the series covered a wide range. Duplicate and triplicate tests were made on several of the steels to ascertain the reliability with which the endurance limit could be determined, and the results thus obtained were well within the limits of the precision of stress adjustment.

27. *Tests*.—The test finally adopted as a standard consists in running the machine for 30 seconds at a speed of 1000 revolutions per minute at a known stress and recording the maximum deflection of the galvanometer. Then, with a series of such readings cor-

responding to a series of stresses, a curve is plotted to show the increase in temperature with increase of stress. The point at which the curve shows a sharp break corresponds to the endurance limit. Typical curves are shown in Fig. 43. In the cases of carbon steels with sorbitic structures and in the case of alloy steels it was found necessary to increase the time of the run, at each stress, from 30 seconds to 2 minutes, since these materials are slower in heating under stress.

28. *Results and Discussion.*—Endurance limits determined by the rise of temperature tests are shown in Table 8. Besides the

TABLE 8
COMPARISON OF ENDURANCE LIMIT FOUND BY ROTATING-BEAM TESTS WITH
THAT FOUND BY "RISE OF TEMPERATURE" TESTS

No.	STEEL	Endurance Limit, lb. per sq. in.	
		Rotating-beam Test	"Rise of Temperature" Test
1	1.20 carbon, normalized.....	50 000	43 000
	sorbitic.....	92 000	88 000
3	0.52 carbon, normalized.....	42 000	41 500
	sorbitic.....	55 000	53 000
4	0.37 carbon, normalized.....	33 000	31 000
	sorbitic, treatment B.....	45 000	49 000
5	Chrome-nickel, treatment A.....	68 000	66 000
	treatment B.....	65 000	65 000
	treatment C.....	67 000	70 000
6	0.93 carbon, pearlitic.....	30 500	28 500
	sorbitic.....	56 000	53 000
	troostitic.....	98 000	90 000
7	3.5 nickel, treatment B.....	63 000	62 500
9	0.02 carbon, as received.....	26 000	26 000
10	0.49 carbon, sorbitic.....	48 000	48 000
50	Cold-drawn, as received.....	41 000	41 500
	annealed at 1300° F.....	29 000	27 500
51	Hot-rolled, 0.18 carbon, as received.....	28 000	27 000
	reduced to 0.48 in.....	35 000	38 500
	reduced to 0.44 in.....	41 000	40 500
9	0.02 carbon, tested on Olsen-Foster reversed-torsion machine.....	12 500	13 000
	tested on Upton-Lewis reversed-bending machine.....	23 000	27 000

values obtained on the special machine there is also included a value for 0.02 carbon steel in torsion and a value in reversed bending. These values were taken from specimens tested in the Olsen-Foster reversed-torsion and Upton-Lewis reversed-bending machines respec-

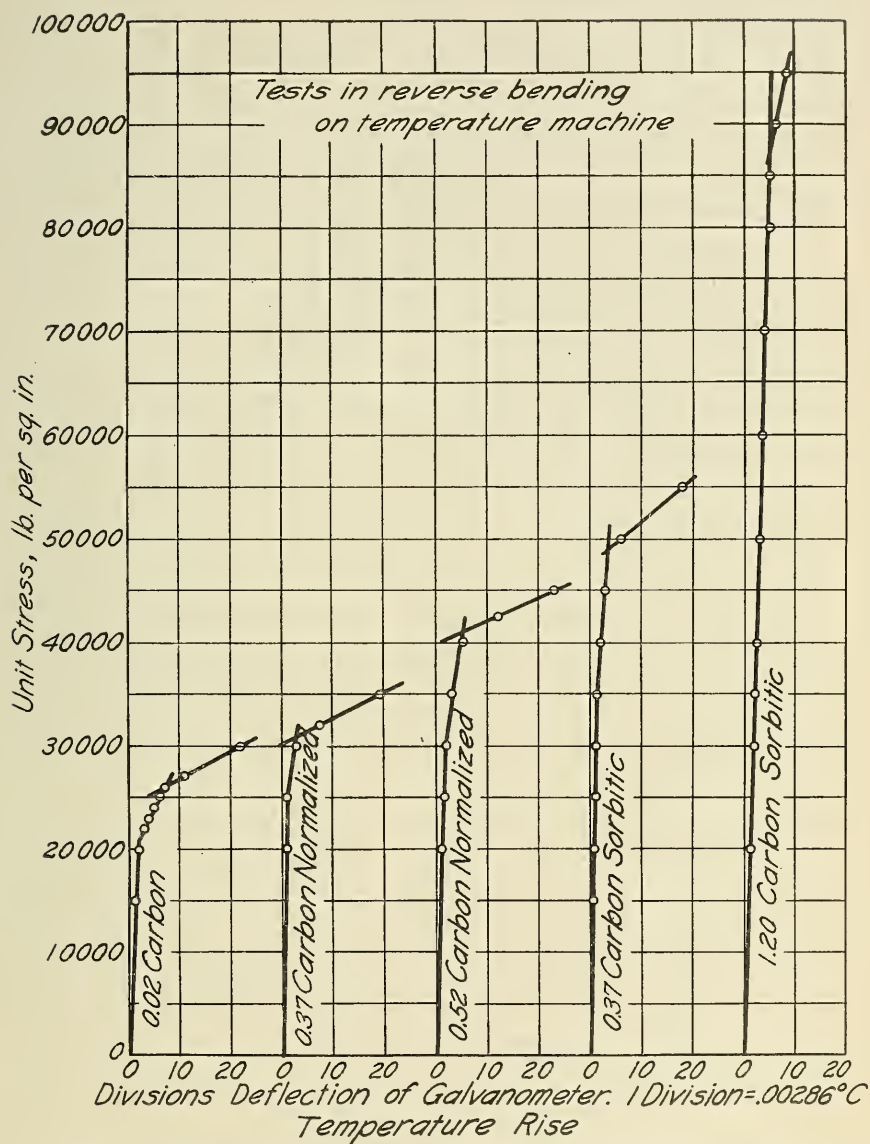


FIG. 43. REPRESENTATIVE DIAGRAMS FOR "RISE OF TEMPERATURE" TESTS

tively, with thermo-couples attached to the specimen. Table 8 also contains for the purpose of comparison the values of endurance limits as determined in the rotating-beam machine and in the temperature machine. This comparison is shown diagrammatically in Fig. 37.

There is a reasonably close coincidence between the endurance limits given by the long-time rotating-beam tests and those given by the temperature tests.

The method of determining the endurance limit from the curve obtained in a temperature test is shown in Fig. 43. The endurance limit is taken at the point shown by the intersection of the lines. The curves in this figure are typical. Most of the curves show a slight break considerably below the endurance limit. It is probable that there is at this point a very slight incipient damage in some weaker constituent, but that the load is still sustained by the stronger constituent and, as shown by actual tests, no failure results until the higher stress is reached. This first break decreases in amount with an increase of the carbon content in the normalized steels, and is also smaller when the steel has been given such heat treatment as to produce a sorbitic structure. It would seem that this first break may depend upon some property of the ferrite crystal and so may decrease in sharpness with a decrease in percentage of ferrite and with a more complete dispersion of cementite, such as occurs in sorbite. This is well shown by the curves in Fig. 43, since these curves represent steels which cover a wide range of carbon content and also show two different heat treatments.

It may be possible that this first break indicates the absolute endurance limit, i. e., the stress under which the material will withstand an infinite number of repetitions. This of course cannot be checked by actual applications of stress, but it is not impossible to conceive that, at the endurance limit determined by tests to 100 000 000 repetitions, the curve may still have a very slight downward trend, which, if the tests were carried to billions of repetitions, might bring the endurance limit to the value shown by the first break of the temperature curve. The characteristic curves also show a very slight but gradual increase in temperature with increase of stress below the endurance limit.

It is believed that a machine of this type, especially in conjunction with two or three rotating-beam machines, is a good piece of equipment for the average college or commercial laboratory, and that with such equipment the time and expense of fatigue investigations or commercial routine fatigue tests would be very materially lessened. With such equipment, using the temperature machine for a preliminary test, and the rotating-beam machine to obtain check results, it would be quite possible to determine endurance limits with a fair degree of accuracy within a few hours.

Problems worthy of further investigation include:

(1) An application of the principle of temperature measurement to a direct tension-compression fatigue machine and the determination of endurance limits under such stress.

(2) A more complete investigation of the first break in the curves by the use of more sensitive couples.

(3) A study of non-ferrous materials.

VI. SUBJECTS FOR FURTHER INVESTIGATION

29. *Effect of Repetition of Stress Not Reversed.*—The repeated-stress tests described in this bulletin have all involved complete reversal of stress. This is the severest type of repeated stress and is quite common in practice. It is, however, evident that many machine parts are subjected to varying stresses in which the variation does not involve a complete reversal. For example, stresses in a railroad rail vary from a maximum in one direction to a stress in the other direction of about one-quarter the magnitude of the maximum stress; chains are subjected to stresses varying from zero to a maximum. Comparatively little work has been done on the study of the effect of range of stress. In general it is estimated that a material which will withstand a completely reversed stress, S_1 , will withstand a stress of one kind only varying from zero to a maximum of S_2 equal to about 1.5 times the magnitude of S_1 , and that values between these limits may be obtained by interpolation.

Tests to determine the effect of range of stress are much needed, and in connection with the present investigation a machine has been designed and preliminary tests have been made under repeated stresses not completely reversed.

30. *Tests on Other Ferrous Metals.*—The tests recorded in this bulletin constitute a reconnaissance in the field of ferrous metals; they by no means constitute a complete study. Many other kinds of ferrous metals could well be studied, especially alloy steels, of which only two have been studied. Vanadium steels, molybdenum steels, and possibly tungsten and manganese steels at once suggest themselves. Methods developed in this investigation, especially the "rise-of-temperature" test, may be expected to shorten greatly the time required for determining endurance limits.

31. *Tests on Non-Ferrous Metals.*—This investigation in its first stages was definitely limited to the ferrous metals. The study should be extended to the non-ferrous metals—to brasses, bronzes, copper, aluminum, aluminum alloys, and possibly magnesium alloys.

Some tests on these metals have been made at commercial and government laboratories, but the data available are insufficient for drawing even tentative conclusions. Some experimental evidence tends to show that a higher number of repetitions of stress is necessary to develop the endurance limit for non-ferrous metals than for iron and steel.

32. *Effect of Heat Treatment.*—In the tests recorded in this bulletin, materials have been tested with various heat treatments. The object of variation was to produce certain characteristic structures in the metals. For many of the metals a much more detailed study of the effect of heat treatment on resistance to repeated stress should be made. It has been found in certain cases that the heat treatment which gives maximum static strength will not give maximum resistance to repeated stress, and many more tests should be made along this line. It is evident that the careful determination of the heat treatment which will give maximum resistance to repeated stress for any steel is a matter of great practical importance. The development of a reliable accelerated test for the endurance limit, such as the "rise of temperature" test, will greatly expedite such a study of heat treatments for ferrous metals.

33. *Direct Tension-Compression Tests.*—Under a bending stress only the outside fibers receive the maximum stress while the inside fibers of the specimens have a lower stress. A series of tests in which some specimens are subjected to direct tension and compression, while others are subjected to reversed bending should throw considerable light on the importance of the resisting ability of the understressed fibers of flexural members. A machine for producing repeated alternations of direct tension and compression has been designed and constructed and is being calibrated and adjusted.

34. *Strength Under Reversed Shearing Stress.*—In this bulletin test results are given for a few steels tested under reversed torsion in the Olsen-Foster machine. These specimens have been subjected to reversed shearing stress, and as pointed out in Section 15, "Some Factors Influencing the Endurance Limit," a ratio of endurance limit in shear to that in reversed bending of about 0.52 is indicated. Of late years it has become evident that shearing stress is frequently

the cause of failure in material, and a much more complete study of resistance to repeated shearing stress should be made. A machine for producing and measuring repeated alternations of shearing stress has been designed and constructed and is being adjusted and calibrated.

35. *Study of Mechanism of Fatigue Failure.*—The work of Ewing, Humphrey, Rosenhain,* and others on the development of slip lines in metals under stress has done much to explain why metals fail in fatigue. These experiments were generally made on soft metals; similar studies should be made on metals in the heat-treated condition, in order to examine the mechanism of fatigue more in detail. In this connection a study of the actual path of fracture in fatigue, with a microscope of high magnification, would be valuable.

36. *Effect of Repeated Understressing.*—Incidental to the study of overstressing in its effect on the endurance limit, discussed in Section 22, a few results were obtained on the effect of subjecting a specimen to a unit stress below the endurance limit and subsequently testing it at higher stresses. The few results observed indicate that such understressing may be beneficial in increasing resistance to fatigue, possibly by readjusting internal stress. Such tests also have a direct bearing on Bauschinger's theory of fatigue failure.

A study is contemplated in which a series of specimens will be stressed to, say, 10 per cent less than the endurance limit, for perhaps 10 000 cycles, and after this treatment subjected to various stresses in the usual manner, to determine whether any change has occurred in the endurance limit. By making several series of tests, using various percentages of understress, and with various numbers of cycles from 10 000 to perhaps 1 000 000, it will be possible to determine the effect of such understressing on the resistance to fatigue, and also the influence which the number of cycles of such understressing has on the result.

37. *Strength of Cast Metal Under Repeated Stress.*—The tests recorded in this bulletin, and nearly all other tests under repeated

* Rosenhain, "Metallurgy; an Introduction to the Study of Physical Metallurgy."

stress, have been made on rolled metal. The extensive use of steel castings and of castings of non-ferrous metals for stress-resisting parts makes some knowledge of the fatigue strength of such materials of importance. It is well known that the elastic strength of castings is low as compared with the ultimate strength, but there are no test results from which to judge whether this means that the strength under repeated stress is also low. Tests on castings would also give data concerning the effect of different crystalline structures on the fatigue strength of metals, since cast metals usually have a crystalline structure markedly different from rolled metals of the same chemical composition.

38. *Repair by Heat Treatment of Structural Damage Due to Repeated Stress.*—The question has often been raised whether a part which had been subjected to many repetitions of stress, a car axle after several years of service, for example, could be made “as good as new” by annealing, possibly followed by heat treatment. So far as is known no data exist on which to base an answer. An experimental study of this problem would quite possibly be closely allied to the study of the mechanism of fatigue failure. This general problem seems to be one of distinct importance.

VII. CONCLUSIONS

39. *Summary of Conclusions.*—The general conclusions to be drawn from the results obtained in the investigation may be summarized as follows:

(1) For the metals tested under reversed stress there was observed a well-defined critical stress at which the relation between unit stress and the number of reversals necessary to cause failure changed markedly. Below this critical stress the metals withstood 100 000 000 reversals of stress, and, so far as can be predicted from test results, would have withstood an indefinite number of such reversals. The name *endurance limit* has been given to this critical stress.

(2) In the reconnaissance tests made in the field of ferrous metals no simple relation was found between the endurance limit and the "elastic limit," however determined. The ultimate tensile strength seemed to be a better index of the endurance limit under reversed stress than was the elastic limit. The Brinell hardness test seemed to furnish a still better index of the endurance limit. The reason why the Brinell test, and, to a less degree, the ultimate tensile strength, seem to be better indices of the endurance limit than the elastic limit is not clear, and this result should be regarded as tentative. Elastic limits determined from compression tests and torsion tests gave no better index than did those from tension tests.

(3) The single-blow impact tests (Charpy tests) and the repeated-impact tests did not furnish a reliable index for the endurance limit under reversed stress of the ferrous metals tested.

(4) Accelerated or short-time tests of metals under repeated stress, using high stresses and consequent small numbers of repetitions to cause failure, are not reliable as indices of the ability of metal to withstand millions of repetitions of low stress.

(5) The endurance limit for the ferrous metals tested could be predicted with a good degree of accuracy by the measurement of rise of temperature under reversed stress applied for a few minutes. This relation is explicable in view of the intercrystalline and intracrystalline slippage under repeated

stress shown by the microscope. It is believed that this test, which is a development of a test proposed by Mr. C. E. Stromeyer, can be developed into a reliable commercial test of ferrous metals under repeated stress. Its applicability to non-ferrous metals has not been investigated.

(6) Abrupt changes of outline of specimens subjected to repeated stress greatly lowered their resistance. Cracks, nicks, and grooves caused in machine parts by wear, by accidental blows, by accidental heavy overload, or by improper heat treatment may cause such abrupt change of outline. Shoulders with short-radius fillets are a marked source of weakness.

(7) Poor surface finish on specimens subjected to reversed stress was found to be a source of weakness. This weakness may be explained by the formation of cracks due to localized stress at the bottom of scratches or tool marks.

(8) Stress above the endurance limits, due to either a heavy overload applied a few times or a light overload applied some thousands of times, was found to reduce somewhat the endurance limit of two ferrous metals tested.

(9) In none of the ferrous metals tested did the endurance limit under completely reversed stress fall below 36 per cent of the ultimate tensile strength; for only one metal did it fall below 40 per cent, while for several metals it was more than 50 per cent. However, these metals were to a high degree free from inclusions or other internal defects; the specimens had no abrupt changes of outline, and had a good surface finish.

(10) It is well known that subjecting steel to a stress beyond the yield point raises the static elastic tensile strength to a marked degree. The effect is less marked on the endurance limit, although some increase was observed for 0.18 carbon steel with the surface polished after being stretched well beyond the yield point. Annealing of commercial cold-drawn screw stock was found to lower its endurance limit somewhat less than it did its static elastic strength.

(11) The test results herein reported indicate the effectiveness of proper heat treatment in raising the endurance limit of the ferrous metals tested. Here again it should be noted that an increase in static elastic strength due to heat treatment is

not a reliable index of increase of endurance limit under reversed stress.

(12) The phenomenon known as "fatigue" of metals under repeated stress might better be called the "progressive failure" of metals. The most probable explanation seems to be that such failure is a progressive spread of microscopic fractures. A nucleus for damage may be a very small area of high, localized stress, due to a groove, a scratch, or a crack; in other cases failure may be due to internal inclusions or irregularities of structure; it may be due to internal stress remaining after heat treatment; it may be due to a grain or group of grains unfavorably placed to resist stress; or failure may begin in the weaker grains of a metal whose structure consists of two or more kinds of grains; or it may, of course, begin in any portion of the metal which, by accidental overload or otherwise, is stressed to the yield point.

APPENDIX A

COMPARISON OF ENDURANCE TESTS BEYOND THE YIELD POINT WITH
ENDURANCE TESTS WITHIN THE YIELD POINT*

In connection with a course in graphic methods given at the University of Wisconsin, use was made of some data from endurance tests, and it was found that the deflections and corresponding cycles for rupture, obtained in tests involving stresses beyond the yield point, plotted as a straight line on logarithmic paper.

The tests in question had been performed on a machine of special design in which the specimen was gripped in the base of the machine, the projecting end of the specimen being deflected back and forth over a free length of 4 inches. By using various deflections it was possible to determine a relation between the deflection of the specimen and the number of cycles of stress necessary to cause rupture.

The fact that this relation was represented by a straight line when plotted on logarithmic paper suggested the possibility of extending the curve in order to show values of cycles for rupture at deflections which were known to be within the elastic limit. By this means it would be possible to get a relation between unit stress and cycles for rupture, because within the elastic limit the formula connecting unit stress and deflection for a cantilever beam is

$$D = \frac{SL^2}{3EC} \text{ in which}$$

D = deflection of the beam in inches,

S = unit stress in pounds per square inch in outer fiber, corresponding to maximum bending moment,

L = length, in inches, over which deflection takes place,

E = modulus of elasticity of the beam material,

C = distance, in inches, from the neutral axis of the beam to the outer fiber.

The correctness of the method of extending the deflection-cycles curve would depend upon whether the relation between deflection

*This appendix is based on former experimental work done by Professor Kommers, mainly at the University of Wisconsin.

and cycles for rupture as found by tests beyond the yield point held also within the yield point. If this proved to be the case, there would be available a method of establishing the relation between any unit stress within the elastic limit, and the number of cycles at this stress which would cause failure. The practical value of this proposed method would lie in the fact that the deflection-cycles curve could be determined in a few hours, thus rendering unnecessary the time-consuming methods ordinarily employed in establishing the relation between unit stress and cycles for rupture.

In order to prove or disprove the correctness of the proposed method it was decided to make tests on a series of three steels varying in carbon content, determining first the relation between deflection and cycles for rupture by tests beyond the yield point. The resulting deflection-cycles curve, plotted on logarithmic paper, would then be extended or extrapolated, and by means of the formula connecting unit stress and deflection a relation between unit stress within the elastic limit and cycles for rupture would be determined. A series of reversed-stress tests on a rotating-beam specimen would then be made to determine by direct experiment the actual relation existing. If these two methods gave the same value for the relation between these quantities, it could be considered that the proposed method was justified.

Table 9 gives the chemical analysis of the material used in the tests. It was received in the form of $\frac{7}{8}$ -inch rounds. Steel No. 1 was annealed at 850 degrees C. for one half-hour, and steels Nos. 2 and 3 were annealed at 800 degrees C. for one half-hour. The tensile strengths of the steels are given in Table 10.

TABLE 9

CHEMICAL ANALYSES OF STEELS FOR SPECIAL SERIES OF TESTS IN THE
WISCONSIN ROTATING-BEAM TESTING MACHINE

Steel No.	Carbon	Manganese	Silicon	Phosphorus	Sulphur
1	0.42	0.59	0.07	0.019	0.031
2	0.56	0.55	0.08	0.023	0.035
3	0.80	0.51	0.12	0.029	0.036

TABLE 10

RESULTS OF TENSION TESTS OF STEELS FOR SPECIAL SERIES OF TESTS
IN THE WISCONSIN ROTATING-BEAM TESTING MACHINE

Steel No.	Proportional Elastic Limit lb. per sq. in.	Yield Point lb. per sq. in.	Ultimate Tensile Strength lb. per sq. in.	Elongation in 2-in. per cent	Reduction of Area per cent
1	38 900	40 000	75 500	32.5	49.3
2	40 300	45 300	87 200	27.0	35.6
3	47 800	53 300	110 300	11.0	11.2

Note: Each value is the average of two test results.

Figs. 5 and 6 show the Wisconsin machine used for making the rotating-beam fatigue tests. The specimen, shown in Fig. 19, is fixed into the shaft of the machine, concentric with it, by means of nuts and set screws. The load on the specimen is applied through a ball bearing by means of a spring balance. It will be seen, therefore, that the specimen is loaded as a cantilever beam and is rotated with the shaft of the machine. The speed used was 1200 revolutions per minute.

When this machine was being tested the unit stress on different specimens was gradually lowered until finally a unit stress was reached which the specimen withstood for 100 000 000 cycles of stress without failure. This result was unexpected, and immediately suggested the probability that a metal had a well defined endurance limit.

The fact that a steel has apparently an endurance limit had a direct bearing upon the experimental work which had been planned. Because experimental data were lacking to prove the contrary, it had been assumed that the inclined straight line obtained when plotting on logarithmic paper unit stresses and cycles for rupture gave the correct relation between those quantities even for large numbers of cycles of stress. While this assumption would be disproved by the existence of a well-defined endurance limit, yet it seemed worth while to continue the experiments for the purpose of determining whether even the inclined part of the *S-N* diagram obtained for stresses below the yield point was related to that obtained for stresses beyond the yield point.

The upper half of Fig. 44 shows the curves obtained from the endurance tests on which the specimens were stressed beyond the yield point, while the lower half shows the curves obtained from the endurance tests in which the specimens were stressed within the elastic limit.

The equations for the straight lines in the upper half of Fig. 44 are as follows:

For steel No. 1,

$$N = \frac{56.9}{D^{2.57}} \quad (1)$$

For steel No. 2,

$$N = \frac{64.27}{D^{2.57}} \quad (2)$$

For steel No. 3,

$$N = \frac{26.18}{D^{2.77}} \quad (3)$$

in which D is deflection of the specimen in inches, and N is the number of cycles for rupture.

From these equations and the equation connecting deflection and unit stress within the elastic limit, taking 29 000 000 pounds per square inch as the value of the modulus of elasticity of steel, the following equations were derived:

For steel No. 1,

$$S = \frac{4\,896\,000}{\sqrt[2.57]{N}} \quad (4)$$

For steel No. 2,

$$S = \frac{5\,151\,000}{\sqrt[2.57]{N}} \quad (5)$$

For steel No. 3,

$$S = \frac{3\,314\,000}{\sqrt[2.77]{N}} \quad (6)$$

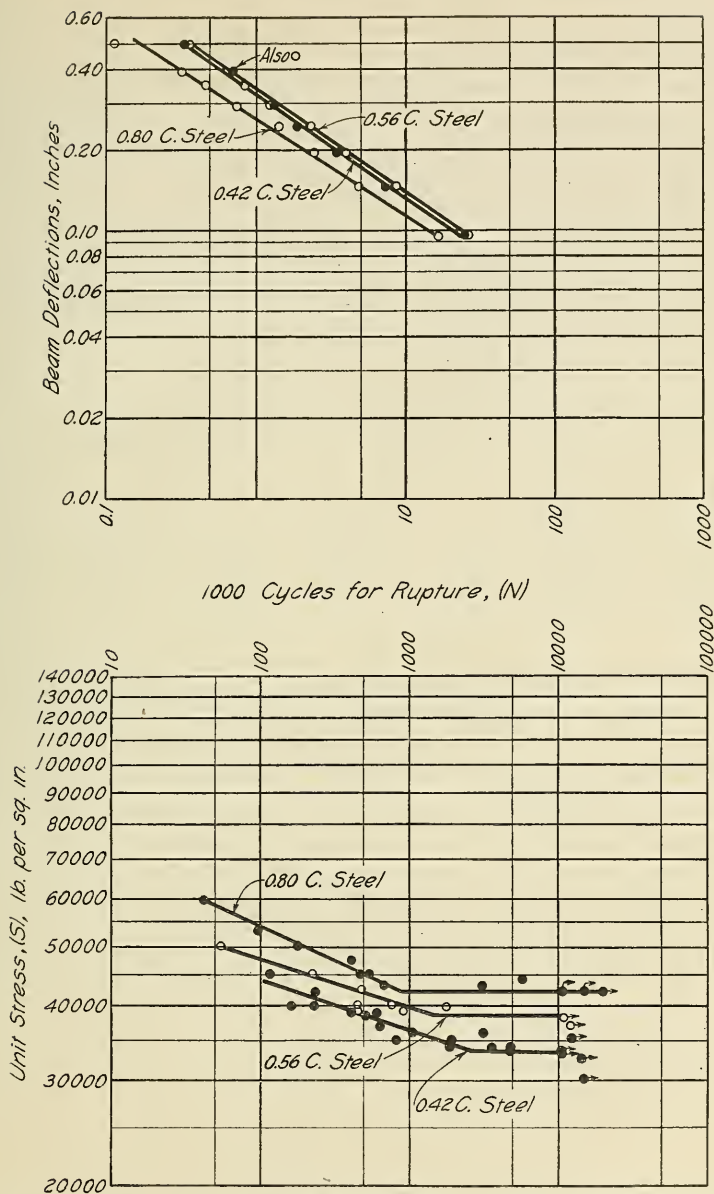


FIG. 44. $S-N$ AND $D-N$ DIAGRAMS FOR TESTS ON WISCONSIN ROTATING-BEAM TESTING MACHINE

Logarithmic abscissas give thousands of cycles for rupture

in which S is the unit stress in pounds per square inch, and N is the number of cycles for rupture.

Table 11 gives certain values of N and the corresponding values of S computed from these equations, and also for purposes of comparison values of S obtained from long-time endurance tests corresponding to the lower half of Fig. 44.

TABLE 11

COMPARISON OF COMPUTED VALUES AND ACTUAL VALUES OF STRESS FOR SPECIAL SERIES OF TESTS IN THE WISCONSIN ROTATING-BEAM TESTING MACHINE

N	Steel No. 1		Steel No. 2		Steel No. 3	
	Computed S	Actual S	Computed S	Actual S	Computed S	Actual S
50 000	72 700		76 500		63 600	58 300
100 000	55 500	44 000	58 400	43 000	51 900	54 000
500 000	29 700	38 500	31 200	38 000	29 000	44 800
1 000 000	22 700	36 500	23 800	36 800	22 600	42 000
2 000 000	17 300	34 500	18 200	34 800	17 600	42 000
3 000 000	14 800	33 500	15 500	33 800	15 200	42 000

This table makes it quite clear that even that part of the S - N curve between the endurance limit and the yield point cannot be predicted from equations such as (4), (5), and (6), derived from short-time tests beyond the yield point. It is evident, therefore, that the relation existing between deflection and cycles for rupture for stresses beyond the yield point is different from that existing within the yield point. Furthermore, calculation shows that there is no constant relation between the computed and the actual values of S given in Table 11. It is also impossible to make the computed values of S agree with the actual values by changing the exponent of N in the equations, for calculation shows that an exponent that will make the values agree for a value of $N=50\,000$ will not do so for a value of $N=3\,000\,000$.

APPENDIX B

FATIGUE PHENOMENA IN METALS

The following pages give the substance of a progress report prepared by the Engineering Division of the National Research Council Committee on Fatigue Phenomena in Metals and published in "Mechanical Engineering" for September, 1919.

This committee of the National Research Council was made the Advisory Committee for the Joint Investigation of the Fatigue of Metals. Those paragraphs of the 1919 progress report which deal with plans for an investigation of fatigue phenomena in metals are not printed here, since they are given in substance either in the parts of this bulletin dealing with test results or in Chapter VI, "Subjects for Further Investigation." It should be remembered that the progress report given in this appendix was written before the main part of the bulletin, and that it is to be expected that the results of the investigation will be found to modify some of the views given in the first progress report.

In this appendix references are made by number to articles listed in the Bibliography given in Appendix D.

Metal parts of machines, such as springs, shafts, crankpins, and axles, occasionally fail suddenly while only subjected to conditions of ordinary service. Not only does failure occur suddenly, but the part about to fail shows no ordinary evidence of weakness. The broken parts when examined are seen to be broken off short, and without general distortion, even though the material may show high ductility in ordinary tests. Such failures are found only in parts subjected to stress repeated many times—to "vibration," as it is sometimes stated—and the phenomena which are involved in the final failure of metal through oft-repeated loading are known as "fatigue" phenomena of metals.

The phenomena of fatigue failure have recently given rise to some perplexing problems in connection with the design and service of airplane-engine crankshafts, the hulls of steel ships, axles and shafts in railway cars, motor cars and trucks, and other machine parts. The question whether structural parts subjected to repeated stress are in danger of fatigue failure has been discussed at considerable length. The danger of fatigue failure seems to be an unimportant factor in determining the safety of structural parts, with the possible exception of parts subjected to reversal of stress. The reason for this is probably found in the relatively small number of loadings which most structures are

called upon to withstand, and in the fact that most of the loadings are below the maximum working value. On the other hand, the danger of fatigue failure is a major factor in determining the safety of many machine parts.

The problem of fatigue of metals engaged the attention of engineers seventy years ago, in connection with car axles and members of iron railway bridges. It was early recognized that high stress tends to cause, or at least to hasten, fatigue failure, and about 1860 Wöhler's famous investigations 132(b) were undertaken to determine the relation between intensity of fiber stress and the ability of materials to resist fatigue under repeated stress. Wöhler's tests occupied some eleven years, and remain to this day the most thorough tests on record. Wöhler investigated metals under direct tension, under bending, and under torsion (shear). For some of his tests the stress varied from zero to a maximum and for others the stress was reversed. The results of Wöhler's tests may be summarized as follows:

- (1) A machine part or structural member may be ruptured by the repeated application of a load which produces a computed fiber stress less than the ultimate strength of the material as determined by a static test.
- (2) The greater the range of stress, the lower the limiting fiber stress to insure against rupture after a very large number of repetitions.
- (3) To insure against rupture after a very large number of repetitions of loading causing complete reversal of stress, the limiting fiber stress is but little greater than one-half the limiting fiber stress for a very large number of repetitions of stress varying from zero to a maximum.

Following Wöhler the famous Bauschinger^a published a series of conclusions on fatigue, and various other investigators, notably Gerber,^{125(b)} and Weyrauch and Launhardt,^{68(d)} gave early interpretations of the experimental results of Wöhler and Bauschinger.

In these earlier experiments several facts seem noteworthy. The prime object of the investigators was to deduce laws of fatigue for railway bridges and car axles. The problem of fatigue in high-speed machine parts had not then appeared. These investigators carried their tests far enough to cover the number of repetitions required by the structures of their day and assumed that having done so they had established an endurance limit. Reading their conclusions carefully, the statement does not seem to be made that material which passed their tests would stand an *infinite* number of repetitions. The term generally used is "indefinite" or "very large," and the number corresponding is from ten to fifty millions. For the problem which they investigated their tests seem to give safe guides for practice, but today, with the advent of modern high-speed machinery, some parts of which must be as light as possible, and the extension of the fatigue problem to such members as the cranks and the connecting rods of gas engines and the shafts of steam turbines, the number of repetitions of stress which a machine member may be called upon to undergo is very much increased. This fact is illustrated by Table 12, which gives a statement of the approximate service required from various structural and machine members.

Investigations have been made in recent years by Howard,⁵⁶ Stanton,^{112, 113, 114, 115} Basquin,⁸ Smith,¹⁰⁷ Eden, Rose and Cunningham,²⁸ Kommers,⁶⁸ Mason,⁷⁴ Moore and Seely,⁸³ and others. The efforts of these investigators

TABLE 12

APPROXIMATE SERVICE REQUIRED OF VARIOUS MEMBERS OF STRUCTURES
AND MACHINES SUBJECTED TO REPEATED STRESS

Part of Structure or Machine	Approximate Number of Repetitions of Stress in the "Lifetime" of the Structure or Machine
Railroad bridge, chord members.....	2 000 000
Elevated-railroad structure, floor beams.....	40 000 000
Railroad rail, locomotive wheel loads.....	500 000
Railroad rail, car wheel loads.....	15 000 000
Airplane-engine crankshafts.....	18 000 000
Car axles.....	50 000 000
Automobile-engine crankshafts.....	120 000 000
Lineshafting in shops.....	360 000 000
Steam engine piston rods, connecting rods and crankshafts.....	1 000 000 000
Steam-turbine shafts, bending stresses.....	15 000 000 000

have been toward the study of modern materials, refinements in methods of testing, and interpretation of results. The limits of actual tests have not been extended to modern requirements, and the problem still remains of obtaining test data for much longer endurance of fatigue than was contemplated by Wöhler. Under the most favorable conditions conceivable such data will be obtained very slowly, and meanwhile there must be faced the problem of determining safe stresses for very large numbers of repetitions by extrapolation from previous test results.

MACHINES FOR TESTING FATIGUE STRENGTH

Fatigue tests cannot readily be carried out with ordinary "static" testing machines. It is, of course, possible to repeat loadings on a test specimen in such a machine, but the process is very slow. Such a machine equipped with an ingenious automatic arrangement for applying and releasing load was used by Van Ornum¹²⁸ in fatigue tests of concrete in compression, but the time required for even a hundred thousand cycles of stress was very great.

A very simple repeated-stress testing machine acts by the application and removal of a weight to the end of the long arm of a simple or compound lever, the specimen carrying load at the short arm. Such a machine was used by Berry in fatigue tests of concrete in compression. In a machine of this type the load must be applied slowly, else there will be inertia forces set up by the impact of the weight as it is let into place.

A common type of repeated-stress testing machine is one in which a calibrated set of springs resist the tensile, compressive, flexural, or torsional stress set up in the specimen, and the deformation of the calibrated set of springs gives a measure of the force or moment acting on the specimen. Fig. 45(a) diagrammatically illustrates this type of machine which was used by Wöhler and has since been used by many other experimenters. The Upton-Lewis machine is of this type and extensive use was made of it in torsion tests carried on by McAdam.^{85(b)} This type of machine permits a fairly high rate of repetition of cycles of stress, and machines which have been run at 1000 repetitions per minute have given results apparently trustworthy.

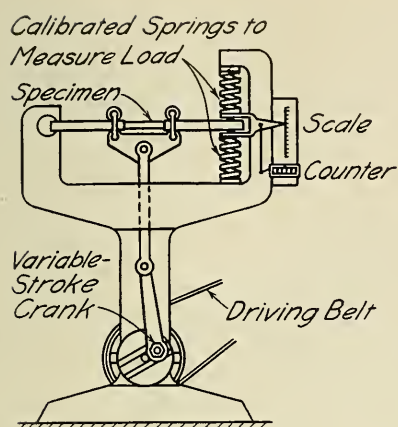
The most common type of machine for reversed bending stresses uses a circular specimen acting as a rotating beam. This type was used by Wöhler, and also by many later investigators. Fig. 45(b) illustrates such a machine. The specimen is in the form of a bar of circular section, to which bending stress is applied by weights. The specimen is rotated by means of a pulley. At any instant the outer fibers are subjected to a stress varying from tension on one side to compression on the other, and the fiber stress at any point passes through a cycle of reversed stress during each revolution. As shown, the specimen is loaded at two symmetrical points of the span, and between these two points the extreme fiber stress is constant for each element along the bar. This type of machine permits high speed of reversal of stress, speeds up to 2000 revolutions per minute having been successfully used.

British experimenters have used repeated-stress testing machines in which varying stress was applied to a specimen by means of the inertia of reciprocating parts. Fig. 45(c) shows such a machine, which can be used at high speeds. However, the speed must be very closely controlled, as the inertia forces vary with the square of the speed. Moreover, friction on the guides causes some slight uncertainty as to the magnitude of stress set up at each stroke of the crank.

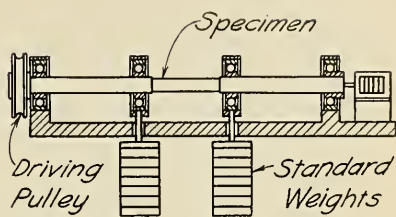
A repeated-stress testing machine depending on centrifugal force to produce cycles of stress is shown in Fig. 45(d). It is evident that as the eccentric weights revolve the specimen will be placed alternately in tension and in compression. This machine has been used by J. H. Smith.^{107(a)} Its characteristics are much like those of the inertia type; in fact, it is a special form of inertia machine.

A type of machine used by Arnold and later by other experimenters is shown in Fig. 45(e). In this machine a specimen is repeatedly given a certain deflection. Usually this deflection is sufficiently large to stress the material well beyond the yield point, and no very definite stress can be computed. This machine is used mainly for short-time tests.

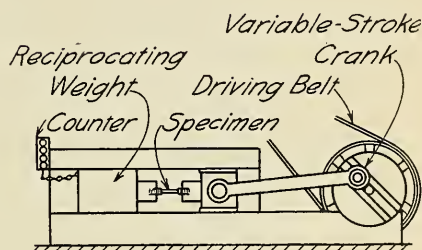
Another short-time-test machine uses the repeated impact of a small hammer. The claim is made that impact loading emphasizes local flaws better than a load which is more gradually applied, and that thus it *indirectly* gives a better index of fatigue strength. Data, however, are lacking to prove or disprove this claim.



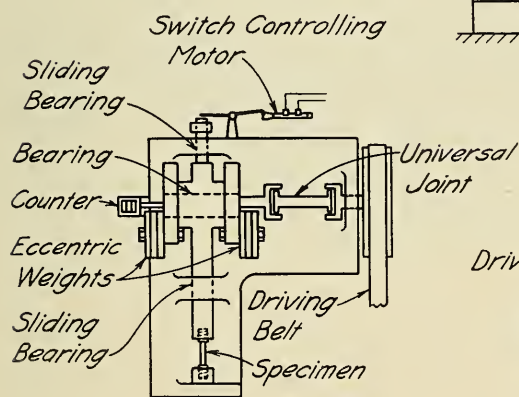
(a)



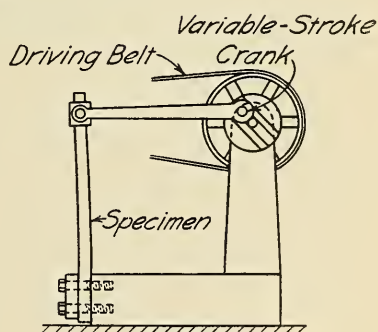
(b)



(c)



(d)



(e)

FIG. 45. DIAGRAMS OF TYPICAL REPEATED-STRESS TESTING MACHINES

Various repeated-stress testing machines have been constructed in which the cycles of stress were set up by the action of an electromagnet energized by alternating current. Usually the stress was measured either by the deflection of a spring or by the deformation of a standard test bar attached to the specimen. The speed of such a machine, however, is usually so high that there seems to be some uncertainty as to whether the successive waves of stress pass through the specimen without interference.

While the microscope can hardly be classified as a testing machine, it has, nevertheless, been of such vital importance in studying fatigue phenomena that it may well be mentioned in this place. Space will not permit of a detailed description of the methods employed in the microscopic examination of metals, but the process involves polishing a small area of the metal, etching the surface with some reagent to bring out the lines of the crystalline structure, and examining and photographing the surface through a microscope, the surface of the metal being illuminated by means of a reflected light.

THE PHENOMENA OF FATIGUE FAILURE

A fatigue failure of a metal, whether it occurs in a test specimen or in a machine part, is characterized by suddenness, lack of warning, apparent brittleness of material, and, in many cases, a fracture with a crystalline appearance over a part of its surface.

This crystalline appearance led to the old theory that under repeated stress metal "crystallized," changing from a ductile "fibrous" structure to a brittle "crystalline" one. This theory, however, has been quite thoroughly demolished as a result of study of the structure of steel under the microscope. As revealed by the microscope the structure of all metals used for structures and machines is crystalline, any "fibrous" structure being caused by inclusions of non-metallic impurities (for example, slag in wrought iron). Microscopic examination of metals under stress shows no change of the general scheme of internal structure, but under sufficiently heavy stress there appears gradual breakdown of the crystals in the structure.

When a ductile metal is deformed cold, the first deformation occurs in the particular grains which either take the most stress or have the lowest elastic limit. Deformation takes place by the slipping of one portion of the grain with reference to other portions. This slipping is shown by the appearance of lines called "slip bands" or "slip lines" extending across crystals and indicating planes of cleavage, as shown in Fig. 46. As the load is increased deformation proceeds and other slip bands are formed, the law being that the most easily deformable grains first show slip bands. Gradually the most favorable planes of slip are exhausted, and further slippage can take place only with the addition of more load.

The failure in ductile metals, subjected to repeated stress takes place with substantially no general deformation. There is, however, considerable local deformation over microscopic areas, evidenced by the appearance of many slip

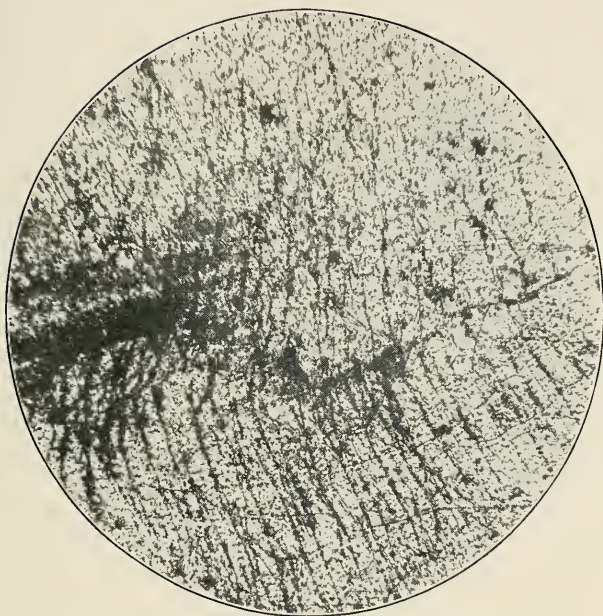


FIG. 46. MICROGRAPH SHOWING SLIP LINES IN IRON

Magnification 80 diameters

bands on a polished surface of the metal after the application of repeated stress. These slip bands appear after a small number of reversals of stress with relatively large loads, and may not appear at all with slight loads. The slip bands may first appear either in the interior of a grain or at the grain boundary. As the number of applications of stress increases more slip bands appear, and those first appearing usually lengthen and widen. Under the microscope and with normal illumination the general surface becomes blacker as the number and width of the slip bands increase. In ductile metals fatigue failure is almost exclusively through the grains themselves rather than at the grain boundaries, and the first slip bands to appear do not necessarily form a part of the final path of rupture. Failure seems to take place by the uniting of slip bands into cracks. When the first grain develops a crack extending across its entire width, added stress promotes the extension of this crack into adjacent grains on both sides, although the orientations of these grains may be and usually are such that the crack must extend itself at an angle to that in the initial grain. The general tendency is for these slip bands to follow the lines of cleavage of the particular grain in which they occur. Often incipient fracture is found in many grains adjacent to the final path of rupture, indicating that had rupture not taken place where it did, it would have soon taken place in some other adjacent part.

Such observations by means of the microscope indicate that *localized deformation* is the primary cause of fatigue failure in ductile metals, but it does not necessarily follow that the formation of one slip band under repeated stress will indicate eventual fracture if the loading is continued; one grain may appear to have a greatly reduced elastic limit because of internal strains or peculiarly unfavorable orientation. It is not certain that there is a limiting load below which fatigue will never take place.

Materials classified as brittle have very little permanent deformation under static stress, and under repeated stress the progressive fracture of brittle material might take place, not by slipping within crystals, but by tensile fracture of crystals. There has been practically no study made of the fracture of brittle materials under repeated stress, and it would be instructive to have tests carried out on brittle amorphous materials such as fused silica and on brittle crystalline materials like marble or tungsten. It is gradually being recognized that the breaking load of a specimen is a complex matter, and depends, among other things, on the time of application of the load. Mere duration of static loading, however, does not have an effect at all comparable with repetition of loading in reducing the breaking load. It seems evident that the distribution of stress in some brittle materials is very much less uniform than in ductile materials, and that fractures in brittle materials start on areas of high stress, whereas in ductile materials the high stresses are relieved by local yielding. A more complete understanding of the mechanism of rupture in brittle materials would doubtless be of great value.

When the action of metal under repeated stress is considered from the viewpoint of the internal strains and accompanying stresses in the material, a

radical difference is seen in the behavior of material under static load and under repeated load. In a general way we may consider any structural or machine part as subjected to static conditions if the load on it is applied gradually and is not repeated more than a few hundred times; the part may be considered as subjected to fatigue if the load on it is applied, say, one hundred thousand times or more; and for intermediate conditions of loading the phenomena characteristic of both kinds of loading would be present.

We must look upon steel as filled with a multitude of minute flaws. These flaws are developed in the solidification of the material. In static testing, steel under stress of about half its ultimate strength passes into a semi-plastic condition, in which there is a gradual flow of the material. Under such conditions the small flaws have almost no effect upon the flow or upon the static strength. When steel is loaded to moderate stresses the yielding is almost entirely elastic, but in general a small portion of it is inelastic, energy being taken up by the steel itself. If the specimen can be loaded a great number of times without heat loss its temperature will increase. If it is set vibrating in a chamber free from air it will stop vibrating in a short time, due to the absorption of energy. In such cases the stress-strain curve appears to be straight and the curve for the removal of the load may be practically identical with that for the application of load, but still these other effects show loss of energy in the steel itself.

This loss of energy is doubtless due to small displacements at these flaws, which are not reversible. Under alternate loadings these displacements are made back and forth. Energy is continuously being absorbed in the location of these small flaws, and it is perfectly natural that they should increase in size. We must look upon these extensions of the flaws as occurring in a great many parts of the steel. If the stresses are small the increase in size of these flaws is practically negligible, but if the stresses are larger the increase is rapid, and later on in the history of the piece under test, very rapid, and finally the strength of the piece is terminated when a sufficient number of these flaws have connected so as to form an area of very great weakness.

For static loads all the above is of little consequence with a ductile metal; but it is of consequence in the case of a brittle metal like cast iron, which has a remarkably low strength in tension in comparison with its compressive strength. Ductile metals may be considered as having a very high value for cohesion with a rather low coefficient of friction—or whatever corresponds to that—so that these metals begin to slide on diagonal planes without actual fracture under high local stress.

When a ductile material is loaded it may be subjected to stresses whose average values for small areas are not very different for parts that are a tenth of an inch apart; but there is a multitude of tiny spots whose fiber stresses are 2, 3, 4, or 10 times the average value,^{61, 92, 21(7)}. This holds so long as elastic conditions obtain. As the applied stress increases some of these stresses increase in like ratio, but not in like increments. At moderate applied stresses these special stresses reach inelastic conditions and slipping occurs. If the average stress is now entirely removed, we may assume that the unloading takes

place in a similar manner. The small spots unload first in an elastic manner, but at a different rate than the remainder of the material. They will unload approximately at the same rates as they used in taking stress, namely, 2, 3, 4, or 10 times the normal rate. For the unloading they have about twice the range of stress that they had in the loading before inelastic action is set up. Some of them will reach the opposite limit and slip part way back again, while some of them may not be subject to this return slip of inelastic action, but will retain in the unloaded state a stress distribution of the opposite kind. Either of these actions will give rise to hysteresis and to slight change of dimensions, usually, however, too small to be detected.

In all of the foregoing the main parts of the material have not been subjected to stresses which give inelastic action. If the loading is repeated without reversal, the spots that slipped on the first unloading will be subjected to further slipping both on loading and on unloading, but the areas that suffered no slipping on the first unloading should show no further inelastic action unless the loading is reversed. If the loading is reversed, however, all the particles that slipped in the first loading will slip on the reverse loading, so that with repetition a larger number of spots undergo this slipping action than is the case with loading which is not reversed. This explains the shorter life under reversed stress than under repeated stresses in one direction only.

If the fractured surface of a "rotating beam" specimen made of ductile metal and broken by repeated stress is examined, it is usually seen to be made up of two parts: (1) near the extreme fibers there is a dark surface with a dull, lusterless appearance, while (2) the remainder of the surface has a bright crystalline fracture. If these are examined more carefully it is found that their principal difference is in the size of the small flat surfaces that constitute the fracture. The center portion of the area has comparatively large surfaces, giving a crystalline effect, while the dull gray portion has very small surfaces of fracture.

An explanation of this is that the flaws in the outer portion of the surface have connected to form an annulus, whose rugged face is roughly at right angles to the axis of rotation. This has doubtless occurred slowly, and has started from many centers, thus giving the rough face. After this slow growth of flaws into an annular fracture has been accomplished the specimen has become very weak and the stresses have become so large at the fracture that they suddenly tear the metal in two on the natural surfaces of cleavage of the crystal grains.

The center portion of this fractured surface does not differ from the crystalline surface at the bottom of a cup in an ordinary static tension fracture, except that the crystalline surfaces are somewhat larger. This is to be explained by the fact that in an ordinary tensile test the material at the fracture has elongated something like 100 per cent, so that the crystal grains have become of smaller cross-section and will naturally show smaller facets on fracture, whereas, in a fracture of the endurance specimen, the material has had no chance to elongate and the crystalline grains have their normal size, which will be

shown in fracture. It is not the crystalline portion of the broken specimen which has failed primarily by repeated stress, but the dull portion. In the crystalline part of the fatigue fracture and in the crystalline part of the static tension fracture the failure seems to be of the same nature, namely, a failure in cohesion.

In considering the phenomena of fatigue failure it may be well to call attention to the fact that there is an intermediate type of failure of ductile material in which both plastic action and the development and spread of microscopic flaws are present. Such failures sometimes occur in staybolts, boiler sheets between rivet holes, and other parts occasionally subjected to very severe local distortion.

LOCALIZED STRESSES UNDER STATIC LOADING AND UNDER IMPACT LOADING

When a machine member or structural part is loaded gradually a state of strain and accompanying stress is set up throughout it. In a general way the distribution of stress is similar to that given by the theory of elastic action which serves as a basis of our formulæ for computing stress and strain. There are, however, many deviations from this distribution due to non-homogeneity of the material and to irregularities in outline such as projecting corners, scratches, and tool marks. When load is applied the general behavior of the piece as indicated by careful measurements of stretch, compression, twist, or flexure conforms to that required by the common theory of elastic action, but there are doubtless many localized strains which cannot be detected, even by the use of delicate micrometer measurements. It is to be recalled that in measuring strains it is necessary to use a gage line of considerable length, with the result that the observed strain is an average value along a relatively long line. The localized stresses, corresponding to these undetected localized strains, are not of any great importance under static load. When the load is increased to such an extent that a considerable portion of the piece is stressed beyond the elastic limit, the distortion of the piece increases abnormally and the piece may be considered to have reached its yield point. After this limit is passed the distribution of stress is much modified, and for parts made of ductile material the abnormal distortion at the yield point usually gives warning of structural damage before complete failure occurs.

Under impact loading, which is merely loading applied in a very short space of time, the action is somewhat similar to that under static loading, except that ductile material may offer a higher resistance to very rapid fracture than it does to fracture occurring gradually through a period of several minutes. Impact fracture, moreover, may emphasize somewhat the localized stresses set up at places where the structure of the material is non-homogeneous, or at places where there are sharp notches or deep scratches in the surface of the piece. Under slowly applied load there is opportunity for considerable adjustment and equalization of stress after the yield point is passed; under impact load there is probably less equalization on account of the rapidity of the action,

and hence the localized stresses are higher and more effective in causing failure. This explanation of the action under impact is given here because repeated stresses also emphasize the effect of high localized stress, though for an entirely different reason.

TESTS AND CRITERIA FOR FATIGUE STRENGTH

It was formerly the common opinion that the determination of the elastic limit of a material by means of a static test in a testing machine gave a reliable test for the fatigue-resisting qualities of the material, and that the material could withstand an infinite number of repetitions of stress lower than this elastic limit. Tests at various laboratories, however, have quite thoroughly disproved this idea, and have thrown grave doubts on the reliability of the elastic limit as an index of fatigue strength. The term "elastic limit" has always been rather loosely used, and covers several quite different stresses.^{79(a)} The value determined for the elastic limit for any material depends on the sensitiveness of instruments used and the accuracy of plotting results, and the elastic limit as determined by such a test in a testing machine is determined by the average behavior of the material over a considerable length, while the process of fatigue failure may be going on over a section so small that it does not appreciably affect the readings of the measuring instruments used. In several laboratories comparative repeated-stress tests of different materials have shown higher fatigue resistance for the material with the lower elastic limit.^{23, 80, 82}

Bauschinger in his classic experiments showed that the elastic limits in tension and compression as determined by ordinary testing-machine tests were variable limits, their value depending on the treatment of the material during fabrication. He called such limits "primitive" elastic limits, and showed that when a specimen is subjected to gradually increasing range of alternating stress there are soon set up two elastic limits in the bar—one in tension and one in compression. He called these limits, which may have values widely different from the "primitive" elastic limit, the "natural" elastic limits, and the range between them the "elastic range." He also showed that a test specimen will stand several million repetitions of this elastic range of stress without failure, and proposed the "natural" elastic limits as indices of the fatigue-resisting strength of the material. J. H. Smith^{107(c)} has developed a somewhat simplified process of determining the elastic range. This elastic range seems a more reliable index of fatigue strength than the ordinary "primitive" elastic limit, but the reliability of indices of fatigue strength based on determinations of any elastic limit by testing-machine tests is open to question on account of the possibility that localized fatigue failure may be in progress without affecting the readings of the instruments used in static tests.

Wöhler used as an index of fatigue strength the "endurance limit" of material as determined from a series of fatigue tests with different intensities of stress. He used the method of plotting values of stress (S) against numbers

of repetitions required for fracture (N) and determining by eye where this S - N curve became "practically horizontal." Other investigators have plotted values of S against values of $1/N$ or of $(1/N)^n$ and by extending the diagram till it intersected the axis of ordinates have determined an assumed endurance limit for an infinite number of repetitions of stress. Both of these methods involve enormous extrapolation of test data. Moreover, widely different endurance limits can be determined from the same test data by different methods of plotting values.⁸³ The tendency to irregularity of test results under low stresses makes the decision whether the S - N curve is horizontal or slightly sloping downward one of very considerable uncertainty.

It has been proposed by various experimenters to compare the fatigue-resisting qualities of different metals by short-time tests with stresses well beyond the yield point of the material. Such tests are quickly and easily made. Under such stresses, however, the action of the material is partly a plastic flow. Such tests give good promise of determining fatigue strength and toughness under occasional overload for parts such as staybolts, which in their ordinary service are subjected to rather severe distortion, but it is not at all certain that such tests give a reliable index of resistance of machine parts under ordinary working stresses.⁸⁰

It has been proposed by various laboratories to compare the fatigue strengths of various materials by comparing their life under repetitions or reversals of some standard stress, usually less than the elastic limit of the material as determined by a static test. A somewhat similar standard proposed is to determine the stress which will cause failure under a given number of reversals. Standard stresses proposed for steel are 38 000 pounds per square inch (reversal) and 25 000 pounds per square inch (reversal). One million reversals has been proposed as a standard "life." These two types of test approach working conditions more closely than do the short-time, high-stress tests described above. However, they determine only one point on a S - N diagram for a material and do not indicate how fatigue endurance changes with change of stress.

A comparative study of fatigue strengths of various materials can be made from a S - N diagram plotted on logarithmic paper. Up to about 1 000 000 repetitions of stress logarithmic S - N diagrams fall quite closely along straight lines, and from the ordinates and slopes of these lines the behavior of materials under various intensities of stress can be studied. Tests may conveniently be made with stresses at about the yield point of the material, at stresses about 20 per cent lower, and at one or two intermediate stresses.

Various other possible tests have been proposed for determining the fatigue-resisting strength of a material, but no test has been proved to be of sufficient reliability to be accepted as a standard. A number of tests, however, seem worthy of experimental study.

The rate of dying out of vibrations in a "tuning fork" specimen of the material has been suggested as a possible index of fatigue strength.¹⁵ It is assumed that the gradual dying out of vibration is due largely to loss of energy

spent in inelastic action in the material, and that such inelastic action is a measure of the fatigue weakness of the material. Test data are lacking to determine the value of this test, but it seems worthy of study.

Tests of magnetic permeability have also been proposed to locate internal flaws in the material and thus indicate its relative fatigue strength. The entire subject of the correlation of the magnetic and the mechanical properties of iron and steel is a promising field of investigation.¹⁹

The rise of temperature under repeated stress has likewise been proposed as a measure of fatigue resistance.^{118(b)} Theoretically, if a specimen is subjected to reversed elastic stress no change in temperature should take place, and it has been proposed to determine the endurance limit for metals at that stress which causes the first noticeable rise in temperature after some thousand or more reversals. A practical difficulty in using this test is to secure proper heat insulation for the specimen. This test seems worthy of study, however, especially if employed in an inertia type of testing machine (see Fig. 45[c] and [d]).

The detection of the appearance and growth of "slip lines" in a specimen subjected to repeated stress gives some promise of furnishing a reliable test for fatigue strength. Slip lines appear long before fracture occurs, and if their appearance or the rate of their spread can be shown to be an index of fatigue strength it seems possible that a feasible laboratory test may be devised. The search for slip lines over any considerable area would, however, be very tedious.

Impact tests, usually on notched bars in bending, have been proposed as an index of fatigue strength. The actions under impact failure and under repeated stress are very different, the first giving a sudden break of the entire cross-section of the specimen, and the second a gradually developing fracture. Both failures, however, seem to be affected by localized flaws or irregularities in outline, and though no definite correlation between fatigue strength and strength to resist impact has been established, yet such tests are worthy of study. Repeated-impact tests have also been proposed to determine fatigue strength, but whether such tests have any advantage over short-time tests under non-impact loads is not known.

In all tests to determine fatigue strength it is of the highest importance to secure uniformity of surface finish between the different specimens to be compared. Probably this can best be done by polishing the surface of the specimens where failure is expected.

There is today no short-time test accepted as a standard test for fatigue strength; but the development of such a test, and the establishment of its reliability, would unquestionably be of very great service to testing engineers.

LOCALIZED STRESS AND ITS INFLUENCE IN PRODUCING FATIGUE

The ordinary formulæ and methods of analysis used in computing the fiber stress in a machine part or structural member are based on the assumption that the material is homogeneous throughout, and that the cross-section

of the member is either constant or that it changes its dimensions so regularly and gradually that there is no appreciable localized fiber stress at sections of rapid change. For structures and machines of ductile material subjected to not more than a few hundred loadings, such assumptions are reliable, because localized stresses do not appreciably affect the general deformation of a member, nor do they under ordinary working conditions cause trouble before the member has been subjected to some thousand or more repetitions of load. For nearly all parts, however, high localized stresses are present. Internal flaws may cause such localized stresses. This is shown by mathematical analysis of stress in plates with holes in them⁶¹ and by direct experiment on such plates.^{92, 214} External irregularities of outline may cause localized stress. Under bending or twisting a member with a sharp reëntrant angle in its outline theoretically develops an infinite stress at the root of the angle,²⁹ and actually both mathematical analysis and direct experiment show that very high localized stress may be caused by sharp grooves or scratches on the surface of a machine part or structural member.

It has been stated above that for parts subjected to a few loadings localized stresses are not of great significance. The case is quite different, however, for parts subjected to thousands of loadings. High localized stress may cause a crack to start, either directly or by "cold-working" the material where the localized stress exists until the material becomes brittle. This crack forms an extension of the discontinuity of the material which caused it, and under repeated stress tends to spread still more rapidly. This tendency is illustrated by the action of a piece of plate glass in which a crack has started. In most cases under any repetition of load the crack spreads, and will cause final fracture of the glass. A fatigue failure under repeated stress is a progressive failure. This spreading of cracks to cause failure explains why under fatigue even ductile materials snap short off. Failure does not involve plastic flow of considerable masses of metal, but only of microscopic masses near the crack, and final fracture comes suddenly just as if the member were cut half off by means of a saw cut and then bent. The importance of avoiding localized high stress in members subjected to repeated stress can hardly be overemphasized. Homogeneity of internal structure, smoothness of external surface, and avoidance of sudden changes of cross-section may be more important in the construction of machine parts subjected to repeated stress than is high static strength of material.

Shoulders of crankshafts and of axles, keyways in shafts, screw threads, and rivet holes are examples of locations where high localized stress is liable to occur.

RELATION BETWEEN MICROSCOPIC STRUCTURE AND FATIGUE

A very large field of investigation and one in which very little systematic work has been done is the study by means of the microscope of fatigue failures in various characteristic structures of metals, especially steels. The following paragraphs are given as a summary of the theory held by present-day metallogra-

phists of the relation of microstructure of metal to its fatigue strength. Many of the details of this theory, however, lack adequate experimental verification.

Annealed steel consisting of ferrite (pure iron) and cementite (iron carbide, Fe_3C) seems to increase in resistance to fatigue with the increase in carbon content, especially when the cementite is present in the form of plates as in lamellar pearlite and as long as the cementite does not surround the grains of pearlite. When the cementite is spheroidized, the elastic limit is greatly decreased and probably the resistance to fatigue is also decreased. As a structural material, therefore, a steel with considerable carbon in the form of spherical globules of iron carbide would have practically no advantages over wrought iron. When, however, the iron carbide is in plates it seems to have a marked effect in raising the elastic limit, and probably increases the resistance to fatigue. We would also expect that complete and large networks of ferrite would lower fatigue resistance.

The same arguments regarding grain size of single constituent metals hold to a certain extent for two component alloys. For example, such experimental evidence as is available indicates that the sorbitic structure in steel is the one which resists fatigue best. This structure is supposed to represent an extreme refinement of grain in which the particles of iron carbide are very small, and hence the particles of ferrite must also be very small. It is true that some of the iron carbide may be in solution in the iron, but it is more probable that the mechanical properties observed can be accounted for by an extreme reduction in the size and by the dispersion of ferrite and cementite particles. When these globules are made larger by heating to a higher temperature than that at which the sorbite was formed, granular pearlite results with reduced fatigue resistance.

When a high-carbon steel is quenched from above the critical range to form martensite, the metal becomes extremely brittle. The normal path of static rupture in brittle martensite is at the old austenite (solid solution of Fe_3C in gamma iron) grain boundaries. The path of rupture in fatigue has, so far as is known, not been ascertained. From certain tests on the resistance to fatigue of chrome-vanadium steel after various heat treatments, Dr. C. M. Olmstead, of the C. M. O. Physical Laboratories, Buffalo, found that the steel in the martensitic state, that is, as quenched, had a very much lower resistance to fatigue than after reheating to about 1100 degrees F. The maximum resistance to fatigue occurred by quenching and reheating to 1000-1200 degrees F. (538-648 degrees C.), and there was very little difference between the specimens tempered at 1000 degrees and at 1200 degrees. There was a marked difference, however, between these and the samples tempered at lower temperatures or those not tempered at all. This is the heat treatment that is commonly given to automobile parts which must withstand fatigue stresses, and which may be subjected to shock. The tempering of springs is done at a little lower temperature, but it is not certain that the spring structure is the one having the highest resistance to continued repetition of stresses.

It seems from the above that martensite is not a suitable material to withstand fatigue stresses, and that some intermediate structure between mar-

tensite and the annealed or normalized structure will have the maximum resistance to fatigue. This structure is called the sorbitic structure and corresponds to that used in automobile springs and other parts of automobiles which must resist fatigue and shock stresses.

FORMULÆ FOR DESIGNING PARTS SUBJECTED TO REPEATED STRESS

All formulæ which have been proposed for designing parts subjected to repeated stress depend upon extrapolation from test results and should therefore be regarded as tentative. Their use is justified only on the ground of necessity. Parts must be designed to resist repeated stress, and even formulæ derived from a confessedly inadequate experimental basis seem better than mere guesswork. Two types of formula have been used.^{68(d), 79(b)}

In many discussions of data of repeated-stress tests, it is assumed that there exists some definite "endurance limit," that is, some stress, greater than zero, which can be repeated an infinite number of times without causing failure of the material. If such a limit exists it is certainly lower than the elastic limit of the material as determined by static tests, for actual failures of materials have occurred under repeated nominal stresses as low as one-quarter of the elastic limit as determined by a static test. Examination of test data indicates that the endurance limit is an assumption rather than a proved fact. It is usually determined by plotting a diagram with stresses as ordinates and number of repetitions producing failure as abscissas and estimating the stress for which the diagram seems to become horizontal. Various other methods have been proposed, but all involve this assumption.⁸³

In 1910 a paper⁸ presented before the American Society for Testing Materials pointed out that an examination of the results of numerous series of repeated-stress tests indicates that for nearly all the range covered the law of resistance to repeated stress may be expressed by the equation:

$$S = KN^{-m} \dots\dots\dots [1]$$

in which S is the maximum unit stress developed in the test piece, N the number of repetitions of stress necessary to cause failure, and K and m are constants depending on the material and somewhat on the manner of making the test. This is known as the "exponential equation for repeated stress."

Another form of expression for the above equation, and frequently more convenient, is:

$$\log S = \log K - m \log N \dots\dots\dots [2]$$

If the logarithms of S and N are plotted, or if the values of S and N are plotted on logarithmic cross-section paper, Equation [2] is represented by a straight line. Fig. 36 shows the S - N diagram given by a series of repeated-stress tests. In Fig. 36(a) ordinary coördinates are used, but in Fig. 36(c) the coördinates are logarithmic. For large values of N the exponential equation, which is represented by the inclined line of the diagram, would give values of S smaller than the observed values; in other words, the exponential formula seems to err on the side of safety.

It will be noted that the use of the exponential formula involves the assumption that any stress if repeated often enough will eventually produce failure of the material. Thus while both the endurance limit and the exponential formula are based on extrapolation from known data, the exponential formula seems to be an assumption on the safe side. The working stresses as developed by the two methods do not differ greatly except for members subject to more than ten million repetitions of stress. Above that number the exponential formula requires lower working stress, but even then the stresses given by the exponential formula are not impracticably low.

While nothing but tentative formulæ can be proposed now,³⁰ some features which a satisfactory formula for fatigue strength should include may be noted. It is probable that such a formula for any material will not depend on ordinary static qualities of the material such as elastic limit or tensile strength. It may depend on some form of elastic limit determined after the material has been put in a "cyclic" or "normalized" state by a number of reversals of stress. Such a formula will quite probably contain factors dependent on the surface finish of the part and upon the uniformity and regularity of its crystalline structure. It will contain a factor dependent on the range of stress during a cycle. Such a formula may contain a factor dependent on the probable number of repetitions of stress which the part may be expected to withstand during a normal period of service, or the result may be an "endurance limit"—a stress which the part is capable of withstanding so many times that even for modern high-speed machinery the number of repetitions may be regarded as infinite.

APPENDIX C

GLOSSARY OF TECHNICAL TERMS

- ALLOY STEEL . . . A steel which owes its characteristic properties chiefly to the presence of an element or elements other than carbon. In the present investigation the two alloy steels tested were nickel steel and chrome-nickel steel.
- ANNEALING . . . See Heat Treatment.
- AUSTENITE . . . See Metallography.
- BENDING MOMENT . . A moment is equivalent to the product of a force multiplied by a distance, and is measured in inch-pounds or in foot-pounds. The bending moment at any cross-section of a piece under flexure measures the tendency to cause flexural failure, and is equal in magnitude to the summation of the moments of the forces on one side of the cross-section.
- BRINNEL TEST . . . See Hardness.
- CEMENTITE See Metallography.
- CHARPY TEST . . . See Impact Test.
- COLD BENDING . . . See Cold Working.
- COLD-DRAWN STEEL . . See Cold Working.
- COLD JUNCTION . . . See Pyrometer.
- COLD-ROLLED STEEL . . See Cold Working.
- COLD STRETCHING . . See Cold Working.
- COLD WORKING . . . Changing the shape of steel parts by compressing, stretching, bending, or twisting, using stresses beyond the yield point and temperatures below the critical range. Cold-drawn steel is finished by being drawn through a die, while cold-rolled steel is finished between rollers.
- COMPRESSION See Deformation.
- CONSTANTAN An alloy of copper and nickel used in thermo-couples.
- CRITICAL RANGE . . . See Metallography.
- CRYSTAL See Metallography.
- CYCLE OF STRESS . . See Repeated Stress.
- DEFORMATION Change of form of a member accompanying the application of external load. The term strain is used in this bulletin as synonymous with deformation. Deformations may be stretches under tension, compressions under compressive load, deflections under bending (or flexure), twists under torsional moment, or detrusions under shear. Twist is a special case of shearing detrusion. The deformation per unit of length over any gage length on a specimen is called the unit deformation or unit strain.

- DETRUSION See Deformation.
- DRAWING See Heat Treatment.
- DUCTILITY Ability to withstand stretch without rupture. Ductility is usually measured by the percentage of elongation after rupture over a gage length laid off on a specimen before stretching, or by the reduction of area of the original cross-section of a specimen when tested in tension.
- ELASTIC LIMIT . . The term "elastic limit" is, unfortunately, used very loosely in general practice. In this bulletin the term elastic limit or set elastic limit is used to denote the highest unit stress at which material will completely recover its form after the stress is removed. Proportional limit, or proportional elastic limit, is used to denote the highest unit stress at which stress is proportional to deformation. The values found for both elastic limit and proportional limit are dependent on the accuracy of apparatus used and the precision of plotting stress-strain diagrams. For practical purposes elastic limit and proportional limit may be regarded as interchangeable terms. The yield point is that unit stress at which material shows a sudden marked increase in the rate of deformation without increase in load. It is usually determined by the sudden drop in the balance beam of the testing machine as strain is applied to the specimen at a uniform rate, or by a sudden increase of deformation which can be seen by the use of a pair of dividers on the specimen. The "FR" point referred to in this bulletin is a special limit determined by very delicate flexure tests. The method of determining it is described in the reference given in connection with the text.
- ELONGATION See Ductility.
- ENDURANCE In this bulletin the term endurance is used to denote the number of cycles of repeated stress withstood by a specimen before failure.
- ENDURANCE LIMIT . The highest unit stress which, applied in cycles of completely reversed stress, can be withstood an indefinite number of times without failure. If an *S-N* diagram for a series of reversed-stress tests is plotted to logarithmic coördinates the endurance limit is the unit stress at which the diagram abruptly changes direction from a sloping line to a horizontal line.
- ENDURANCE STRENGTH A general term denoting ability to resist repeated stress; synonymous with fatigue strength.
- EXTENSOMETER . . . An instrument for measuring small changes of length of specimens under tension. Sometimes the term is loosely

- used to denote any instrument for measuring small deformations of material.
- FATIGUE The action which takes place in material causing failure after a large number of applications of stress. Failures due to fatigue are characterized by their suddenness, and by the absence of general deformation in the piece which fails.
- FATIGUE LIMIT . . See Endurance Limit.
- FATIGUE STRENGTH. See Endurance Strength.
- FERRITE See Metallography.
- FERROUS METALS . Metals whose principal ingredient is iron. The common ferrous metals are: wrought iron, cast iron, malleable cast iron, and steel.
- FILLET A curved surface at the junction of two different sized parts of a machine member. The use of a fillet prevents a sharp shoulder at the junction.
- FLAW In this bulletin this term is used to denote any defect in metal, either a surface defect or a defect in grain structure.
- FLEXURE See Deformation.
- “FR” POINT . . See Elastic Limit.
- GRAIN See Metallography.
- HARDNESS The term hardness is used with a variety of meanings. In this bulletin it is used to denote resistance to penetration. The two common tests for hardness are the Brinnel test and the scleroscope test. In the Brinnel test a hardened steel ball of a standard diameter is forced against the surface of a test specimen, using a standard pressure. The diameter or the depth of the resulting impression is an inverse measure of the hardness. In the scleroscope test a small weight fitted with a diamond point is allowed to fall from a standard height upon the surface of the specimen, thus causing a minute indentation. The height of rebound is a measure of the hardness.
- HEAT TREATMENT . Heat treatment of steel is the proper control of heating and cooling so as to produce the desired structure (pearlitic, sorbitic, etc.), and includes:
- ANNEALING, which consists in very slow cooling from above the critical range, and which gives a large-grained, soft, pearlitic structure;
- NORMALIZING, which consists in cooling from above the critical range in still air, and which gives a fine-grained, pearlitic structure;
- OIL QUENCHING, which consists in cooling from above the critical range by cooling in oil at room temperature, and which yields steel of sorbitic or troostitic structure, depending

on the carbon content (certain special alloy steels yield a martensitic structure or even an austenitic structure with oil quenching);

WATER QUENCHING, which consists in cooling from above the critical range by cooling in water at room temperature, and which yields steel of martensitic, troostitic, or sorbitic structure, depending on the carbon content (certain special alloy steels yield a martensitic or an austenitic structure with water quenching);

DRAWING, which consists in reheating quenched steel to a temperature slightly below the critical range and then cooling. This process tends to bring martensitic, troostitic, or sorbitic steel towards the pearlitic state, and, by varying the temperature of drawing, it is possible to control the state of the steel with a good degree of precision.

Other liquids are sometimes used for quenching steel; such as molten lead, molten barium chloride, ice water, and brine.

Alloying elements, including carbon, slow up the transition period so that high-carbon steels and alloy steels are more susceptible to heat treatment than are low-carbon steels. See Metallography.

HYSTERESIS, MECHANICAL . . . If load is applied to a specimen, and is removed, then, if the specimen is perfectly elastic under the stress caused by the load, the energy expended in loading the specimen is all given back when the load is removed. If the specimen is not perfectly elastic under the stress caused by the applied load, then some of the energy applied is dissipated as heat. This dissipated energy is called mechanical hysteresis, and is measured by the area of the loop between the stress-strain diagram for application of load and the stress-strain diagram for release of load.

IMPACT TEST . . . A test in which a specimen is subjected to a very suddenly applied load. In the machines commonly used for making impact tests of steel specimens the sudden load is applied by the blow of a swinging pendulum. The three testing machines in common use for this test are the Charpy, the Izod, and the Olsen. Impact tests in flexure are usually made on small beams of steel with a sharp notch in them, because an unnotched beam would bend without fracturing; such tests being sometimes called notched-bar tests. Impact-tension tests are also sometimes made. In the three machines mentioned above enough energy is stored in the raised pendulum to break the specimen with a single blow. Impact tests using repeated blows

to fracture the specimen are also made. A well known machine for making such tests is the Stanton machine.

IRON For the distinction between iron and steel, see Steel.

IZOD TEST See Impact Test.

LOGARITHMIC COÖRDINATES . . . Coördinates for plotting results of tests in which the scale used is proportional to the logarithms of the values plotted. Logarithmic coördinates have been found useful in plotting the results of repeated-stress tests.

MARTENSITE . . . See Metallography.

MARTENSITIC . . . See Metallography.

METALLOGRAPHY . . Metallography deals with the physical state and the proximate constituents of a metal or an alloy. It has to do with the physical grouping, distribution of constituents, and relative dimensions of the substances as revealed by microscopic examination. It may be characterized as a study of the anatomy of metals.

Steel is an alloy the essential constituents of which are iron and carbon, the latter being the controlling element. The carbon exists in steel as a carbide of iron, Fe_3C , to which the name cementite is applied. The free iron or ferrite, together with the cementite, has the power of forming a conglomerate called pearlite, a very intimate mechanical mixture composed of about 7 parts of ferrite to one part of cementite. If molten iron is cooled there is formed first a solution of carbon in molten iron, then as the metal solidifies the carbon exists as cementite in solid solution in the iron. This solid solution is called austenite, and it crystallizes into imperfect crystals or grains. With further cooling the steel passes through a critical or transformation range of temperature (extreme range about 1650 degrees F. to 1250 degrees F.), and the two constituents of the metal pass successively through several transition stages, namely: martensite, in which long needle-like crystals are formed, giving a very hard and brittle substance; troostite, in which dark colored masses resembling sorbite (see below) are surrounded by a groundwork of martensite, the troostitic state yielding a substance hard but tougher than the martensitic; sorbite, in which cementite and ferrite are in a state resembling an emulsion, yielding a substance fairly hard and very tough; and pearlite, in which ferrite and cementite exist, usually in stratified layers or bands. If the steel has a carbon content of about 0.90 per cent all the grains will be pearlite; if the carbon content is lower than 0.90 per cent there will be grains of pearlite and grains of ferrite; if the carbon

content is greater than about 0.90 per cent there will be grains of pearlite and grains of cementite.

The presence of carbon or of other alloying elements slows down the process of transition. By varying the rate of quenching steel the transition process may in general be halted at any desired stage, and the resulting cooled steel may be given any desired characteristic structure. See Heat Treatment, Micrograph.

- MICROGRAPH . . . Micrographs are obtained by polishing the surface of a metal, etching the polished surface with a suitable reagent to bring out the metallographic structure, then reproducing, usually by photographic methods, the appearance of the surface as seen through a microscope. Photomicrograph and microphotograph are terms sometimes used for micrographs made by a photographic process.
- MILLIVOLTMETER . . An electrical instrument for measuring small electric potentials. Used for measuring the small voltages of thermocouples developed by changes of temperature.
- NON-FERROUS METALS . . . Metals in which iron is not a constituent, or is only present in very small quantities.
- OVERSTRESS . . . Stress above any given limit of safety, used in this bulletin in general, to denote stress above the endurance limit.
- PEARLITE See Metallography.
- PEARLITIC See Metallography.
- PYROMETER An instrument for measuring high temperatures. In this investigation the pyrometers used were of the thermoelectric type, which utilizes the electromotive force generated by a junction of two dissimilar metals when exposed to heat. In each pyrometer there were two junctions each made by welding together wires of the two dissimilar metals, platinum and platinum-rhodium for example. One junction is then exposed to the temperature to be measured and is called the hot junction; the other junction, which is opposed to the first-named junction, is kept at a constant temperature and is called the cold junction. A millivoltmeter, or other instrument for measuring electromotive force, is attached by conductors to the free ends of the opposed junctions, and by its reading indicates the electromotive force generated and hence the temperature of the hot junction.
- RANGE OF STRESS . See Repeated Stress.
- REPEATED STRESS . This is the general term used to denote any regular variation of stress applied to a member a large number of times. Repeated stress includes variations in magnitude of stress in one direction, variations from stress in one

direction to a smaller stress in the opposite direction, and variations from stress in one direction to equal stress in the opposite direction. This last-named case of repeated stress is called reversed or alternating stress. In repeated stress the periods of variation are repeated again and again, and each complete period of variation is called a cycle of stress. The algebraic difference between the maximum stress and the minimum stress during a cycle of stress is called the range of stress.

SHEAR, SHEARING STRESS . . . See Stress.

SLIP BANDS, SLIP LINES . . . Minute cracks which, under the action of destructive repeated stress, form within the crystals of metal. See Fig. 46.

S-N DIAGRAM. . . A diagram showing the results of a series of repeated-stress tests. Values of unit stress are plotted as ordinates, and values of corresponding numbers of cycles of stress to cause failure are plotted as abscissas. In this bulletin logarithmic coördinates have been used for plotting *S-N* diagrams.

SORBITE. See Metallography.

SORBITIC See Metallography.

STATIC TEST A test of a specimen in which the rate of application of load is so slow that it may be regarded as zero. The term refers in general to a test made with an ordinary testing machine.

STEEL The term steel is used to denote any ferrous metal with a carbon content less than about 1.7 per cent, which is made by a process involving complete fusion. Thus in this bulletin Steel No. 9, which chemically is almost pure iron, and which is called Armco Iron by the manufacturers, is designated as 0.02 carbon steel. Wrought iron has a low carbon content, and is made from a pasty mass at a temperature below complete fusion. Ferrous metals with a carbon content higher than about 1.7 per cent are called cast iron.

STRESS An internal force which resists the destructive action of external force. Stresses are always accompanied by strains or deformations, and there are tensile stresses, compressive stresses, and shearing stresses. Torsion on a specimen is resisted by shearing stress. At any point on a stressed member the stress per unit area is called the unit stress. Sometimes the term stress is used to denote unit stress.

TESTING MACHINE . . In general, any machine for applying stress to specimens of material, and measuring the applied stress. The term

is ordinarily applied to a machine for applying stress at a slow rate and measuring the stress by means of a weighing scale.

TOUGHNESS . . . In this bulletin the term toughness is used to denote a combination of strength and ductility in a material.

TROOSTITE . . . See Metallography.

TROOSTITIC. . . See Metallography.

TWISTING MOMENT. The twisting moment on any cross-section of a member in torsion is equal to the sum of the twisting moments of the forces acting on one side of the cross-section. See Bending Moment.

ULTIMATE TENSILE STRENGTH . . . The highest unit stress carried by a tension specimen in a test to rupture. In computing ultimate tensile strength the original area of cross-section of the specimen is used.

APPENDIX D

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Supplementary Note.—Since the preparation of the manuscript of this bulletin an important piece of research in the fatigue of metals has been reported from the British National Physical Laboratory by H. J. Gough. This report is given in The Engineer, London August 12, 1921, and an abstract is given in "Mechanical Engineering," October, 1921.

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THE DISTRIBUTION OF THE FORMS OF SULPHUR IN THE COAL BED

BY

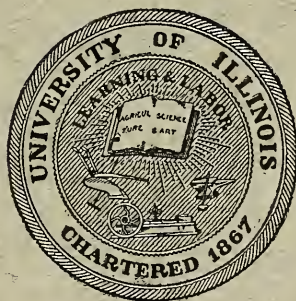
H. F. YANCEY

AND

THOMAS FRASER

ILLINOIS COAL MINING INVESTIGATIONS COÖPERATIVE AGREEMENT

(THIS REPORT WAS PREPARED UNDER A COÖPERATIVE AGREEMENT BETWEEN THE
U. S. BUREAU OF MINES, THE ENGINEERING EXPERIMENT STATION OF
THE UNIVERSITY OF ILLINOIS, AND THE ILLINOIS STATE
GEOLOGICAL SURVEY DIVISION)



BULLETIN No. 125

ENGINEERING EXPERIMENT STATION

PUBLISHED BY THE UNIVERSITY OF ILLINOIS, URBANA

EUROPEAN AGENT

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THE Engineering Experiment Station was established by act of the Board of Trustees of the University of Illinois on December 8, 1903. It is the purpose of the Station to conduct investigations and make studies of importance to the engineering, manufacturing, railway, mining, and other industrial interests of the State.

The management of the Engineering Experiment Station is vested in an Executive Staff composed of the Director and his Assistant, the Heads of the several Departments in the College of Engineering, and the Professor of Industrial Chemistry. This Staff is responsible for the establishment of general policies governing the work of the Station, including the approval of material for publication. All members of the teaching staff of the College are encouraged to engage in scientific research; either directly or in coöperation with the Research Corps composed of full-time research assistants, research graduate assistants, and special investigators.

The volume and number at the top of the front cover page are merely arbitrary numbers and refer to the general publications of the University of Illinois; *either above the title or below the seal is given the number of the Engineering Experiment Station bulletin or circular which should be used in referring to these publications.*

The present bulletin is issued under a coöperative agreement between the Engineering Experiment Station of the University of Illinois, the State Geological Survey, and the United States Bureau of Mines. The reports of this coöperative investigation are issued in the form of bulletins by the Engineering Experiment Station, the State Geological Survey and the United States Bureau of Mines. For bulletins issued by the Engineering Experiment Station, address Engineering Experiment Station, Urbana, Illinois; for those issued by the State Geological Survey, address State Geological Survey, Urbana, Illinois; and for those issued by the United States Bureau of Mines, address the Director, United States Bureau of Mines, Washington, D. C.

UNIVERSITY OF ILLINOIS
ENGINEERING EXPERIMENT STATION

BULLETIN No. 125

MAY, 1921

THE
DISTRIBUTION OF THE FORMS OF SULPHUR
IN THE COAL BED

ERRATA

- Page 11, par. 2, line 7: *for* "mine" *read* "nine."
- Page 12, line 1: *for* "certified" *read* "confirmed."
- Page 24, lines 10 and 12: *for* " $1.30(1.00-x)5x$ " *read* " $1.30(1.00-x) + 5x$."
- Page 29, par. 2, line 5: *omit* "and the dotted line fine disseminated pyritic sulphur."
For data on fine disseminated pyritic sulphur see Tables A, C, G, J, P, S, and T. Appendix I.
- Page 30, line 4: *for* "Fig. 6" *read* "Fig. 5."
- Page 34, lines 2 and 3: *for* "Fig. 6: A, B, C, D, E, and F, and Fig. 7: G, H, I, J, K, and L" *read* "Fig. 6: A, B, C, D; Fig. 7: E, F, G, H; and Fig. 8: I, J, K, L."
- Page 38, par. 4, line 3: *for* "Fig. 13, U, and V" *read* "Fig. 13, T, and U."
- Page 44, Table 6: *for* "on moisture-free basis" *read* "on 'as received' basis."
- Page 66, lines 5 and 6: *for* "Fig. 1, Charts A, B, C, D, E, and F, and in Fig. 2, Charts G, H, I, J, K, and L" *read* "Fig. 6, Charts A, B, C, and D; Fig. 7, Charts E, F, G, and H; and Fig. 8, Charts I, J, K, and L."

ENGINEERING EXPERIMENT STATION

PUBLISHED BY THE UNIVERSITY OF ILLINOIS, URBANA

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THE DISTRIBUTION OF THE FORMS OF SULPHUR IN THE COAL BED

I. INTRODUCTION

1. *Preliminary Statement.*—This bulletin is a report of investigational work carried out by the United States Bureau of Mines under the terms of a coöperative agreement with the Engineering Experiment Station of the University of Illinois and the Illinois State Geological Survey. Previous work published by the Engineering Experiment Station on the forms of sulphur in coal was conducted in the Chemical Laboratory of the University of Illinois by S. W. Parr and A. R. Powell, who developed successful methods for determining quantitatively the different chemical forms of sulphur in coal. Their methods, described in a previous bulletin,* have been applied in the present work to the study of the manner of occurrence and distribution of the different forms of sulphur in the coal bed.

2. *Object of the Work.*—The experimental work was undertaken as a part of the United States Bureau of Mines plan of research on coal preparation, and the application of the information secured to the problem of producing cleaner coal was the principal objective. The solution of this problem must of necessity rest to a considerable extent upon an intelligent knowledge and appreciation of the physical and the chemical forms in which sulphur occurs in a particular coal, and how it is distributed through the coal in the bed. This information is essential in forming an opinion on the extent to which any given coal may be cleaned by washing, that is, by a gravity separation in water. The forms of sulphur occurring in the coal are sulphur as iron pyrite, sulphur in organic combination with the coal substance itself, and sulphate sulphur present chiefly as calcium sulphate. Pyrite occurs both in macroscopic and microscopic form.† Thiessen has found microscopic globules of pyrite in coals from many different fields in the United States, and even in peat. Pyrite has also been shown to exist in this form in Japanese‡ coals.

* "A Study of the Forms in which Sulphur Occurs in Coal," Univ. of Ill. Eng. Exp. Sta., Bul. 111, 1919.

† Am. Inst. Min. Eng., Bul. 153, 2431, 1919.

‡ Iwasaki, Colliery Guardian 120, 797, 1920.

Pyrite or pyritic sulphur is not uniformly distributed in coal. It is well known that the variation in total sulphur content of the different benches or horizontal sections of a coal bed may be quite marked. This of course might be entirely due to the heterogeneous distribution of iron pyrite. The distribution of organic sulphur has been given but little attention, because it exists in an amicroscopic form and must be determined by chemical methods. Until recently no satisfactory methods have been available. Some earlier work,* done since the publication of Parr and Powell's methods, led to the tentative conclusion that the organic sulphur content of a given coal varies but little, and that, at least, it is much more uniformly distributed than pyritic sulphur. The distribution of these forms of sulphur has an important bearing on the practice of coal washing, the dry separation of impurities from coal, and the use of coal for coking purposes.

The fact that organic sulphur has some bearing on the washability of a coal has long been recognized. Some coal washing reports have contained an approximation of the figure for the percentage of this form of sulphur, in some cases with fine pyrite included under the caption "fixed sulphur," "combined sulphur," or "organic sulphur." Bolling,† in a paper on "Chemical Control of Coal Washers" describing methods used at the plant of the Nova Scotia Steel and Coal Co., gives the following figures determined by a sink and float test on a calcium chloride solution of 1.35 specific gravity: "Raw coal mixture, total sulphur 2.30 per cent; by calcium chloride, total sulphur 1.10 per cent; $(2.30 - 1.10) = 1.20$ per cent pyritic sulphur, organic sulphur 1.10 per cent." While this method does not give the correct values for organic sulphur, it does give a figure which is of value in coal washing work.

Carl Wendell,‡ in an article on coal washing, states concerning combined sulphur that many times in examining a new washing apparatus he had been assured by the manufacturer that the machine would produce the desired results, this conclusion being based on two figures only; namely, total sulphur content of the coal and combined sulphur content. Wendell adds further, "Invariably I have found that this premature statement was wrong."

* Fraser and Yancey, Am. Inst. Min. Eng., Bul. 153, 1822, 1919.

† Eng. and Min. Journal, Aug. 29, 1908.

‡ Univ. of Ill. Technograph, page 133, April, 1915.

His comment concerning the value of such hastily formulated opinions is fully warranted. The instances cited indicate that in some cases organic sulphur has been one of the factors considered in connection with the reduction of sulphur in coal by washing processes. On the other hand the opinion is sometimes expressed that the organic sulphur of coal is negligible in amount. There is a need for more accurate and specific information on this subject.

It is apparent that the possibility of cleaning coal of sulphur by processes of dry separation such as selective mining, hand picking at the face and in the tippie, or by mechanical devices, depends upon the way in which the sulphur, particularly the pyritic sulphur, is distributed throughout the coal.

This factor also has an important bearing in connection with the coking of coal. Uniformity of sulphur content is a valuable attribute of a coal to be used for coking or for gas manufacture. Some of the Illinois mines nominally rated as low sulphur coal mines cannot be depended upon to ship day in and day out a product consistently low in sulphur.

All of the mechanical or gravitational methods of cleaning coal depend upon the removal of particles of impurities of high specific gravity from coal which is of lower specific gravity. In this connection it is important to know whether organic sulphur segregates as does pyritic sulphur. In case concentrations of organic sulphur were found to exist it would be desirable to associate such occurrences with other impurities or specific recognizable conditions.

3. *Sources of Information.*—The observations set forth in this bulletin are based on data secured by:

1. Visual examination and sampling of coal in situ at mines in three different beds in the central district; namely, Illinois No. 6 Coal at the Middlefork Mine of the United States Fuel Company at Benton, Illinois, and Kentucky beds No. 9 and No. 12 at mines No. 7 and No. 8 of the West Kentucky Coal Co. in Union and Webster Counties of western Kentucky. Sectional bench samples were taken in a number of places in each of these mines and analyzed for total sulphur, organic sulphur, pyritic sulphur, and fine disseminated sulphur.

2. Investigations incident to a survey of the coal pyrite resources of the central district, made by the United States Bureau of

Mines and the State Geological Surveys of Illinois, Indiana, Ohio, Tennessee, Missouri, and Iowa, in 1918. A large number of mines were visited by the field men sent out by these agencies. Notes were taken on the occurrence of sulphur, and estimates were made of the available tonnage of pyrite which could be recovered and cleaned for industrial uses. The records of this work present quite a fund of information on the distribution of coarse pyrite in the various coal beds throughout the district.

3. Visual examination of the coal faces in mines in connection with other investigational work and routine face sampling.

4. *Acknowledgments.*—The investigation was carried out under the general direction of Mr. E. A. HOLBROOK, Assistant Director, and Mr. GEORGE S. RICE, Chief Mining Engineer, of the United States Bureau of Mines. Professors S. W. PARR, Head of the Division of Industrial Chemistry, and H. H. STOEK, Head of the Department of Mining Engineering of the University of Illinois, have contributed many helpful and valuable suggestions throughout the course of the work. Mr. C. A. MEISSNER, Chairman of Coke and Coal Washing Committees of the United States Steel Corporation, has followed the progress of the investigation with cordial coöperation. Messrs. W. H. CLINGERMAN, President, and THOMAS MOSES, General Superintendent, of the United States Fuel Co., made the arrangements for the work at the Middlefork mine, and through their local organization at the mine gave every assistance. Mr. D. M. LEWINS, Chemist of the United States Fuel Co. at the Middlefork mine, rendered valuable aid. The field work on western Kentucky coals was carried out at mines of the West Kentucky Coal Co., aided by the active coöperation of Mr. T. E. JENKINS, Vice-President and General Manager of this Company.

II. GENERAL DATA ON SULPHUR DISTRIBUTION

5. *Chemical Forms in which Sulphur Occurs in Coal.*—Sulphur occurs in coal in both organic and inorganic combination. In the central and eastern coal fields of the United States the major inorganic sulphur compound is iron pyrite or marcasite, both of identical chemical composition. Sulphate sulphur is ordinarily present in freshly mined coal chiefly as calcium sulphate, (gypsum), occurring in thin white flakes along the cleavage planes. Calcite also occurs in a similar form. Because of the marked color contrast between the gypsum and the coal an exaggerated idea of the relative amount of the total sulphur present in the sulphate form is easily given. Freshly mined coal may contain as much as 0.1 per cent sulphur as sulphate, but as a rule the amount of this form is less than 0.05 per cent. Stored coal increases in sulphate sulphur because of the oxidation of its iron pyrite. Length of storage, temperature, moisture, and the size of coal stored appear to be the principal factors governing the rate of oxidation and the consequent formation of sulphate sulphur, so that after long storage the sulphate sulphur content may exceed that of the pyritic sulphur. This sulphate sulphur is water-soluble and if the coal is stored in the open a large part will be leached out.

It has long been believed that a varying though considerable proportion of the total sulphur content of coal is present in organic combination with the coal substance. The investigations of Wheeler and of many other workers have confirmed this view. Prior to 1873 Wormley* observed that some coals contained too little iron to combine with all the sulphur present to form iron pyrite. He gives total sulphur and total iron determinations of mine coals, and in no case is sufficient iron present to unite with all of the sulphur as ferric sulphide. The investigations of Bradbury,† Kimball,‡ Drown,§ Fischer,§ M'Callum,** Parr,†† Powell,‡‡ and Wibaut and Stoffel¶¶ have

* Geol. Survey of Ohio, 1, 361, 1873.

† Chem. News, 38, 147, 1878.

‡ Trans. Am. Inst. Min. Engr., 8, 185, 1880.

§ Trans. Am. Inst. Min. Engr., 9, 656, 1881.

§ Zeit. angew. Chem., 764, 1899.

** Chem. Engr., 11, 27, 1910.

†† Univ. of Ill. Eng. Exp. Sta., Bul. 111, 1919.

‡‡ J. Ind. Eng. Chem., 12, 887, 1920.

¶¶ Rec. trav. chim., 38, 132, 1919.

certified the existence of sulphur in organic combination in coal. It appears that the organic sulphur is united with two different types of coal constituent, and is classified as humus organic sulphur and resinic, or perhaps more properly merely as phenol soluble sulphur.

6. *Physical Forms of Pyrite.*—Iron pyrite may be found in coal in particles of various sizes and various shapes. Thiessen,* through the microscopic study of thin sections, found that all the coals he examined contained very small globules or particles of pyrite varying in diameter from a few microns to a hundred microns. At the other extreme it is not uncommon to find, in the roof slate of the No. 5 coal in Indiana, pyrite boulders several feet in diameter. For the purpose of studying the distribution of the pyrite in coal it has been found convenient to classify it according to the size of particles in which it occurs, under three headings, namely, microscopic pyrite, fine disseminated visible pyrite, and coarse pyrite. The lines of division into these groups are of course merely arbitrary. As the particles exist in the bed they grade in size all the way from the very smallest to the largest.

The microscopic pyrite consists of minute spherical globules invisible except under the microscope. This form of sulphur is not removable by mechanical processes.

The fine disseminated visible pyrite occurs in several forms. This term is used to designate pyrite infiltrations coarse enough to be visible to the naked eye, yet so fine and intimately mixed with the coal substance that their separation is difficult. Thin film-like coatings sometimes occur on the joint planes, giving a natural cleavage face of a lump of coal the appearance of being gold painted. A fraction of a per cent of sulphur in this form may make a car of coal look very dirty. Pyrite may occur in the form of small grains disseminated through the coal. Charcoal or mother-coal is sometimes so thickly impregnated with pyrite grains of this sort that it has a brownish color and is so hard that it interferes with the use of undercutting machines. The miners call this material "black jack." Clay and shale partings may also be impregnated with pyrite particles in the same manner.

The coarse pyrite occurs in a wide variety of shapes, the most common of which is perhaps the lens; circular or elliptical in plan and

* Loc. cit.

lenticular in cross section, thick in the center and gradually thinning to an edge around the perimeter. These lenses are commonly from $\frac{1}{2}$ in. to 3 in. in greatest thickness and about 1 ft. across, although lenses as much as 3 or 4 ft. thick and several hundred feet long are sometimes found at the top or bottom of a seam. In some beds the small lenses occur always at the same horizon along certain bedding planes. They may be almost edge to edge, forming a practically continuous sheet of varying thickness. In other cases the lenses are distributed promiscuously through the bed and may be found in any position.

Another somewhat similar form of pyrite deposit, much less common, is thinnest in the center with the thick part near the circular rim.

Balls or nodules of sulphur are roughly spherical in shape. This form is often found in the roof or floor. The term nodules is usually applied to the small ones of 2-4 in. in diameter. The larger sizes are more often called niggerheads or balls. Where they occur in the roof they are sometimes 4 or 5 ft. in diameter. The large ones quite often consist of a mixture of iron carbonate and pyrite or they may have a core of limestone with a pyrite shell. The small nodules are usually bright crystalline pyrite and approach more closely pure iron pyrite (FeS_2) in chemical composition. Pyrite deposits which appear as continuous bands in a working face may be described as flat lenses of great lateral extent, such that they may be continuous over large areas. They commonly vary from the thickness of a knife edge up to 2 or 3 in. Locally, thin parallel bands sometimes occur interlaminated with thin bands of coal in such a manner as to necessitate picking out and discarding a large proportion of coal with the pyrite. Similarly with the thin facings of coal described under fine disseminated pyrite, large lumps of coal sometimes are so thickly impregnated with these film-like flakes of pyrite as to justify discarding the lumps as a whole.

Pyrite also occurs in vertical or inclined veins in some coals although this form is not very common. The thicker and more prominent veins are called by the miners "spar sulphur."

Local groups of veinlets that appear to arise from a common center are called cat-faces. The center is often a small ball of irregular outline. The pyrite of cat-faces is a brassy yellow in color and often of a flaky porous structure. Bands of pyrite of a similar porous

or cellular structure are commonly found in Southern Illinois coals. A lump of such forms seems to be built up of small thin plates of pyrite put together with spaces intervening so that the apparent specific gravity of the whole lump is much lower than that of a piece of solid pyrite. Apparent specific gravity determinations on samples from the No. 6 bed in Franklin County, Illinois, gave results as low as 2.9.

Pyritized vegetable fossils are not uncommon although they do not constitute any considerable part of the pyrite of coal. They are usually in the form of fragments of branches or leaves.

The manner in which the pyrite in any coal occurs in these various forms determines to a large extent the ease with which it may be separated from the coal by handpicking or washing. Solid balls and lenses of the massive amorphous structure and brassy color are readily removed underground as they are easily seen and they usually break freely from the surrounding coal. Lenses of a dark stony gray color are not so easily picked out at the face. They are much more difficult to see and, as the boundary between coal and pyrite is usually indefinite, the lens stick fast to the coal.

Of the interbedded continuous bands, those which are solid clean pyrite usually separate easily from the coal along a natural bedding plane. Sometimes the coal and pyrite fall apart during mining. Bands of this sort which are $\frac{1}{2}$ in. or more in thickness are usually left in the mine. Thinner bands are broken up in shooting down the coal, so that fragments become mixed with the coal and are difficult to pick out. A washer, however, will remove these small pieces.

The grey laminated bands are much more difficult to remove. There usually is no natural cleavage between the band and the adjacent coal. The thin laminae of pyrite merge indistinguishably into the coal and it is necessary to discard a large amount of adhering coal in order to get rid of them by picking. If this material is sent to the washer, a large proportion of the pyrite is slimed in crushing, which increases the difficulty of its removal by washing.

This is also true of the cat-faces and pyrite facings in joint cracks. In order to improve the appearance of a car of coal, lumps which contain only two or three per cent sulphur in this form are sometimes discarded in the tippie. These cat-faces and pyrite facings are the most troublesome visible forms of sulphur for the washerman. When the coal is crushed for washing, even as fine as $\frac{1}{4}$ -in. size, a

large part of this sulphur adheres to the coal particles, and a large part of that which breaks free is slimed or broken into thin scales which have a tendency to float off with the washed coal.

The impregnations of fine grains of pyrite are of course inseparably mixed with the coal. All the coals used in the coal washing tests at Urbana were found to contain a considerable proportion of pyritic sulphur in this form. Samples of coal, finer than $\frac{1}{8}$ -in. maximum size, carefully hand-picked to eliminate all pieces showing a trace of impurity on the surface, still contained pyritic sulphur. Analyses of three such samples of a Williamson County, Illinois, coal are given in Table 1.

TABLE 1
PYRITIC SULPHUR IN HAND-PICKED SAMPLES OF A WILLIAMSON COUNTY,
ILLINOIS, COAL

Values given in table in per cent, on a moisture-free basis

Sample No.	Ash	Pyritic Sulphur	Total Sulphur
1	3.58	1.27	2.06
2	4.82	1.06	1.86
3a	4.27	0.84	1.63

a. Float on solution of 1.3 sp. gr.

7. *Distribution of Coarse Pyrite in Various Beds.*—If a large proportion of the sulphur in a coal is concentrated in lenses, balls, etc., of solid pyrite, it may be possible by careful picking in the mine and tipple to ship a low sulphur product even though the sulphur content of the coal in the bed may amount to several per cent. Ordinarily, however, coals which contain a large amount of coarse pyrite are also high in organic and finer pyritic sulphur. The distribution of the coarse pyrite in the bed is also of importance; for instance, if, throughout the mine, the lenses and balls are concentrated at a certain definite horizon, or if a certain bench contains a much larger percentage of sulphur than the rest of the seam, hand-picking or selective mining is facilitated.

In some of the Central District coal seams the manner of distribution of iron pyrite deposits in the bed is well defined and typical over the entire area of the seam.

Balls or nodules in the roof and extending down into the top coal are characteristic of the Illinois No. 5 seam practically throughout the district in Indiana, Illinois, and Kentucky. Often there is a discontinuous band of pyrite lenses about the middle of this bed.

The No. 6 coal in Illinois naturally divides itself into three benches. The top coal and the bottom coal each comprising from one eighth to one sixth the thickness of the bed are separated from the middle coal by definite persistent partings, the upper one usually of mother-coal and the lower one, called the blue band, of shale sometimes impregnated with pyrite. As a rule the narrow upper and lower benches contain the larger part of the pyrite of the seam.

In the No. 2 coal of the Longwall district of Illinois the sulphur balls and lenses are confined largely to the upper half of the bed.

In the Danville district, where the No. 7 coal is mined, pyrite is usually in the form of lenses of the gray stony variety. They are distributed promiscuously through the bed.

Some interesting notes on the coarse pyrite deposits of the Ohio coals are given by W. M. Tucker* who did the field work in that state in connection with coal pyrite survey of 1918. The Middle Kittanning No. 6 coal contains everywhere a parting near the middle of the bed of pyrite or shale, known as the middle band. This seam in Perry County, Ohio, shows a band of lenses about 13 in. from the top of the coal. In the Upper Freeport No. 7 coal, deposits of pyrite in the form of large boulders are found imbedded in the coal along the sides of the sandstone horsebacks.

* Economic Geology, 14, 198, 1919.

III. FIELD AND LABORATORY METHODS

8. *General Plan of Investigation.*—In outlining the plan for the work it seemed that the only way to study the distribution of the forms of sulphur in the coal bed was to take sectional samples of the horizontal benches of the seam in the mine at the working faces. So far this plan has been extended to the examination of three beds of coal; the No. 6 bed in Illinois and the No. 9 and No. 12 beds in Kentucky. It has not been possible to study each bed at more than one mine, but it is hoped that the No. 6 bed in Illinois may be examined in several other localities, in order to determine whether there is a characteristic distribution of the forms of sulphur throughout the bed. Obviously then, the results here given for the beds examined at one location are not representative of the entire bed with regard to sulphur content, and must not be so interpreted.

The Illinois No. 6 bed was sampled at the Middlefork mine of the United States Fuel Co., near Benton, Franklin County, Illinois. A large and excellently equipped washery* is maintained at the mine for washing the entire output. The daily tonnage varies from 2400 to 2800 for the mine, and the washery operates about ten shifts per week. The washed coal is used for metallurgical purposes.

In Western Kentucky the No. 9 and No. 12 beds were examined; the No. 9 bed in Union County at Mine No. 8 and the No. 12 bed in Webster County at Mine No. 7 of the West Kentucky Coal Co. The No. 12 coal, a fairly low sulphur coal, is used extensively as a gas coal and also as smithing and steam coal. The bed No. 9 coal is much higher in sulphur, and finds sale chiefly for domestic and steam purposes.

At each mine the seam was divided into horizontal benches, and each bench was sampled separately, beginning at the bottom and continuing to the top of the seam. This information was desired not only to show the distribution of the forms of sulphur in the bed, but also for the purpose of determining whether a relation existed between pyritic and organic sulphur. Additional data on the possible relationship was secured by taking additional samples at places in the

* J. R. Campbell, Am. Inst. Min. Eng., Bul. 153, 1779, 1919.

mine which showed the coal intergrown with lenses, bands, and cat-faces of pyrite in such a manner that none of the coal was more than 1 or 2 in. distant from the pyrite itself. At the Middlefork mine seven tippie samples were obtained, each representing a day's output of about 2500 tons of raw coal. A sample of the washed coal representing one day's operation of the washery was taken in order to secure information as to the form or forms of sulphur eliminated by a mechanical process such as washing. Two samples of mother-coal were taken at the Middlefork mine. Determinations of total pyritic and organic sulphur were made on all of these samples, and the results are shown, both graphically and in tabular form, in this bulletin.

In addition to the samples taken for chemical analysis, six were secured in the Middlefork mine for examination of the microscopic pyritic sulphur. These were taken at the middle and near the top of the seam. Microphotographs of thin cross-sections of these samples are given. A small development map showing where the samples were taken in the Middlefork mine is included. A table showing the pyritic and organic sulphur content of several eastern and central district coals is included for the purpose of giving an idea of the relative amounts of these important sulphur forms contained in various coals.

A new form of float and sink apparatus for use in making tests on fine coal has been developed and is described in this paper. It was used in determining the amount and the variations of fine disseminated pyrite in the bench samples from two of the mines.

9. *Method of Collecting Samples.*—The face samples were taken by the Bureau of Mines method for sampling coal in the mine.* A vertical channel about 4 in. wide by 3 in. deep was cut in a clean fresh face, and the resulting cuttings caught on a canvas. At each place selected for sampling the face was marked off, before cutting the sample, into from four to eight horizontal benches, and each bench was sampled separately, beginning at the floor. Each sample was broken up with the mortar and pestle of the Bureau of Mines sampling kit, then screened and reduced to 4-lb. size by coning and quartering. In the Middlefork mine a portable hand crusher† was used to reduce the coal to a size small enough to permit .

* J. A. Holmes, Bureau of Mines, Technical Paper No. 1.

† S. W. Parr, Illinois Geological Survey, Bul. 29, 19, 1914.

reduction by coning and quartering. It crushes the coal to about $\frac{1}{4}$ -in. maximum size. The crusher is of the burr type, made of aluminum and steel, with detachable handle and legs.

Fig. 1 is a photograph of the coal face in the No. 6 Illinois bed and is introduced to give a better idea of the method of sampling used. The total thickness of the bed was first measured. Then the division of the seam into benches was indicated by white horizontal chalk marks made on the coal face. In the Middlefork mine, No. 6 bed, the seam was divided into from four to eight benches at the sampling locations, and each bench was sampled separately. Sectional bench samples were collected in this manner at twelve working faces in the mine. The top sample represents in most cases the first 10 or 12 in. of coal down to the natural bedding plane. The miner designates this bench as the top coal. The bottom sample represents the coal from the floor up to, but not including, the blue band. This band, sometimes shale and often pyrite of a thickness of 2 or 3 in. generally occurs from 10 to 15 in. above the floor. The intervening portions of the bed were divided into from two to six sections, making the divisions, wherever practicable, at natural partings. The blue band was always thrown out. With this exception the samples cover everything from the top to the bottom of the seam, and represent fairly what is hoisted and sent to the washer. Tables A to L in Appendix I give the vertical thickness of the horizontal sections, show the impurities occurring in a section, and indicate whether they were included in the sample.

In the mine operating in the No. 12 bed in western Kentucky, samples were taken at six locations. Here 1 to 2 ft. of the top coal is left up for roof. This was sampled at two locations in the mine, and the remainder of the seam was divided into four approximately equal benches, represented by a corresponding number of samples. The division into benches was made in such a manner as to include in one sample all of a characteristic shaly band occurring a little below the middle of the seam. In the No. 8 mine of the West Kentucky Coal Co., which operates in the No. 9 bed, the seam was divided into four nearly equal benches, four samples being taken at two locations.

The samples of raw coal, washed coal, and refuse were taken in the manner prescribed for the sampling of deliveries.* The gross

* G. S. Pope, Bureau of Mines, Tech. Paper 133, 197.

samples were collected by taking shovelfuls at twenty minute intervals from the raw coal, washed coal, and refuse conveyors, respectively.

10. *Preparation of Samples for Analysis.*—The 4-lb. samples were allowed to air dry in the laboratory. They were then crushed to $\frac{1}{4}$ -in. size and a sample cut out for the sink and float test. The final samples, crushed in the Braun disc pulverizer to pass a 100-mesh screen, were put into 4-oz. glass bottles sealed with rubber stoppers.

11. *Methods Used for Determination of Forms of Sulphur.*—Total sulphur determinations were made by Eschka's method, which is so extensively used for estimating total sulphur in coal that a description of the analytical procedure need not be given here.

The pyritic sulphur was determined by the method of Parr and Powell* by extracting 1 gm. of 100-mesh coal with a mixture of 20 cc. of concentrated nitric (sp. gr. 1.42) and 60 cc. of water. This dilute nitric acid has a specific gravity of 1.12. It is allowed to cool to room temperature before starting the extraction. The extraction is best conducted in a 300 cc. Ehrlenmeyer flask. Some coals are difficult to "wet" by the dilute nitric acid, but by stoppering the flask and shaking vigorously for a minute, this is readily accomplished. The extraction process was always allowed to proceed for four days at room temperature; a vigorous shaking being given the flask once or twice each day during the period.

After extraction, the coal was filtered off and the filtrate evaporated to dryness on the water bath. The residue was then dissolved in about 5 cc. of 1 to 1 hydrochloric acid, diluted with water to about 100 cc., brought to the boiling point, and filtered. Ordinarily, the pyritic sulphur determinations were made by determining the sulphur in the filtrate rather than the iron, but determinations of both iron and sulphur were made on several samples from each mine in order to find if iron and sulphur were present in the same ratio as in iron pyrite. The values obtained by calculating the sulphur from the iron and by determining sulphur direct, agreed very well indeed for the coal from each of the beds.

Organic sulphur was not directly determined. The values shown for this form of sulphur represent the difference between the total and the pyritic sulphur.

* Loc. cit.

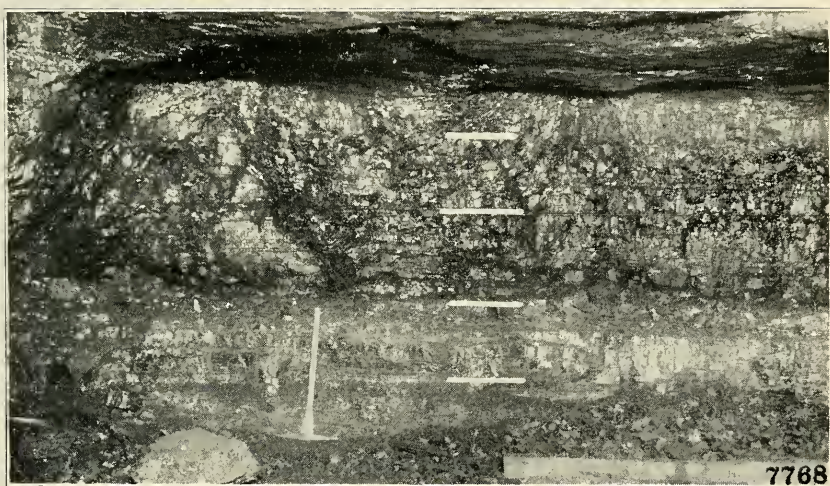


FIG. 1. FACE OF COAL IN ILLINOIS NO. 6 BED.
SHOWING DIVISION OF SEAM INTO BENCHES FOR SAMPLING

Sulphate sulphur was found to be so low in the samples examined that it has been entirely disregarded in tabulating the variations of the forms of sulphur in the coal bed. Only ten samples were examined for sulphate sulphur. It was extracted by boiling 5 gms. of coal for one hour with 50 cc. of hydrochloric acid (sp. gr. 1.2) diluted to 200 cc. with water. The filtrate was evaporated nearly to dryness, diluted, and the sulphates precipitated by adding an excess of a 10 per cent solution of barium chloride in the manner prescribed in the Eschka method for total sulphur. The highest value obtained for sulphate sulphur in the samples was 0.04 per cent, and this value was on a sample containing six per cent of total sulphur which had been mined three months.

12. *Determination of Fine Pyritic Sulphur.*—The term fine disseminated pyritic sulphur is used in this report to designate fine pyrite particles so intimately mixed with the coal that they are not broken free from it by crushing to the finest size at which coal is ordinarily washed, namely, through $\frac{1}{4}$ -in. round hole screen; and sufficiently small not to render the coal particles in which they happen to occur heavy enough to be separable by gravity methods. This is an arbitrary definition which seems perfectly logical from the practical view point in connection with coal washing. The determinations were made by subjecting samples of the coal, of $\frac{1}{4}$ -in. maximum size, to sink and float tests using a zinc chloride solution of a specific gravity 0.05 higher than the average specific gravity of the coal, analyzing the float portion for pyritic sulphur, and correcting for the weight of sink removed in order to express the results as a percentage of the original raw coal sample for comparison with the other analytical data.

As determined by this method the figure for fine disseminated pyritic sulphur percentage does not account for all pyrite particles below any certain fixed size, as some very small pieces, notably of the form which occurs as thin plates in the tiny joint cracks of the coal, may be broken free from the coal and, sinking in the float and sink bath, be thus discarded, while other pieces, possibly larger, which stick fast to or are included in coal particles, may float. The size of the largest pyrite particle which could be included may be calculated as follows; assuming the extreme case of a fragment of the maximum size in the sample consisting of clean coal with an attached or included pyrite particle just large enough to increase the specific gravity of the piece by 0.05,

and assuming that the specific gravity of the clean coal is 1.30, the specific gravity of the raw coal particle in question is then $1.30 + 0.05 = 1.35$.

Let x = volume of pyrite in this raw coal particle of unit volume.

1.00 = total volume of the raw coal particle.

$1.00 - x$ = volume of the clean coal part.

$1.30 (1.00 - x)$ = mass of the clean coal part.

$5x$ = mass of the pyrite part, assuming specific gravity of pyrite as 5.00.

Then $1.30 (1.00 - x) + 5x$ = total mass of the raw coal particle.

Also 1.35 = total mass of the raw coal particle.

$$1.30 (1.00 - x) + 5x = 1.35.$$

$$3.7x = 0.05.$$

$$x = 0.0135 \text{ volume of pyrite part of raw coal particle.}$$

$$\sqrt[3]{0.0135} = 0.26 \text{ size pyrite cube in unit cube of raw coal.}$$

$$0.26 \times \frac{1}{4} \text{ in.} = 0.065 \text{ in. size of maximum pyrite cube in } \frac{1}{4}\text{-in. cube of raw coal.}$$

$$0.065 \text{ in.} = 10 \text{ mesh.}$$

Sixty-five thousandths of an inch in diameter, or through 10-mesh screen, therefore, is the maximum size of pyrite particle that could be included in the float part of the sample in determining fine pyrite sulphur by this method.

For making the determinations a new sink and float apparatus illustrated in Fig. 2 was used. This was designed for use in the coal washing laboratory for control work and preliminary testing when small coal is being handled. For coarse coal of jig size the apparatus developed some years ago by G. R. Delamater is used. This has been very widely used for coal washery control and the method has proved very satisfactory for the size of coal ordinarily washed in jig washers. In fact it is often referred to as the "Standard" sink and float apparatus.

In working with small coal, such as some washeries are now handling, through a $\frac{1}{4}$ -in. screen, or through a $\frac{1}{8}$ -in. screen with a large proportion of slime, and in experimental work in oil coagulation and flotation of coal, where even smaller sizes are handled, a need has arisen for an apparatus in which all the solution used as well as the coal sample is divided into a float portion and a sink portion, and each screened or filtered with the respective float and sink portions of the coal sample.

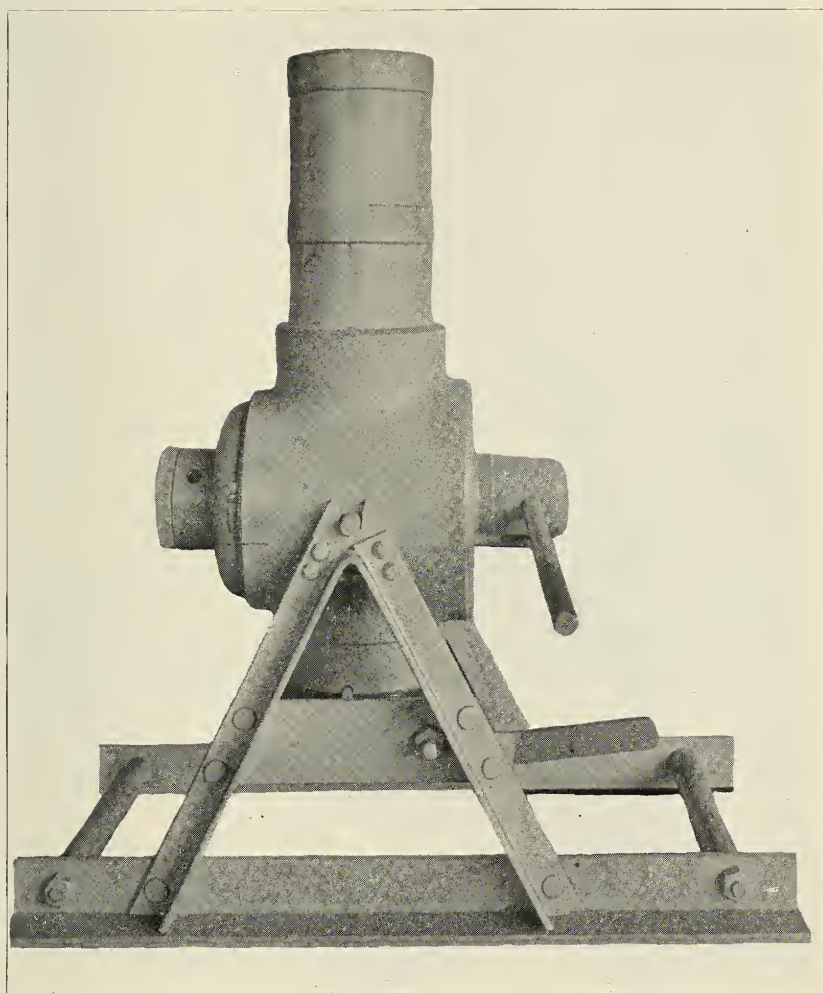


FIG. 2. NEW SINK AND FLOAT TEST APPARATUS, ASSEMBLED

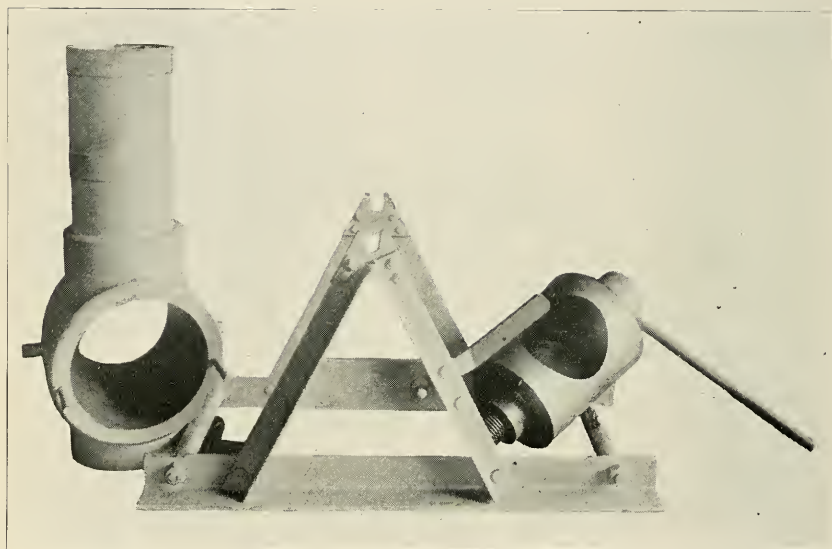


FIG. 3. NEW SINK AND FLOAT TEST APPARATUS, DISSEMBLED, SHOWING
CONSTRUCTION

The apparatus illustrated accomplishes this purpose. It is shown assembled ready for use in Fig. 2. The three separate pieces which make up the apparatus are shown in Fig. 3, and details of construction of the barrel in Fig. 4. The apparatus was made in the laboratory

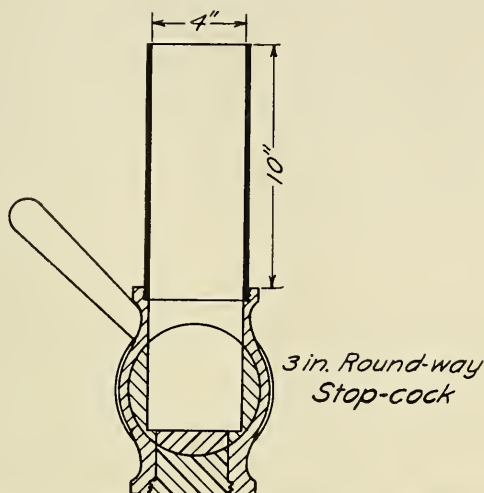


FIG. 4. DETAILS OF BARREL OF NEW SINK AND FLOAT TEST APPARATUS

shop, of a 3-in. round-way stop-cock valve, a piece of 4-in. galvanized pipe, and a tilting frame. It is so simple in construction and operation that very little description is necessary.

The bore of the valve plug was cut out to 4-in. size at one end and almost through to the other side of the plug. A cylindrical wooden core was then inserted, and babbitt was poured in around this to fill up the corners in the interior of the plug and stop up the small end of the bore so that the valve plug, instead of a 3-in. round hole through it, has a 4-in. cylindrical well in it, as shown in Fig. 3. In the valve body one opening was cut out to 4 in. in diameter and a galvanized pipe 10 in. long and 4 in. in inside diameter was soldered on; the opposite opening was closed with babbitt. The 4-in. galvanized pipe and the 4-in. well in the valve plug form the container for the sink and float bath. For convenience in manipulation this barrel was pivoted on a tilting frame fitted with a spring at the bottom for holding the barrel rigid when in the vertical position, and a stop for the valve handle to facilitate lining up the well in the valve plug with the bore of the galvanized pipe.

In making a sink and float test the valve handle is turned over against the stop, forming a continuous cylinder of the pipe and valve, which is filled to within about 2 in. of the top with a solution of the desired specific gravity. The coal sample is then immersed in the solution and stirred till thoroughly wetted. It is then allowed to stand undisturbed for a short time to permit the heavy particles to sink to the bottom and the light particles to rise to the top. The valve handle is then turned through 180 degrees, care being used to avoid jerking or jarring the apparatus. This separates the heavy particles in the well of the valve plug from the light particles floating in the upper part of the galvanized cylinder.

By tilting the barrel the float coal and solution is then poured out into a fine screen or a filter, the solution is drained off for use in another determination, and the adhering particles of float coal in the apparatus are flushed out on the filter with a small stream of water. The valve is then turned back to the open position, and the sink coal and solution is poured upon another screen or filter in the same manner. The products are then washed with water on the screens or filters to remove all trace of the solution used. By using this apparatus with filters for washing the products all the sample is recovered and there is no loss of fines by suspension in the solution. The small volume of solution used makes the operation with vacuum filters fairly short. On samples from which the slime has been removed a 100-mesh screen is used instead of the filter. This apparatus was designed to make the separation just above the top of the sink in the cylinder so that practically all of the solution carrying particles in suspension goes with the float. The float, therefore, includes particles of the same density as the solution and lighter.

Samples as large as 200 gms. are handled in one operation by an apparatus of the size described. If it is desired to use larger samples they should be treated in 200-gm. portions.

For the determination of fine disseminated pyrite, mixtures of chloroform and alcohol and solutions of zinc chloride in water were used. Chloroform and mixtures of chloroform and alcohol are more suitable for sink and float tests on fine coal because they are not so viscous as is a zinc chloride solution of equal density. However, evaporation losses when using chloroform make the method more expensive than when a zinc chloride solution is used.

IV. DATA AND DISCUSSION

13. *Distribution of Forms of Sulphur in the Coal Bed.*—The data on the distribution of the total sulphur, organic sulphur, total pyritic sulphur, and fine disseminated sulphur content through the vertical section of the bed at various places in the three mines examined are shown graphically by means of charts, Figs. 6, 7, 11, and 13. The exact figures for the percentage of each form of sulphur in each bench at each place where a sample was taken are given in tabular form, with a description of the visible impurities in each bench, in Appendix I.

In the charts, distance from the roof or top of the coal is laid off on the ordinate axis and the percentages of total, pyritic, and organic sulphur are laid off as abscissæ. The dash line represents total sulphur, the unbroken line pyritic sulphur, the dot-dash line organic sulphur, and the dotted line fine disseminated pyritic sulphur. The straight vertical lines showing per cent sulphur represent the average values for the forms of sulphur in benches whose relative thicknesses are represented by the lengths of the lines. The horizontal offsets in these vertical lines are due to variations in the values for the forms of sulphur occurring in adjacent benches, and do not mean that the sulphur content changes abruptly at these points. The sulphur percentages plotted in the graphs represent values for moisture-free coal.

Middlefork Mine

The No. 6 coal at the Middlefork mine near Benton, Illinois, averages about 7 ft. in thickness, though it varies from 6½ to over 8 ft. It shows everywhere the characteristic shale blue band of the No. 6 coal, from 9 in. to a foot and a half above the floor. In some places in this mine the blue band is pyrite or shale impregnated with pyrite grains. A sample of the blue band analyzed 6.39 per cent sulphur and 84.4 per cent ash. The seam here divides naturally into three benches, the lower bench separated from the middle bench by the blue band and the upper bench or top coal usually separated from the middle part of the bed by a very thin parting of charcoal or mother-coal, or simply by a well defined

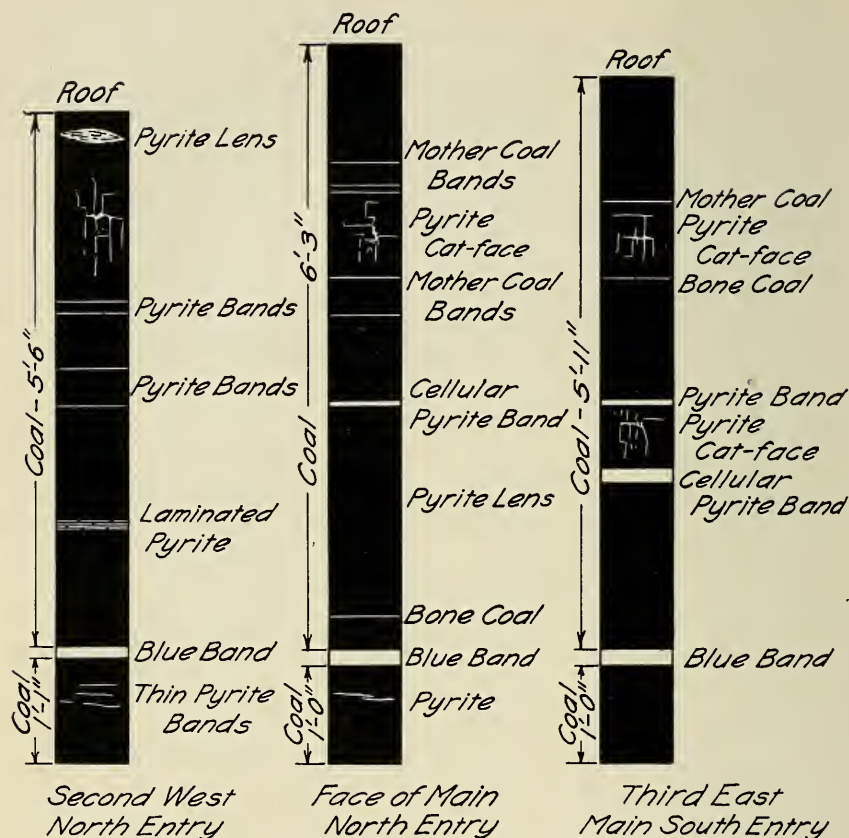


FIG. 5. SECTIONS OF ILLINOIS NO. 6 BED

bedding plane from 12 to 18 in. below the roof. Typical sections of the No. 6 bed showing the occurrence of coarse pyrite infiltrations and other visible impurities at three locations where samples were taken are given in Fig. 6. The moisture content of the coal from this mine averaged 6.45 per cent.

The pyrite occurs chiefly in the form of cat-faces, small flakes or plates and joint fissures, and thin bands less than $\frac{1}{4}$ in. in thickness. Some thicker bands and lenses of solid pyrite also are found, but a large proportion is in the form of very thin bands, veinlets, or plates of a cellular or friable structure, so that it is very difficult to separate from the coal by hand-picking, and it slimes badly when crushed for washing.

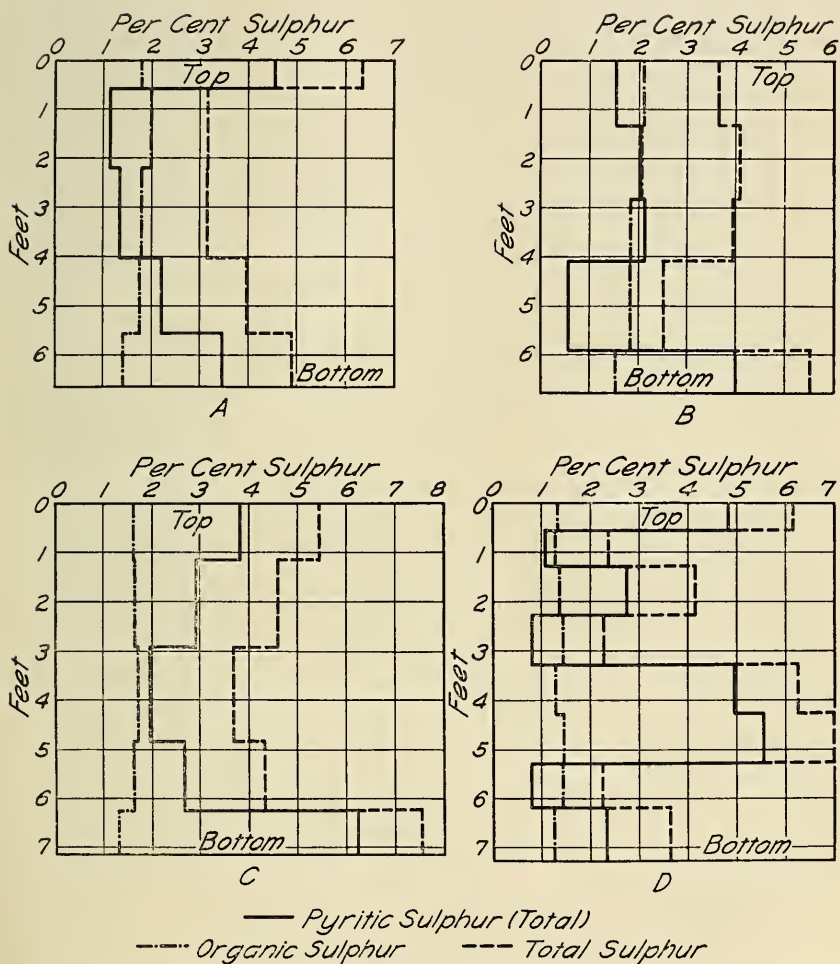


FIG. 6. DISTRIBUTION OF FORMS OF SULPHUR IN THE COAL BED,
MIDDLEFORK MINE

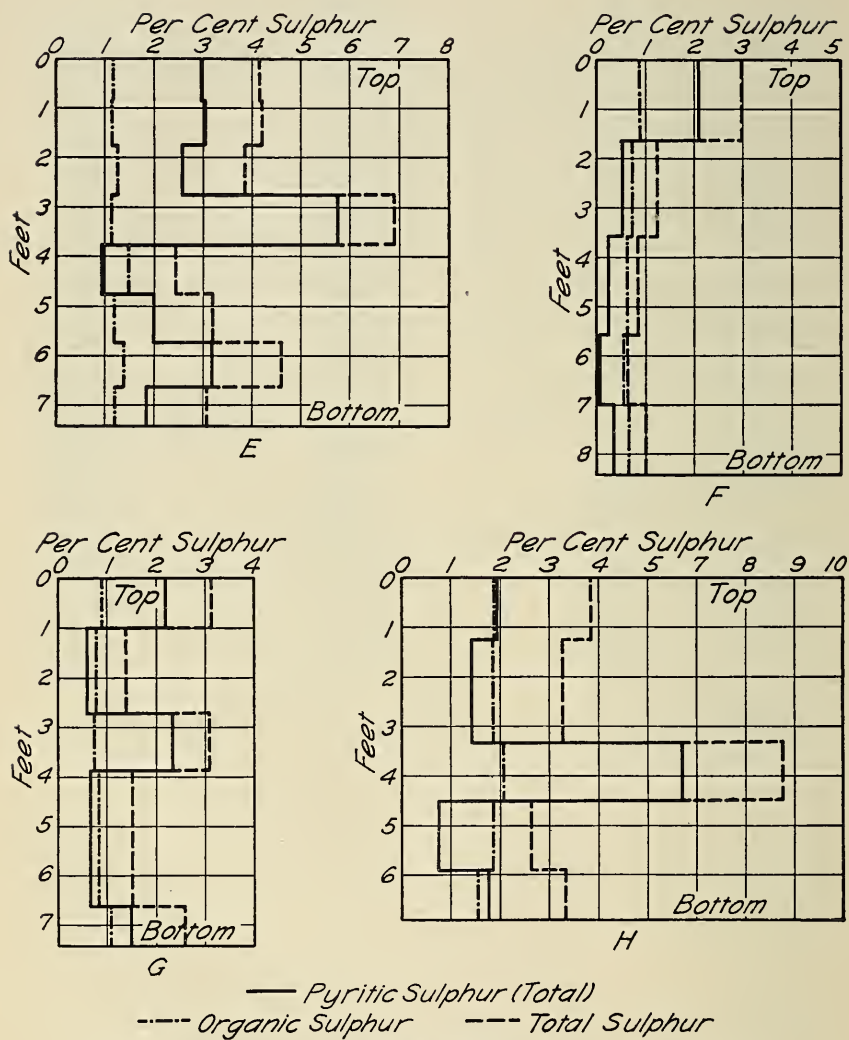


FIG. 7. DISTRIBUTION OF FORMS OF SULPHUR IN THE COAL BED, MIDDLEFORK MINE

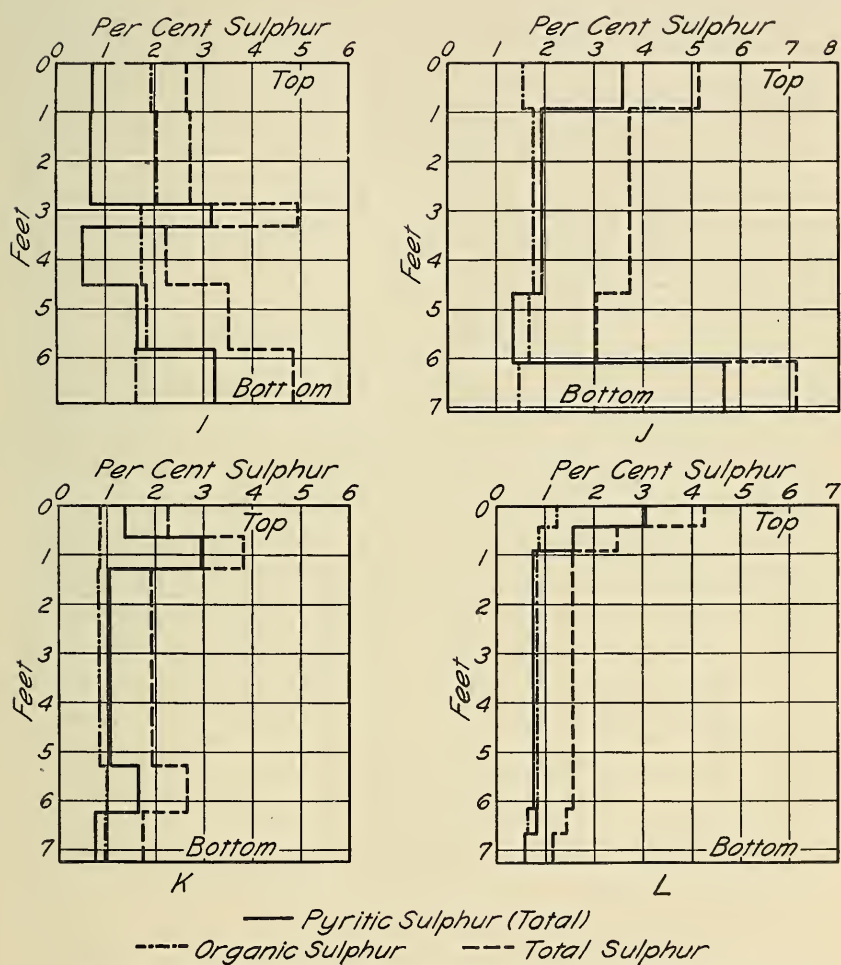


FIG. 8. DISTRIBUTION OF FORMS OF SULPHUR IN THE COAL BED,
MIDDLEFORK MINE

The distribution of the forms of sulphur occurring in this mine are plotted in Fig. 6; A, B, C, D, E, and F, and Fig. 7; G, H, I, J, K, and L. Each lettered chart represents a vertical section of the coal bed at one of the places sampled. A small development map of the mine, Fig. 9, shows by corresponding letters the location at which each

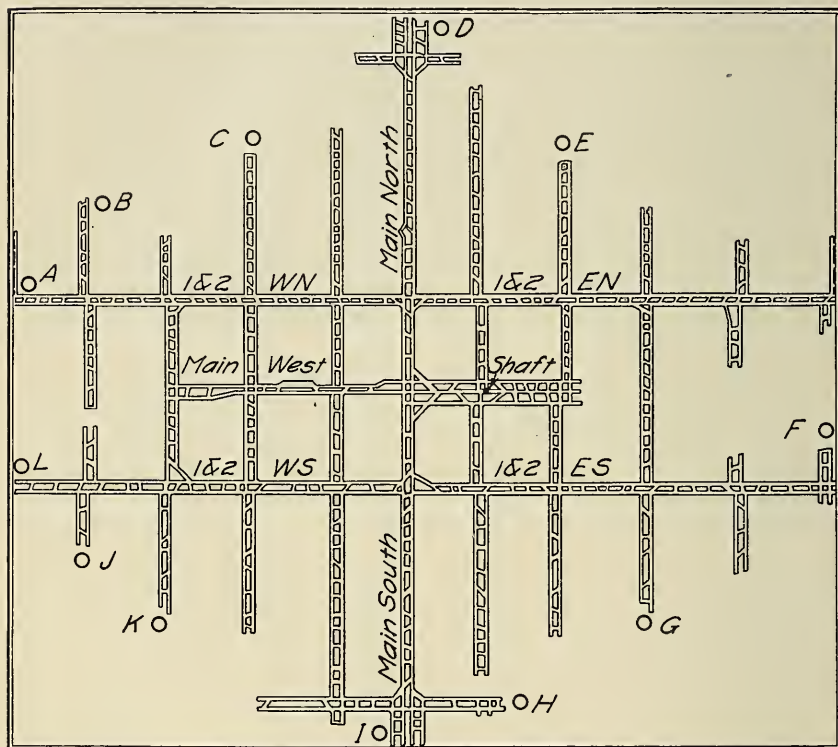


FIG. 9. SKELETON MAP OF MIDDLEFORK MINE

set of samples was taken. The same letters are also used to designate the tables in Appendix I which give the analyses of samples taken at these locations.

It is hardly necessary to point out the great variations in pyritic sulphur content in the vertical span of the seam. This is shown most clearly in the graphs. In practically every section the total sulphur and the pyritic sulphur are much higher in the top coal and in the coal below the blue band than in the middle benches of the bed. The

average total sulphur content of all the top coal samples was 4.17 per cent; of the bottom coal, 3.83 per cent; and the average content of the middle benches between the top coal and the blue band was 3.05 per cent.

The organic sulphur content is shown in the graphs by the dot-dash line. While this does not run uniform from top to bottom of the seam the variations are not large. As compared with the very great changes in percentage of pyritic sulphur, the organic sulphur is quite uniform. The greatest divergence in any one section is from a minimum of 0.57 per cent to a maximum of 1.25 per cent. Considering the mine as a whole, however, it cannot be said that the organic sulphur is constant and uniformly distributed, as the samples varied from this low percentage, 0.57, to as high as 2.10, thus showing the maximum variation over the mine of one and a half per cent. The theory has been advanced that the organic sulphur content of a given bed of coal may be constant over large areas and characteristic of that particular bed. The results of these investigations, however, would indicate that the uniformity of organic sulphur distribution is confined to very limited areas in the bed and that even for a single mine as a whole the variations in organic sulphur content may be considerable.

The average percentages of the different forms of sulphur in the three natural benches of the coal are given in Table 2. It is to be noted that the coal below the blue band is somewhat lower in organic sulphur content than either the top or middle coal.

It will be observed from the graphs that, though in general the top coal and bottom coal benches are much higher in total sulphur than the middle part of the bed, the pyritic sulphur and consequently the total sulphur locally is very erratic in occurrence, and, at some places sampled, sections of considerable thickness near the middle of the face are very high in sulphur. This is only partly explainable as due to chance inclusion, in a comparatively small section, of a large lens or band of pyrite. As a comparison of the graphs with the notes in Appendix I will show, some benches that showed no conspicuous concentrations of visible pyrite analyzed high in pyritic sulphur.

The two samples of mother-coal taken at this mine were much higher than the average coal in pyritic sulphur and ash but very low in organic sulphur. These are samples of mother-coal partings about 2½ in. thick, occurring near the roof. One was taken near the face of the first East South Entry near F on the map and the other at the face of the first West South Entry near L.

TABLE 2
SULPHUR FORMS IN TOP, MIDDLE, AND BOTTOM BENCHES OF No. 6 COAL IN
MIDDLEFORK MINE

Values given in table in per cent, on a moisture-free basis

LOCATION	Total Sulphur	Pyritic Sulphur	Organic Sulphur	Ash
North side of mine:				
Average top coal samples.....	5.16	3.55	1.61	10.3
Average middle benches.....	3.81	2.20	1.61	12.1
Average below blue band.....	4.96	3.50	1.36	13.8
South side of mine:				
Average top coal samples.....	3.47	2.14	1.33	8.2
Average middle benches.....	2.48	1.23	1.25	9.7
Average below blue band.....	3.04	1.95	1.09	11.3
Mine average:				
Top coal samples.....	4.17	2.72	1.45	9.1
Middle benches.....	3.05	1.65	1.40	10.7
Below blue band.....	3.83	2.63	1.20	12.3

TABLE 3
FORMS OF SULPHUR IN MOTHER-COAL SAMPLES
Values given in table in per cent, on a moisture-free basis

Sample No.	Location	Total Sulphur	Pyritic Sulphur	Organic Sulphur	Ash
56	1st E. S.	6.63	6.62	0.01	39.3
57	1st W. S.	20.85	20.48	0.37	32.3

Although the samples were very high in pyritic sulphur it was not apparent to the eye, and Sample No. 57 carrying 20.48 per cent pyritic sulphur was not conspicuously higher in visible pyrite than Sample No. 56 with only 6.62 per cent pyritic sulphur.

There has been considerable speculation as to the composition and origin of mother-coal layers in coal beds. Other investigators who present analyses of this substance, have generally concluded that mother-coal, while different in composition from the adjacent coal in the same bed, is a good grade of fuel not appreciably higher in ash and sulphur content. E. A. Holbrook* gives the analysis of a sample taken from a ½-in. band of mother-coal in the No. 6 bed in Williamson County as follows: "Volatile matter, 9.75 per cent; fixed carbon, 87.47 per cent; ash, 1.72 per cent; and sulphur, 0.90 per cent. This indicates that mother-coal is a high grade coal of different composition than the rest of the seam." T. E. Savage† in discussing the vegetable matter of Illinois coal beds says that the mineral charcoal generally contains but little, if any, more ash than the average coal of the bed in which it occurs. Analysis of a sample of mother-coal and of a sample of ordinary coal, both from the No. 6 bed in Williamson County, are given as follows:

	Volatile Matter	Fixed Carbon	Ash
Coal	34.91	52.03	10.72
Mother-Coal	23.96	64.28	10.18

On the other hand the samples of mother-coal taken at the Middlefork mine in connection with the present work contained much larger percentages of both ash and pyritic sulphur than the average coal of the bed. In view of these divergent results it is impossible to make any generalizations. Additional information on the composition of

* Univ. of Ill. Eng. Exp. Sta., Bul. 88, 62, 1916.

† Journal of Geology 22, 76, 1914.

mother-coal is desirable. The most striking characteristic of the two samples examined is that they contained practically no organic sulphur.

West Kentucky Coal Co., Mines Nos. 7 and 8

The No. 12 or Caney Fork seam in Kentucky, sampled at the No. 7 mine of the West Kentucky Coal Company, is a high grade low sulphur coal used to a considerable extent as producer gas and by-product coal. This seam varies from 7 to 9 ft. in thickness and carries in most places a characteristic shaly bench, consisting of parallel bands of shale interbedded with coal and bone-coal, one third the height of the coal above the floor and varying from 6 to 12 in. in thickness. From 6 in. to 2 ft. of top coal is left up in the room to protect the roof. The moisture content of the coal from this mine averaged 6.21 per cent.

The visible pyrite occurs mainly as thin bands from the thickness of paper up to $\frac{1}{4}$ in. with only occasional lenses and cat-faces. Sections at three places sampled showing occurrence of visible impurities are given in Fig. 10. The lenses and cat-faces can be picked out to a certain extent, but the larger part of what pyrite occurs in this mine is too fine for hand-picking.

The No. 9 coal sampled at Mine No. 8 has been correlated with the No. 5 bed mined in Saline County, Illinois. It has, at the No. 8 mine where sampled, a thickness of about 5 ft. It is higher in both sulphur and ash than the No. 12 coal and is sold only for steam and domestic fuel. Visible pyrite occurs mainly in the form of interbedded sheets or bands, varying in thickness up to $1\frac{1}{2}$ in. Those which are as much as $\frac{1}{2}$ in. in thickness generally hold together well enough to be largely removed by hand-picking. A large proportion of the sulphur that is concentrated in definite bands could probably be eliminated by washing.

The data on distribution of sulphur in these beds, secured by analysis of the sectional samples taken, are presented graphically in Fig. 11, N, O, P, Q, R, and S; and Fig. 13, U and V. The locations represented by Fig. 11; N, O, P, Q, R, and S, referring to the No. 7 mine, are shown by corresponding letters on the development map, Fig. 12.

The great variations in pyritic sulphur content in the vertical span of the seam are shown most clearly in the graphs, where the un-

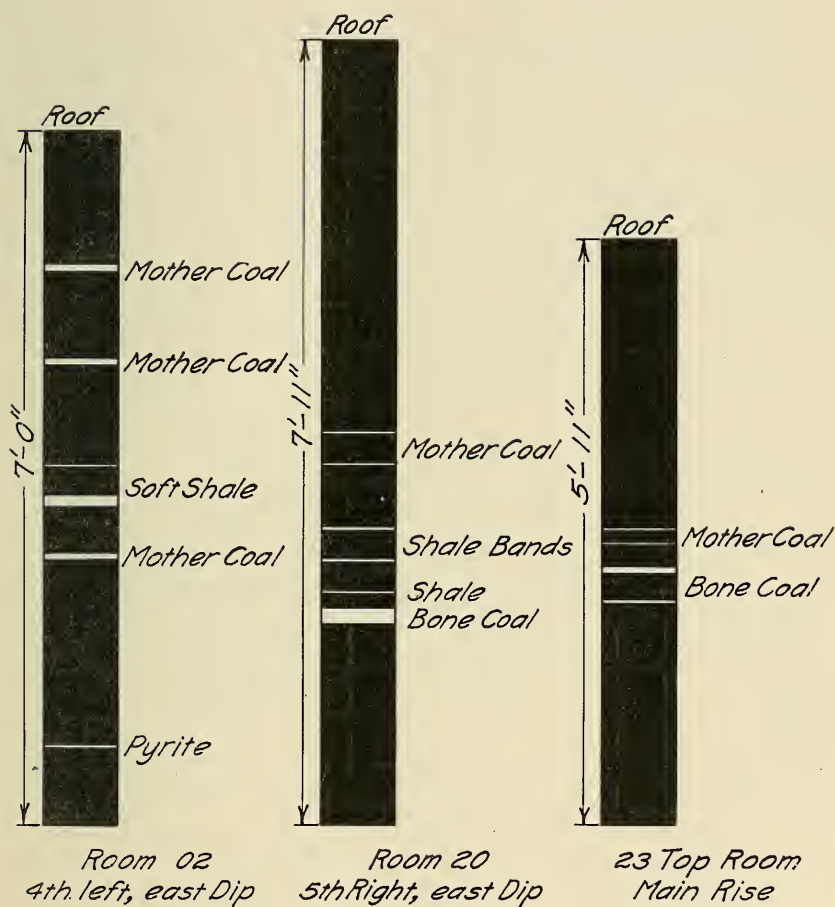


FIG. 10. SECTIONS OF KENTUCKY No. 12 BED

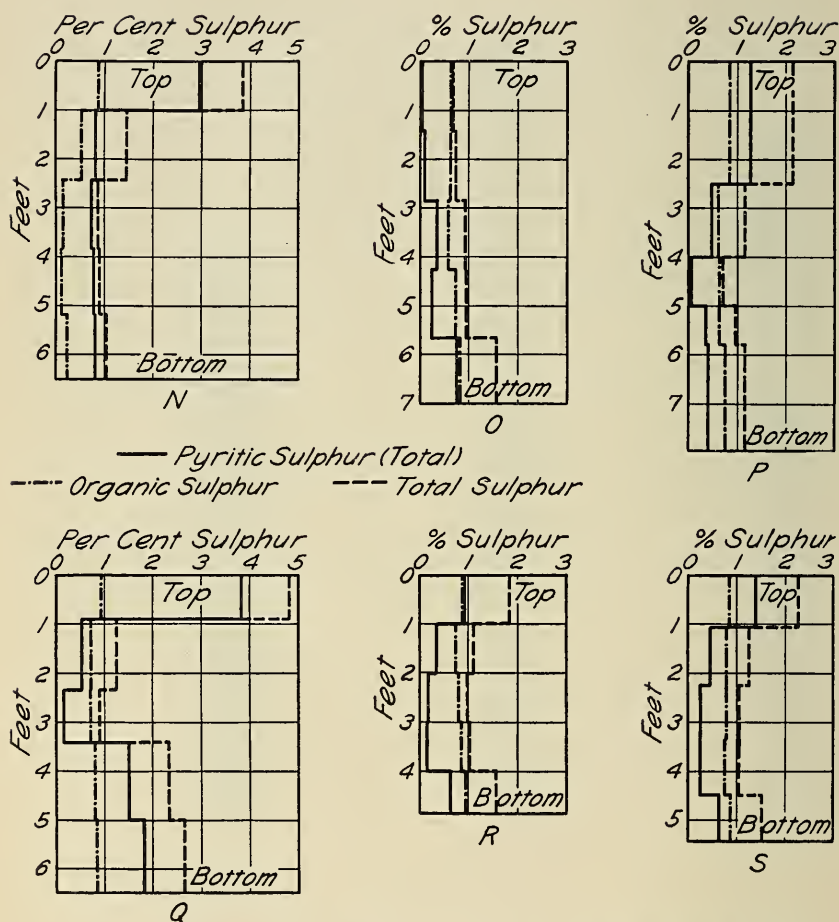


FIG. 11. DISTRIBUTION OF FORMS OF SULPHUR IN COAL BED, MINE No. 7,
 WEST KENTUCKY COAL CO.

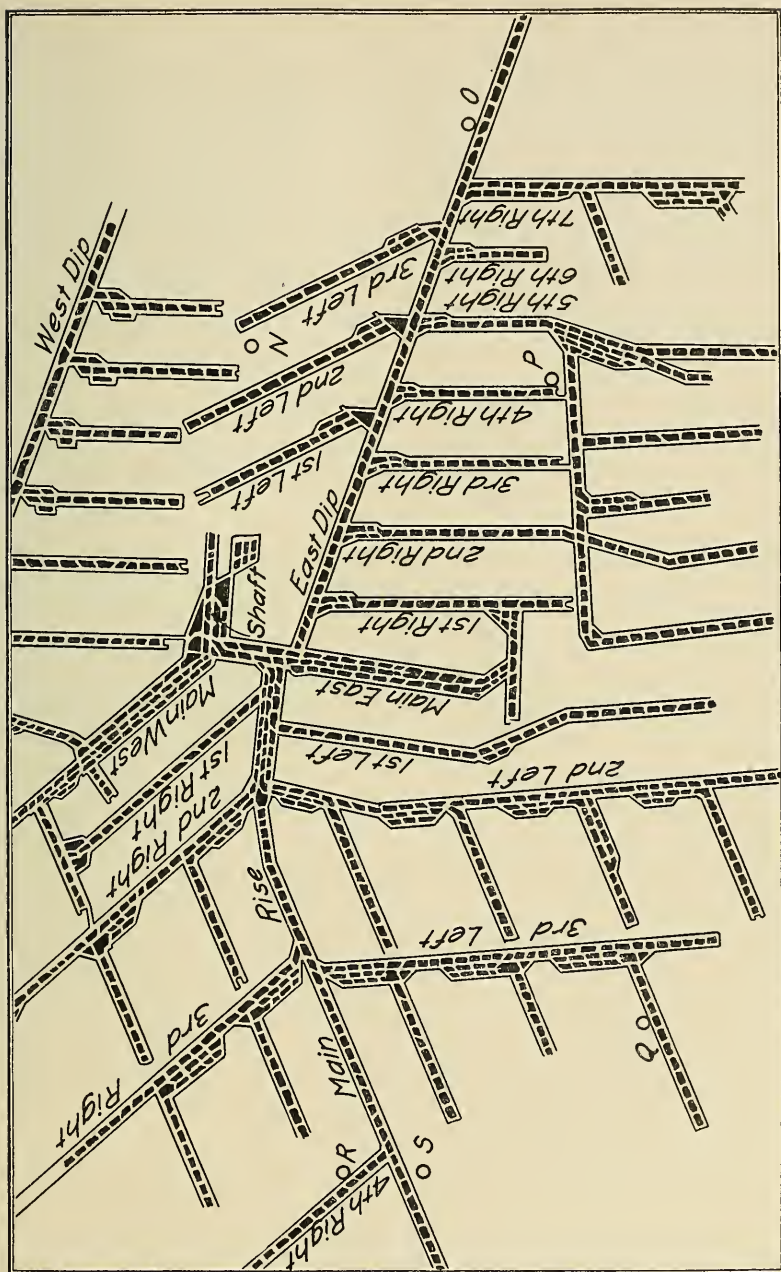


FIG. 12. SKELETON MAP OF MINE No. 7, WEST KENTUCKY COAL CO.

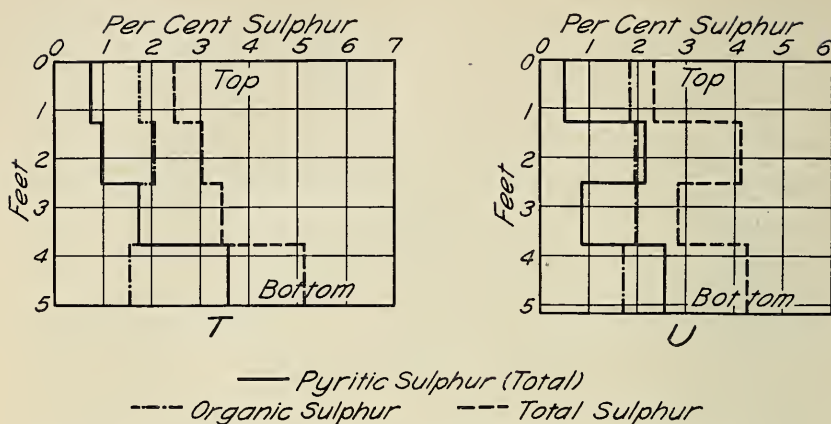


FIG. 13. DISTRIBUTION OF FORMS OF SULPHUR IN COAL BED, MINE NO. 8, WEST KENTUCKY COAL CO.

broken line represents the pyritic sulphur content. At almost every place sampled in the No. 7 mine, the total sulphur and the pyritic sulphur content are considerably lower in the middle part of the bed than in the top and bottom coal. At each place the bed was divided into five approximately equal benches for sampling, and as a rule the middle bench was lowest in sulphur, the two adjacent above and below were higher, and the top coal and bottom coal were highest. At the two places where samples of the roof coal were taken, it proved to be higher in sulphur than any other part of the bed in the same location.

The average sulphur content of the different benches of the seam throughout the mine are given in Table 4. Considering only that part of the seam which is mined, the upper section of an average thickness of 1 ft. 3 in. has an average sulphur content of 1.85 per cent. The bottom coal to an average height of 1 ft. 3 in. above the floor averages 1.58 per cent sulphur, and the intervening middle part of the bed averaging about three and a half feet has an average sulphur content of 1.09 per cent. The weighted average of all the samples taken in the mine is 1.48 per cent sulphur and 9.12 per cent ash.

Two sets of sectional samples taken in the No. 8 mine indicate that in this seam the highest total sulphur and pyritic sulphur content are in the bottom coal and that the sulphur content decreases more or less uniformly from the floor to the roof.

TABLE 4
AVERAGE OF SULPHUR FORMS IN BENCHES OF No. 12 KENTUCKY BED
West Kentucky Coal Co. Mine No. 7

Values given in table in per cent, on a moisture-free basis

Sample	Average Thickness Ft. In.	Total Sulphur	Pyritic Sulphur		Total Sulphur Float on 1.40	Organic Sulphur	Ash
			Total	Fine			
Top Coal Samples	1 0	3.46	2.57	0.74	1.69	0.91	16.55
	1 3	1.85	1.04	0.28	1.03	0.77	7.32
	1 2	1.03	0.31	0.21	0.96	0.73	8.23
	1 2	0.97	0.26	0.23	1.09	0.72	14.63
Bottom Coal Samples	1 4	1.24	0.45	0.23	1.00	0.79	7.86
	1 3	1.58	0.74	0.32	1.21	0.84	8.76
Average		1.48	0.70	0.34	1.17	0.78	9.12

TABLE 5
AVERAGE OF SULPHUR FORMS IN BENCHES OF No. 9 KENTUCKY BED
West Kentucky Coal Co. Mine No. 8

Values given in table in per cent, on a moisture-free basis

Sample	Average Thickness Ft. In.	Total Sulphur	Pyritic Sulphur		Total Sulphur Float on 1.40	Organic Sulphur	Ash
			Total	Fine			
Top Coal Samples	1 3	2.39	0.61	0.33	2.18	1.79	10.25
	1 3	3.56	1.56	0.56	2.00	2.00	7.46
	1 3	3.14	1.29	0.70	2.79	1.85	9.90
Bottom Coal Samples	1 4	4.70	3.08	1.29	3.17	1.63	14.49
Average		3.46	1.65	0.73	2.69	1.81	10.61

The organic sulphur content is shown in the graphs by the dot-dash line. While this does not run uniform from top to bottom of the seam the variations, as shown in the graphs, are not large. As in the Illinois bed No. 6 coal, the organic sulphur content is quite uniform. As compared with the very great changes in the percentage of pyritic sulphur, the greatest divergence in any one section in the No. 7 mine is from a minimum of 0.72 per cent to a maximum of 0.97 per cent. A closer approach to uniformity of the values for organic sulphur in the different benches at a given location is not obtained by calculating organic sulphur content on a moisture, ash, and pyritic sulphur free basis. The fine disseminated pyrite like the organic sulphur is much more uniform and constant in amount throughout the height or thickness of the bed than is the total pyritic sulphur. These two forms, the organic and the fine disseminated pyritic sulphur, may be regarded as very largely fixed or irremovable sulphur, the percentage of which is more or less constant and characteristic of the bed. The irregularities in the distribution of the total sulphur are due mainly to the concentration of pyrite in bands or lenses which are largely removable by hand-picking or to a greater extent by washing.

Mine No. 7 in the Caney Fork, or No. 12 seam, was sampled on June 17, 1918, by W. B. Plank of the United States Bureau of Mines to determine the suitability of this coal for bunker coal. At that time four channel face samples were taken and designated as follows: Sample G taken at the face of the tenth room, off second south entry, off the seventh right east dip entry, 3000 ft. east of the shaft; Sample H taken at the face of the twenty-fourth room, off third left, off the east dip entry, 1400 ft. northeast of the shaft; Sample I taken at the face of the eleventh room, off the fourteenth south entry, off the second

TABLE 6
ANALYSES OF KENTUCKY BED NO. 12 COAL FROM MINE No. 7

Values given in table in per cent, on a moisture-free basis

Sample	Moisture	Volatile	Fixed Carbon	Ash	Sulphur	B. t. u.
G	4.49	35.32	49.97	10.22	1.04	12 540
H	4.61	36.55	51.67	7.17	1.08	13 010
I	6.78	33.43	46.39	13.40	1.21	11 710
J	5.99	33.06	52.35	6.60	1.08	12 900
Average	5.47	34.59	50.09	9.35	1.10	12 540

left main rise, 5100 ft. southeast of the shaft; and Sample J taken at the face of the third south entry, off the third left entry, off the main rise, 2800 ft. south of the shaft. Analyses of these samples are given in Table 6.

14. *Relation of Pyritic and Organic Sulphur.*—It is apparent from the graphs showing the distribution of the sulphur forms that in the beds examined organic sulphur is not concentrated around or associated with segregations of iron pyrite, as the curves for organic and pyritic sulphur show no similarity. A high pyrite content is not accompanied by high organic sulphur content. On the contrary, in the majority of cases, in going from one bench or section of the bed to the next one adjacent above or below, an increase in pyritic sulphur is accompanied by a decrease in organic sulphur and vice versa. This is true almost without exception in the north half of the Middlefork mine, where the samples secured averaged much higher in sulphur than in the south side of the mine. This can hardly be interpreted as supporting the idea that organic sulphur contributes to the formation of pyritic sulphur, however, as this tendency is not nearly so evident in the south side of this mine nor in the other two mines examined.

These graphs show how important is the element of organic sulphur in coal and what large proportion of the total sulphur of these coals it makes up. Of the thirty samples collected in the West Kentucky No. 7 mine, in only seven does the pyritic sulphur content exceed the organic sulphur content. Organic sulphur, which is often considered as constituting a negligible percentage of the total sulphur of coal, exceeds the pyritic sulphur in twenty-three samples out of the thirty collected in this mine. In the No. 8 mine of this company, the coal is higher in organic than in pyritic sulphur in five of the eight samples collected. Of sixty-six regular bench samples collected in the Middlefork mine, twenty-one carried more organic than pyritic sulphur.

The total sulphur in the average for the No. 7 mine is distributed between the pyritic and organic forms as follows:

	Per cent
Pyritic sulphur	47.3
Organic sulphur	52.7
Sulphate sulphur	Trace

For the average of the samples collected in No. 8 mine the proportions are:

	Per cent
Pyritic sulphur	47.7
Organic sulphur	52.3
Sulphate sulphur	Trace

At the Middlefork mine the distribution is as follows:

	Per cent
Pyritic sulphur	60.5
Organic sulphur	39.5
Sulphate sulphur	Trace

The sulphate sulphur content of the coal from each of these mines is less than 0.02 per cent in the freshly mined samples.

The graphs of Figs. 14, 15, 16, and 17 were plotted to show any possible relation that might exist in the occurrence of pyritic and of organic sulphur. The pyritic sulphur values were plotted as abscissæ and per cent organic sulphur as ordinates. Fig. 14, repre-

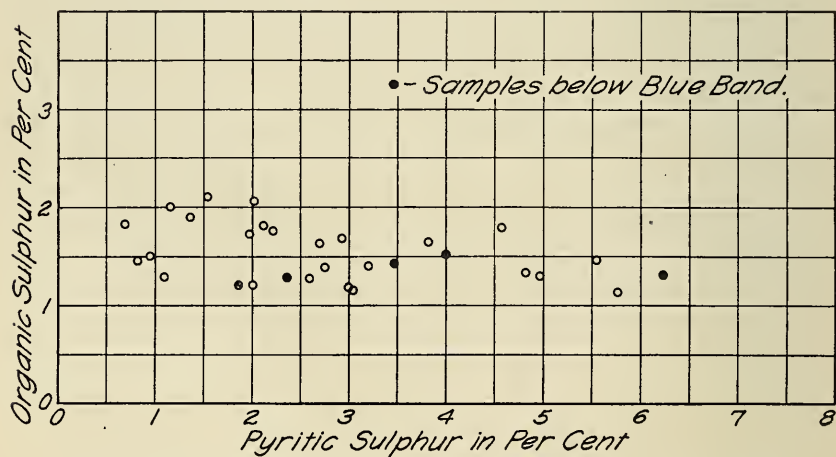


FIG. 14. RELATION OF PYRITIC AND ORGANIC SULPHUR IN THE NORTH HALF OF MIDDLEFORK MINE

senting the samples taken in the north side of the Middlefork mine, shows a general tendency toward lower organic sulphur in the samples high in pyrite. The other graphs, however, show no definite tendency either way in the other half of this mine or in the other two mines

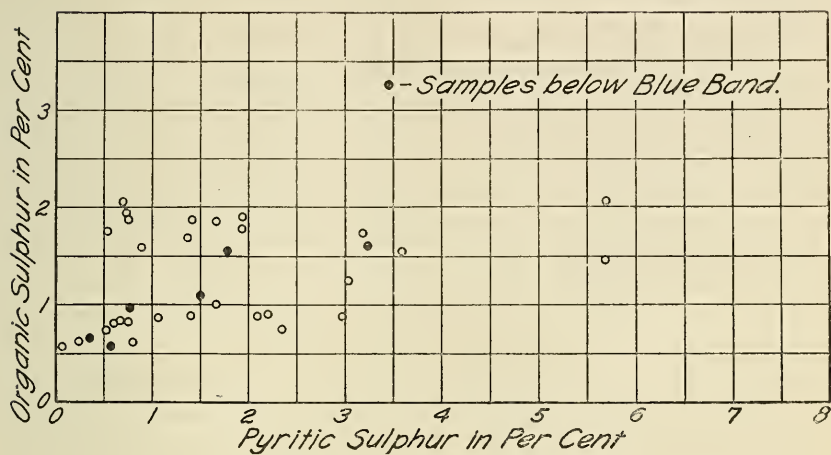


FIG. 15. RELATION OF PYRITIC AND ORGANIC SULPHUR IN THE SOUTH HALF OF MIDDLEFORK MINE

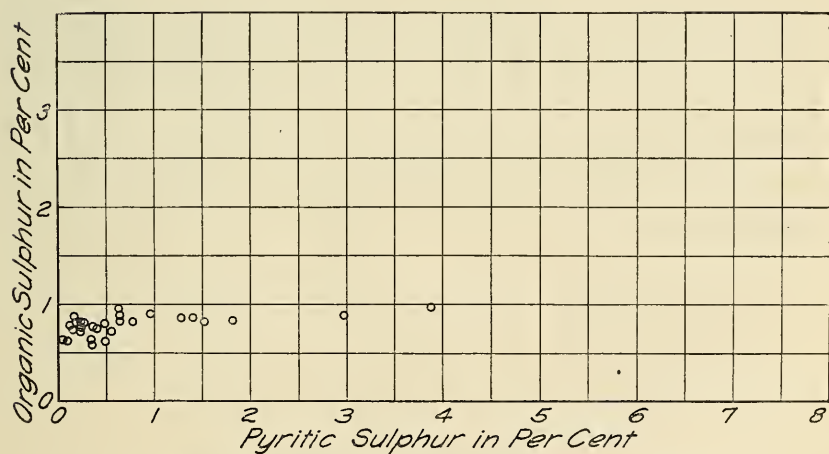


FIG. 16. RELATION OF PYRITIC AND ORGANIC SULPHUR IN MINE NO. 7, WEST KENTUCKY COAL CO.

examined. On the whole the data secured cannot be interpreted as showing any definite relationship between organic sulphur and pyritic sulphur.

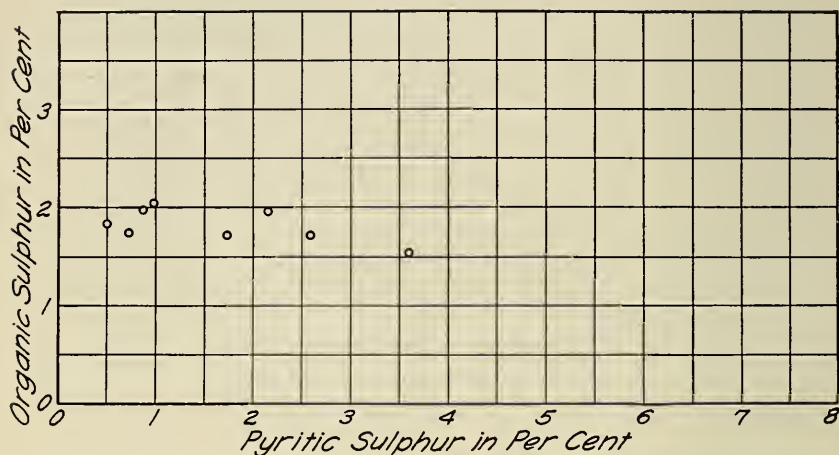


FIG. 17. RELATION OF PYRITIC AND ORGANIC SULPHUR IN MINE No. 8,
WEST KENTUCKY COAL CO.

In order to secure additional data on the possible relation of organic sulphur to pyritic sulphur, a number of special samples were taken of coal immediately surrounding or interbedded with bands or cat-faces of pyrite. These samples were found to be about average or below the average in organic sulphur content. There is no evidence of a concentration of organic sulphur in the coal immediately adjacent to pyrite inclusions. Description and analyses of these samples are given in Table 7.

TABLE 7
SULPHUR FORMS IN SAMPLES HIGH IN PYRITE
Values given in table in per cent, on a moisture-free basis

Sample No.	Vertical Thickness Ft. In.	Total Sulphur	Pyritic Sulphur	Organic Sulphur	Ash	Remarks
15	0 7	6.36	4.58	1.78	19.0	Section 7 in. by 1 ft. 9 in. on both sides, and along lens of pyrite. Lens varies from 0 to 1½ in. thick.
15		2.83	0.88	1.95		Float on chloroform, 1.48 sp. gr.
20A	0 9	8.70	7.86	0.84		Section 9 in. by 1 ft. 9 in. on both sides, and along lens of hard amorphous pyrite and cat-faces of crystalline cellular pyrite.
25A	0 4	8.11	7.63	0.48		Section 2 ft. 9 in. along a 1½-in. to 2-in. band of crystalline flaky pyrite.
42	1 0	7.01	5.54	1.47		¾-in. lens of pyrite.
42		2.69	1.19	1.50		Float on chloroform, 1.48 sp. gr.
52	1 0	6.90	5.76	1.14	14.0	Vertical vein pyrite ¼ in. by 5 in. and two cat-faces.
56A		7.54	6.28	1.26		Strip 6 in. by 2 in. intergrown with veins, bands, and cat-faces of crystalline pyrite.
56A		2.73	1.43	1.30		Float on chloroform, 1.48 sp. gr.
63	0 5½	4.93	3.19	1.74	11.7	5½ in. by 2 ft. 5 in. along high sulphur.
68	1 2	7.76	5.70	2.06	16.5	¾-in. band hard pyrite, 1½-in. band cellular pyrite, and small cat-face.
71	0 6	6.47	4.98	1.49	20.1	1 in. to 2 in. on each side and between two ½-in. bands of pyrite, 3 ft. from bottom.

15. *Forms of Sulphur in Raw and Washed Coals.*—It is evident that if organic sulphur segregates with or is concentrated around pieces of pyrite, bone-coal, or shale, which are all of higher specific gravity than the coal itself, it will be removed along with these impurities as the refuse in the washing operation. On the other hand, if the occurrence of organic sulphur is independent of the distribution of these impurities, their separation should not greatly affect the amount of organic sulphur found in the washed coal, except as a slight increase due to the removal of non-coal impurities, inorganic in nature and containing no organic sulphur. This increase in the organic sulphur content of the washed coal would of course be dependent upon the amount of inorganic impurities removed, and only in unusual cases would the increase be appreciable.

In order to obtain data on this question and on the forms of sulphur in raw and washed coal, seven samples of run of mine raw coal were obtained at the Middlefork mine, in addition to the face samples collected in the mine. Each raw coal sample represents a day's production for the mine, which varies between 2400 and 2800 tons. A sample of washed coal representing one day's operation of the washery, on an average day, was secured for the purpose of comparing the forms of sulphur present in the raw and the washed coal. Table 8 shows the forms of sulphur present in the raw and the washed coal from the Middlefork mine and washery, together with the average for the face samples collected in the mine.

Table 8 shows the average sulphur values for about 17 500 tons of coal representing seven days' production. The sulphur content varies

TABLE 8

FORMS OF SULPHUR IN RAW AND WASHED COALS FROM THE MIDDLEFORK MINE

Values given in table in per cent, on a moisture-free basis

Sample No.	Total Sulphur	Pyritic Sulphur	Organic Sulphur
72 Raw Coal.....	3.68	2.42	1.26
73 Raw Coal.....	3.20	1.90	1.30
74 Raw Coal.....	3.22	1.99	1.23
75 Raw Coal.....	3.59	2.07	1.52
76 Raw Coal.....	3.33	1.93	1.40
77 Raw Coal.....	3.27	2.08	1.19
78 Raw Coal.....	2.77	1.55	1.22
Average for raw coal.....	3.29	1.99	1.30
Average of face samples for mine.....	3.30	1.94	1.38
Washed coal.....	2.25	0.92	1.33
Refuse.....	13.45		

from day to day, but the monthly average for total sulphur is about 3.25 per cent. The average of samples 72 and 78 inclusive, here shown as 3.29 per cent sulphur, therefore represents closely the average production of the mine for total sulphur. Attention is directed to the comparison of the average values for the different forms of sulphur in the run of mine coal with the average for the sectional face samples collected in the mine. For total sulphur the values are 3.29 and 3.30 per cent, for pyritic sulphur 1.99 and 1.94 per cent, and for organic sulphur 1.30 and 1.38 per cent, respectively. This indicates that the sectional face samples collected in the mine are, as a whole, representative of the mine production.

It is interesting to note that the values for organic sulphur in the run of mine raw coal, and in the sample washed coal, are nearly identical; namely, 1.30 per cent for the raw coal and 1.33 per cent for the washed coal. These figures are shown in Table 8 given above. Though the washed coal sample does not necessarily represent the product obtained by washing the identical coal of the run of mine samples, these figures must be taken to show that little or no organic sulphur is removed by the washing process.

Similar data* have been secured as to the effect of washing on forms and distribution of sulphur on two other coals. Table 9 shows the results for a coal from White County, Tennessee, and for samples representing a day's run at a 1200 ton commercial washery in Williamson County, Illinois. The five washed coals are the various trade sizes produced at this washery.

TABLE 9
FORMS OF SULPHUR IN RAW AND WASHED COALS
Values given in table in per cent, on a moisture-free basis

Source and Grade of Coal	Total Sulphur	Pyritic Sulphur	Sulphate Sulphur	Organic Sulphur
White County, Tennessee:				
Raw coal.....	4.87	3.59	0.11	1.17
Table-washed coal.....	3.02	1.84	less than 0.01	1.18
Jig-washed coal.....	3.80	2.61		1.19
Williamson County, Illinois:				
Raw coal.....	1.83	1.04	Trace	0.79
No. 1 washed coal.....	1.81	1.05		0.76
No. 2 washed coal.....	1.56	0.78		0.78
No. 3 washed coal.....	1.57	0.82		0.75
No. 4 washed coal.....	1.57	0.81		0.76
No. 5 washed coal.....	2.33	1.57		0.76

* Fraser and Yancey, Am. Inst. Min. Eng., Bul. 153, 1822, 1919.

These results indicate that only pyritic and sulphate sulphur are removed by washing. Iron sulphate resulting from the oxidation of pyrite is soluble in the wash water. Gypsum occurring in thin plates is difficult to separate with the refuse. From these considerations it is believed that the sulphate sulphur of the White County, Tennessee, raw coal resulted from the oxidation of pyrite, rather than from the presence of calcium sulphate, since sulphate sulphur is almost entirely absent in the clean washed coal.

An inspection of Tables 8 and 9 indicates that organic sulphur is distributed more or less uniformly through the coal substance and that it is not concentrated with or around the pyrite or shale deposits, or other heavy material which is removed by coal dressing processes such as washing. The values for organic sulphur in the washed coal agree quite closely with the organic sulphur content of the corresponding raw coal. There is no marked evidence of noticeably higher values for this sulphur form in the washed products than in the raw coals, because none of the three coals listed in the tables ran abnormally high in non-coal impurities, the removal of which would cause an appreciable concentration of organic sulphur in the washed coal. Because of the presence of large amounts of such impurities in the raw coal, washed coal sometimes contains a higher percentage of total sulphur than the original raw coal.

In estimating the washability of a coal the organic sulphur content is an important consideration. The chief value in experimental coal washing of the determination of this constituent lies in finding a value below which there can be no reduction of sulphur content by mechanical processes of separation. For example, if the coal from a given mine averages two per cent total sulphur and one per cent organic sulphur, it would of course be impossible to secure a washed product carrying less than one per cent sulphur. Although this is a self evident fact, it is of such importance in determining the washability of a coal that attention is directed to it. Furthermore, in all coals a part of the pyritic sulphur will remain in the washed coal, the amount depending, as has already been pointed out, upon the physical form in which the pyrite occurs in that particular coal. Thus in every case the minimum sulphur content that may be obtained in the clean coal is well above the organic sulphur content of the raw coal. All the pyrite occurring in the form of microscopic particles, and a large part of that included under the classification "fine dis-

TABLE 10
PYRITIC AND ORGANIC SULPHUR IN VARIOUS COALS
Values given in table in per cent, on a moisture-free basis

Location of Mine	Coal Bed	Total Sulphur	Pyritic Sulphur	Organic Sulphur	Organic Sulphur as Per Cent of Total Sulphur
*Clearfield Co., Pa.....	C & D	3.48	2.77	0.71	20.4
White Co., Tenn.....	Sewanee	4.87	3.59	1.17	24.0
Pike Co., Ky.....	Freeburn	0.46	0.13	0.33	72.0
Williamson Co., Ill.....	No. 6	1.83	1.04	0.79	43.2
Greene Co., Ind.....	No. 4	1.66	0.89	0.77	46.4
Franklin Co., Ill.....	No. 6	3.29	1.99	1.30	39.5
Webster Co., Ky.....	No. 12	1.48	0.70	0.78	52.6
Union Co., Ky.....	No. 9	3.46	1.65	1.81	52.5
†McDowell Co., W. Va.....	Pocahontas No. 3	0.55	0.08	0.46	83.7
†Letcher Co., Ky.....	Elkhorn	0.68	0.13	0.51	75.0

*H. F. Yancey and Thomas Fraser, Coal Industry 3, (1919) 36.

†A. R. Powell, J. Ind. Eng. Chem., 12, (1920) 889.

seminated pyrite," is not removable, and for practical purposes in connection with coal washing may be considered as fixed sulphur.

16. *Organic Sulphur in Various Coals.*—The relatively high proportion of sulphur occurring in the organic form in many coals has not been generally recognized. Table 10 shows the total, pyritic, and organic sulphur values for a number of well known coals representing both Eastern and Central District fields. For each coal the percentage of the total sulphur in organic combination is tabulated. It will be noted that the percentage of the total sulphur present as organic sulphur varies from 20.4 per cent in a coal from Clearfield County, Pennsylvania, to 83.7 per cent in a coal from McDowell County, West Virginia.

17. *Microscopic Pyrite.*—At the Middlefork mine six samples of coal were taken for the examination of the microscopic pyritic sulphur.

The thin cross-section and photographs of these samples were made at the Pittsburgh Station of the Bureau of Mines by Dr. Reinhardt Thiessen, Microscopist of the Bureau of Mines, who has made a study of the microscopic forms of sulphur in various Eastern and Central District coals.* All of the coals that he examined were found to contain varying amounts of sulphur in the form of very small globules or particles of pyrite. "These particles are seen in thin section as roughly rounded opaque dots. When isolated, they are generally shown to be roughly spherical in shape with a rough outer surface. They vary in diameter from a few microns to a hundred microns†, the majority measuring from 25 to 40 microns, relatively few exceed the latter diameter; they are therefore very small objects. . . . The amount of pyrite occurring in this form varies considerably in different beds from which coals have been examined, and also in different samples from the same bed, or even in different parts of the same section. A section without these pyrite particles is rarely obtained. So far, no regularity has been discovered." Photographs of the thin sections made from the Middlefork coal samples are shown in Figs. 18 to 24. The photographs are all magnified approximately 200 times. The microscopic pyrite is represented by black areas, more or less circular in shape, with a

* Am. Inst. Min. Eng., Bul. 153, 2431, 1919.

† A micron is the thousandth part of a millimeter or one twenty-five thousandth of an inch.

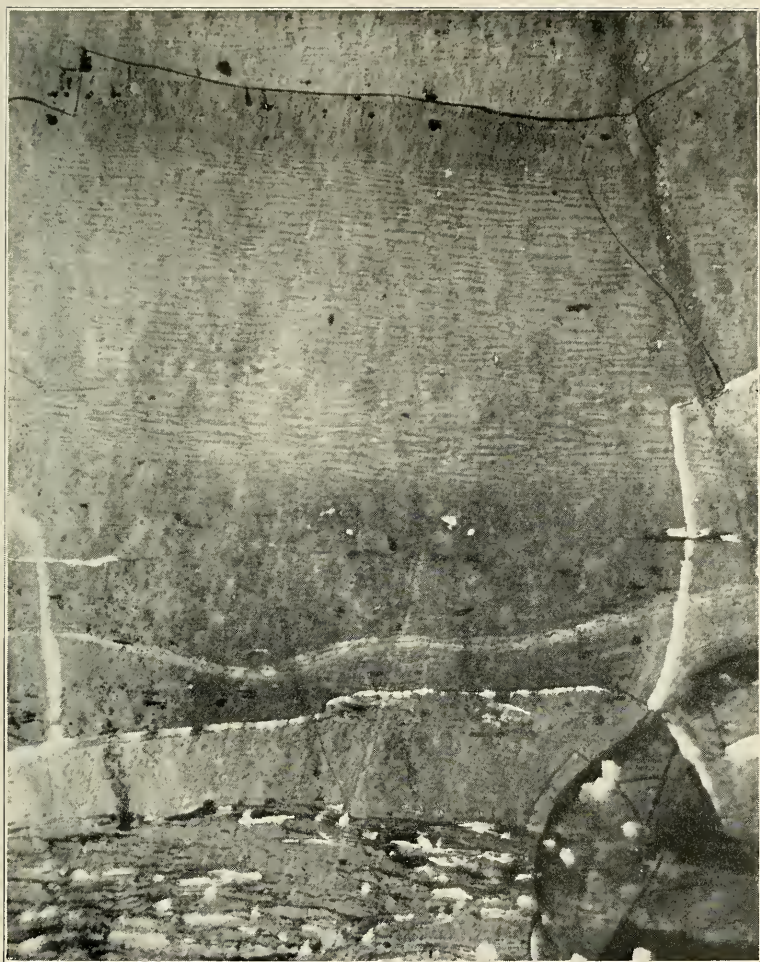


FIG. 18. MICROPHOTOGRAPHS OF SAMPLES OF ILLINOIS NO. 6 COAL,
SHOWING MICROSCOPIC PYRITE



FIG. 19. MICROPHOTOGRAPHS OF SAMPLES OF ILLINOIS No. 6 COAL,
SHOWING MICROSCOPIC PYRITE



FIG. 20. MICROPHOTOGRAPHS OF SAMPLES OF ILLINOIS NO. 6 COAL,
SHOWING MICROSCOPIC PYRITE

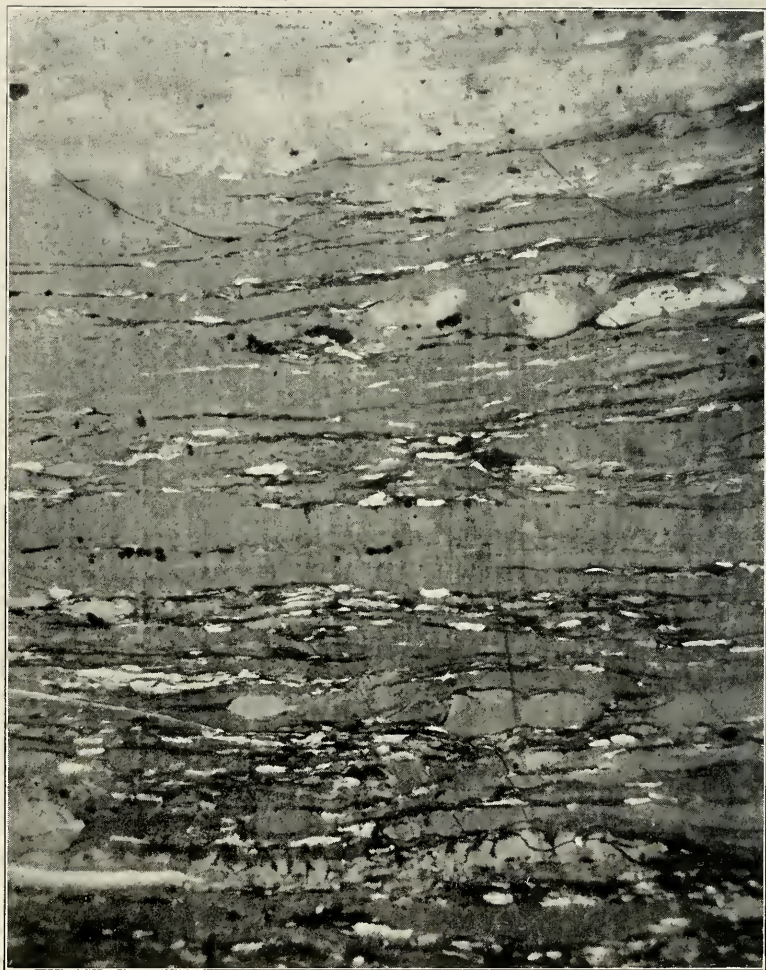


FIG. 21. MICROPHOTOGRAPHS OF SAMPLES OF ILLINOIS NO. 6 COAL,
SHOWING MICROSCOPIC PYRITE



FIG. 22. MICROPHOTOGRAPHS OF SAMPLES OF ILLINOIS No. 6 COAL,
SHOWING MICROSCOPIC PYRITE

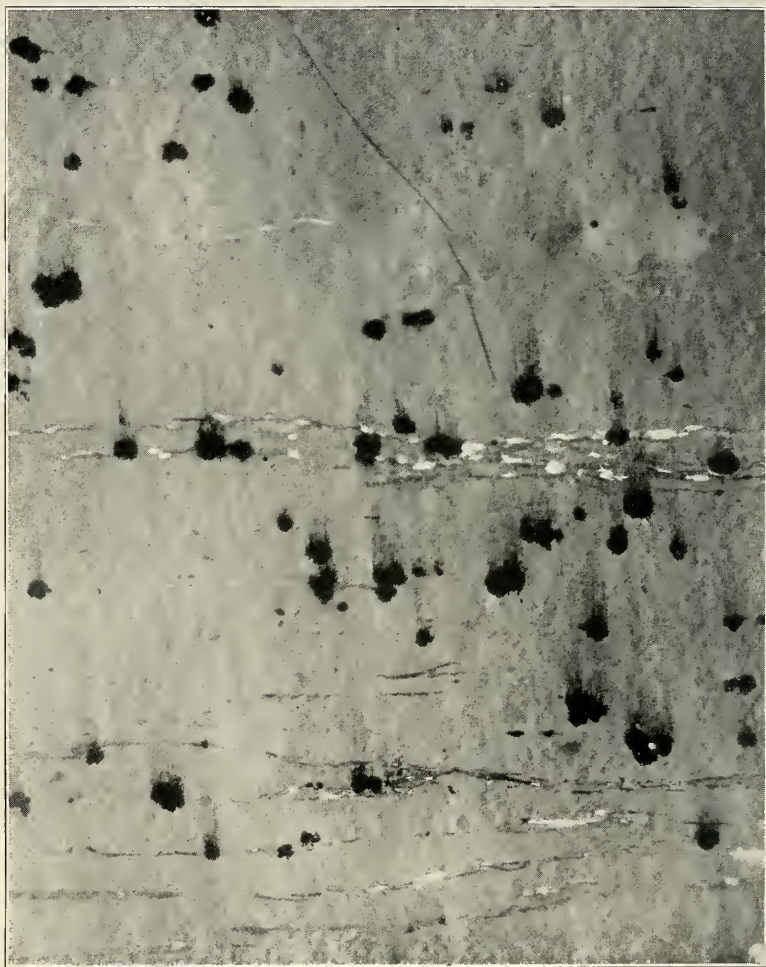


FIG. 23. MICROPHOTOGRAPHS OF SAMPLES OF ILLINOIS NO. 6 COAL,
SHOWING MICROSCOPIC PYRITE

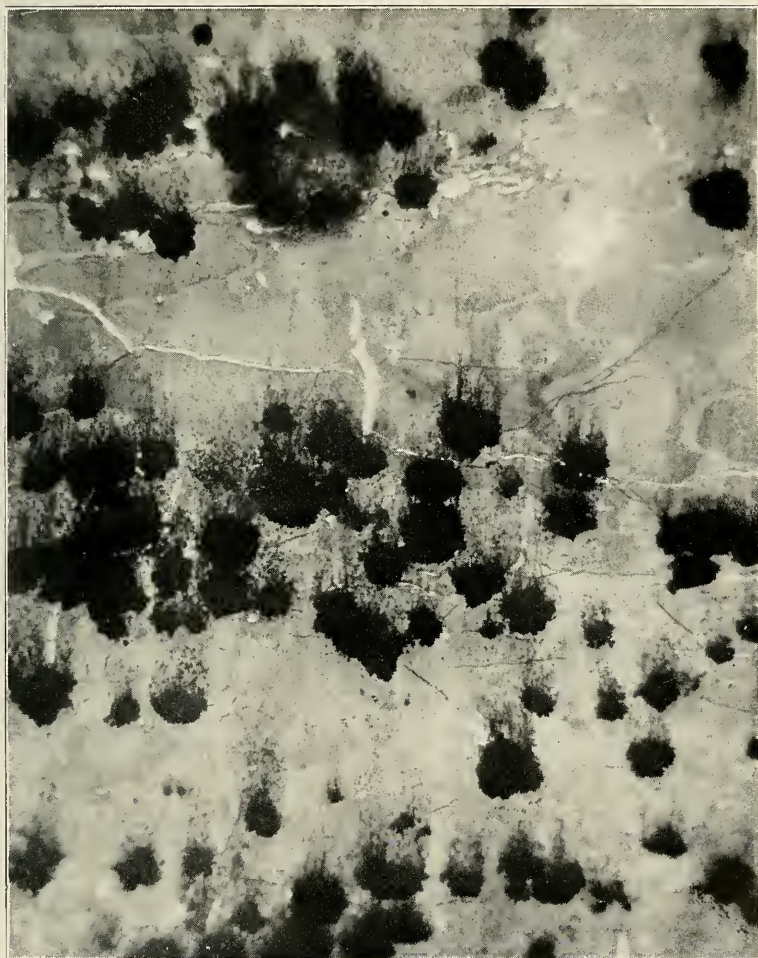


FIG. 24. MICROPHOTOGRAPHS OF SAMPLES OF ILLINOIS NO. 6 COAL,
SHOWING MICROSCOPIC PYRITE

streak on one or more sides. The streak was made during the preparation of the thin cross-section and, of course, is not originally present in the coal. The photographs are cross-sections, that is, they are perpendicular to the bedding.

The pyrite globules vary in diameter from a few microns to about 60 microns. The average diameter is 20 microns. Since the opening in a 100-mesh standard screen is 147 microns it will be seen that a 100-mesh grain of coal might contain a considerable number of such globules.

Samples of the top and middle benches of the coal were taken for this work. Five photographs were made of sections of the top coal. Two of these photographs are included here, Figs. 18 and 19. The top coal showed less microscopic pyrite than any of the samples of middle coal, though in the great majority of cases, it is higher in total and pyritic sulphur than the middle coal. Either the samples of top coal were not representative of this condition, or a smaller amount of the pyritic sulphur of the top coal occurs in the microscopic form.

The middle coal shown in Figs. 20 and 21 was taken at the same location as the top coal. The face at this location was not visibly high in sulphur. Cat-faces and bands and veinlets of pyrite were not unusually numerous. However, samples of middle coal shown in Figs. 22, 23, and 24 were taken at a location which was visibly somewhat higher in pyrite, and the photographs show this condition with respect to microscopic pyrite. The average for this location is represented in Fig. 23.

V. SUMMARY OF CONCLUSIONS

A study of the data given in this bulletin suggests the following summary of conclusions.

18. *Distribution of Pyritic Sulphur.*—Extreme irregularity of distribution is characteristic of the pyritic sulphur of coal in the bed.

1. The irregularities in the distribution of total sulphur are due mainly to the concentration of pyritic sulphur in coarse bands or lenses.

2. Fine disseminated pyrite is much more uniformly distributed than is coarse pyrite.

3. Microscopic pyrite was found in all the samples examined. The amount of pyritic sulphur in this form cannot be accurately determined because it varies widely even in the same thin section.

19. *Distribution of Organic Sulphur.*—As compared with the very great changes in pyritic sulphur content the vertical distribution of organic sulphur at a given location in the same bed is quite uniform. The results of this investigation, however, would indicate that in high sulphur coals the uniformity of organic sulphur distribution is confined to very limited areas in the bed, and that taking even a single mine as a whole the variations in organic sulphur content may be considerable, although it is much more uniformly distributed than is the pyritic sulphur. In low sulphur coals the variations in the distribution of organic sulphur are less and the values obtained for organic sulphur approach uniformity, even for considerable areas.

20. *Relation of Pyritic and Organic Sulphur.*—There is little evidence of a definite relationship in the occurrence of organic and pyritic sulphur. High pyritic sulphur in a bench or section of a bed is not indicative of a correspondingly high organic sulphur content.

The proportion of the total sulphur that is in organic combination in various coals (Table 10) varies within wide limits. In low sulphur coals organic sulphur makes up a greater percentage of the total sulphur content than it does in high sulphur coals.

Organic sulphur makes up a greater proportion of the total sulphur content of coal than is generally recognized. In the 104 face

samples taken in three coal beds, organic sulphur exceeded pyritic in 49 samples.

21. *Practical Applications.*—In estimating the washability of a coal the organic sulphur content is an important consideration. The chief value in experimental coal washing is the determination of this constituent lies in finding a value below which there can be no reduction of sulphur content by mechanical processes of separation, since the organic sulphur content of coal is not reduced by such methods. Furthermore, in all coals a part of the pyritic sulphur will remain in the washed coal, the amount depending upon the physical form in which the pyrite occurs in that particular coal. Thus in every case the minimum sulphur content that may be obtained in the clean coal is well above the organic sulphur content. All of the pyritic sulphur occurring in the form of microscopic pyrite, and a large part of that included under the classification of "fine disseminated pyrite," is not removable, and for practical purposes in connection with coal washing may be considered as "fixed" sulphur.

The organic sulphur content of some coals is sufficiently high to limit seriously the extent to which these coals can be cleaned of sulphur by washing.

In some coal beds partial concentration of the pyritic sulphur in the form of thick bands or lenses at certain definite horizons makes it possible to clean the coal by selective mining or hand-picking.

APPENDIX I

TABLES SHOWING FORMS OF SULPHUR IN SECTIONAL SAMPLES

United States Fuel Co., Middlefork Mine, Illinois, No. 6 Bed.—The following Tables A to L give the figures for total sulphur, organic sulphur, total pyritic sulphur, and fine disseminated pyritic sulphur in the sectional samples taken at the Middlefork mine near Benton, Illinois. These same data have been presented graphically in Fig. 1, Charts A, B, C, D, E, and F, and in Fig. 2, Charts G, H, I, J, K, and L, lettered to correspond with the designation of the tables. The locations at which the samples were taken in the mine are shown by corresponding letters on the map, Fig. 9. Table M gives the weighted average of the forms of sulphur for the bed at the various locations in the Middlefork mine. Below the analyses of each section is given, in parentheses, a statement of the visible impurities occurring in the section. These impurities were included in the sample except where otherwise specified.

TABLE A
FORMS OF SULPHUR IN SECTIONAL SAMPLES
Middlefork Mine—Illinois Bed No. 6
Second West North at Mouth of Tenth North Entry
Values given in table in per cent, on a moisture-free basis

Sample No.	Vertical Thickness Ft. In.	Total Sulphur	Pyritic Sulphur		Total Sulphur Float 1.40	Organic Sulphur	Ash	Remarks
			Total	Fine				
15	0 7	6.36	4.56	0.72	2.67	1.80	19.0	Section 7 in. by 1 ft. 9 in. on both sides and along lens of pyrite. Lens varies from 0 to 1½ in. thick; most of pyrite itself cleaned of coal and discarded; 4 in. of top coal not sampled. Two ½-in. pyrite bands, one small cat-face. One ½-in. band and one ¼-in. band of pyrite. One ½-in. band laminated pyrite 1 ft. 3 in. up from blue band. Three knife-edge sulphur bands.
14	1 7.5	3.15	1.15	0.55	2.60	2.00	7.6	
13	1 10	3.16	1.36	0.49	2.36	1.80	10.8	
12	1 6.5	3.97	2.21	0.62	2.64	1.76	17.9	
11	1 1	4.90	3.47	1.18	3.03	1.43	15.4	
Average		3.88	2.12	0.67	2.62	1.76	13.3	

Note: Location of section shown on mine map at (A); data shown graphically in Fig. 6A. Total thickness of bed, 6 ft. 8 in.

TABLE B
FORMS OF SULPHUR IN SECTIONAL SAMPLES
Middlefork Mine—Illinois Bed No. 6
Room 11, off the Seventh North First West North Entry
Values given in table in per cent, on a moisture-free basis

Sample No.	Vertical Thickness Ft. In.		Total Sulphur	Pyritic Sulphur	Organic Sulphur	Ash	Remarks
10	1	4	3.63	1.53	2.10	8.3	1/4-in. band pyrite.
9	1	6	4.08	2.02	2.06	8.4	Two 1/8-in. bands pyrite; 1 veinlet 1/8-in. by 2 in.
8	1	3	3.93	2.11	1.82	10.2	Two 1/8-in. pyrite bands; cat-face 4 sq. in.
7	1	10	2.51	0.68	1.83	14.3	Knife-edge veins of pyrite.
6		10.5	5.52	4.00	1.52	15.3	
Average			3.72	1.82	1.90	11.1	

Note: Location of section shown on mine map at (B); data shown graphically in Fig. 6B. Total thickness of bed, 6 ft. 9 1/2 in.

TABLE C
FORMS OF SULPHUR IN SECTIONAL SAMPLES
Middlefork Mine—Illinois Bed No. 6
Face of Fourth North, off First West North Entry
Values given in table in per cent, on a moisture-free basis

Sample No.	Vertical Thickness Ft. In.	Total Sulphur	Pyritic Sulphur		Total Sulphur Float 1.40	Organic Sulphur	Ash	Remarks
			Total	Fine				
5	1 2	5.47	3.82	1.45	3.42	1.65	13.3	1/8-in. pyrite band.
4	1 9	4.60	2.92	1.19	2.99	1.68	9.5	Pyrite cat-face 4 sq. in. and 1/8-in. band pyrite.
3	1 11	3.70	1.97	0.59	2.44	1.73	11.2	Diagonal pyrite veinlet 1/8-in. by 4 in.
2	1 5	4.32	2.69	0.91	2.87	1.63	17.2	Two 1/4-in. bands mother-coal.
1	10.5	7.55	6.23	1.26	3.17	1.32	18.0	
Average		4.82	3.17	1.14	2.91	1.65	13.2	

Note: Location of section shown on mine map at (C); data shown graphically in Fig. 6C. Total thickness of bed, 7 ft. 1 1/2 in.

TABLE D
 FORMS OF SULPHUR IN SECTIONAL SAMPLES
 Middlefork Mine—Illinois Bed No. 6
 Face of Main North Back Entry

Values given in table in per cent, on a moisture-free basis

Sample No.	Vertical Thickness Ft. In.	Total Sulphur	Pyritic Sulphur	Organic Sulphur	Ash	Remarks
47	7	6.16	4.82	1.34	10.0	
46	8.5	2.39	1.09	1.30	8.3	1/8-in. mother-coal.
45	1 0	4.14	2.75	1.39	8.9	1/4-in. pyrite cat-face, two 1/8-in. bands mother-coal.
44	1 0	2.28	0.82	1.46	6.5	Two 1/2-in. bands mother-coal.
43	1 0	6.26	4.96	1.30	13.8	One 3/4-in. band cellular pyrite.
42	1 0	7.01	5.54	1.47	19.1	5/8-in. lens of pyrite.
41	1 0	2.26	0.81	1.45	13.8	1/2-in. bone-coal.
40	1 0	3.64	2.35	1.29	10.8	1/8-in. pyrite.
Average		4.24	2.88	1.36	11.6	

Note: Location of section shown on mine map at (D); data shown graphically in Fig. 6D. Total thickness of bed, 6 ft. 9 in.

TABLE E
FORMS OF SULPHUR IN SECTIONAL SAMPLES
Middlefork Mine—Illinois Bed No. 6
Thirty Feet Out by Face of Third North, off First East North Entry
 Values given in table in per cent, on a moisture-free basis

Sample No.	Vertical Thickness Ft. In.	Total Sulphur	Pyritic Sulphur	Organic Sulphur	Ash	Remarks
55	10	4.17	2.98	1.19	10.7	
54	11	4.20	3.04	1.16	10.1	
53	1 0	3.87	2.59	1.28	10.2	Vertical pyrite streak $\frac{1}{4}$ -in. by 3 in.
52	1 0	6.90	5.76	1.14	14.0	Vertical vein pyrite $\frac{1}{4}$ -in. by 5 in.; 2 cat-faces.
51	1 0	2.45	0.95	1.50	8.6	$\frac{1}{4}$ -in. mother-coal; small cat-face.
50	1 0	3.20	2.00	1.20	14.8	$\frac{3}{8}$ -in. band pyrite.
49	10.5	4.60	3.20	1.40	13.6	$\frac{1}{2}$ -in. hard pyrite.
48	9.5	3.09	1.87	1.22	9.7	
Average		4.08	2.95	1.27	11.5	

Note: Location of section shown on the mine map at (E); data shown graphically in Fig. 7E. Total thickness of bed, 7 ft. 3 in. Top coal not included.

TABLE F
 FORMS OF SULPHUR IN SECTIONAL SAMPLES
 Middlefork Mine—Illinois Bed No. 6
 Face of Tenth North, off First East South Entry
 Values given in table in per cent, on a moisture-free basis

Sample No.	Vertical Thickness Ft. In.	Total Sulphur	Pyritic Sulphur	Organic Sulphur	Ash	Remarks
20	1 8	2.98	2.09	0.89	9.0	One $\frac{1}{4}$ -in. band black shale. Two $\frac{1}{8}$ -in. bands and one $\frac{1}{4}$ -in. band of mother-coal.
19	1 11	1.25	0.52	0.73	7.0	
18	2 0	0.85	0.24	0.61	7.9	
17	1 5	0.62	0.06	0.56	10.1	
16	1 5	1.00	0.35	0.65	7.9	
Average		1.35	0.66	0.69	8.3	

Note: Location of section shown on the mine map at (F); data shown graphically in Fig. 7F. Total thickness of bed, 8 ft. 2 in.

TABLE G
FORMS OF SULPHUR IN SECTIONAL SAMPLES
Middlefork Mine—Illinois Bed No. 6
Fifth South, off First East South Entry

Values given in table in per cent, on a moisture-free basis

Sample No.	Vertical Thickness Ft. In.	Total Sulphur	Pyritic Sulphur		Total Sulphur Float 1.40	Organic Sulphur	Ash	Remarks
			Total	Fine				
25	1 0	3.13	2.2	0.84	1.81	0.91	7.7	Three vertical veinlets of pyrite.
24	1 8.5	1.40	0.60	0.59	1.44	0.80	8.0	One 1/4-in. shale band.
23	1 2	3.09	2.34	0.70	1.50	0.75	6.5	One small cat-face.
22	2 9	1.51	0.67	0.48	1.41	0.84	11.1	One 1/2-in. band and one 3/4-in. band of mother-coal.
21	9.5	2.59	1.50	0.75	1.95	1.09	10.5	
Average		2.07	1.22	0.62	1.54	0.85	9.2	

Note: Location of section shown on mine map at (G); data shown graphically in Fig. 7G. Total thickness of bed, 7 ft. 5 in.

TABLE H
FORMS OF SULPHUR IN SECTIONAL SAMPLES
Middlefork Mine—Illinois Bed No. 6
Last Crosscut Between Third and Fourth East, off Main South Entry
Values given in table in per cent, on a moisture-free basis

Sample No.	Vertical Thickness Ft. In.	Total Sulphur	Pyritic Sulphur	Organic Sulphur	Ash	Remarks
70	1 3	3.84	1.94	1.90	8.2	
69	2 1	3.28	1.41	1.87	9.5	$\frac{1}{2}$ -in. mother-coal; $\frac{1}{8}$ -in. bone-coal; small cat-face; $\frac{1}{4}$ -in. mother-coal.
68	1 2	7.76	5.70	2.06	16.5	$\frac{3}{4}$ -in. band hard pyrite; $1\frac{1}{2}$ -in. cellular pyrite; one cat-face.
67	1 5	2.62	0.75	1.87	12.3	
66	1 0	3.33	1.78	1.55	12.0	Blue band pyritized.
Average		4.01	2.15	1.86	11.4	

Note: Location of section shown on the mine map at (H); data shown graphically in Fig. 7H. Total thickness of bed, 6 ft. 11 in.

TABLE I
FORMS OF SULPHUR IN SECTIONAL SAMPLES
Middlefork Mine—Illinois Bed No. 6
Last Crosscut Between Main South and Back West
Values given in table in per cent, on a moisture-free basis

Sample No.	Vertical Thickness Ft. In.	Total Sulphur	Pyritic Sulphur	Organic Sulphur	Ash	Remarks
65	1 0	2.67	0.73	1.94	8.6	Two $\frac{1}{8}$ -in. streaks bone-coal.
64	1 10.5	2.75	0.70	2.05	5.7	One $\frac{1}{4}$ -in. band and 2 cat-faces 1 in. by 2 in., $\frac{1}{2}$ -in. mother-coal.
63	5.5	4.93	3.19	1.74	11.7	Coal taken 5 $\frac{1}{2}$ in. by 26 in. along lens of pyrite.
62	1 2	2.28	0.53	1.75	12.3	Two $\frac{1}{8}$ -in. bands bone-coal and $\frac{1}{8}$ -in. band mother-coal.
61	1 4	3.51	1.66	1.85	14.5	One $\frac{1}{2}$ -in. band and two $\frac{1}{4}$ -in. bands bone-coal.
60	1 1	4.85	3.24	1.61	12.5	$\frac{1}{8}$ -in. band sulphur.
Average		3.28	1.43	1.85	10.4	

Note: Location of section shown on mine map at (1); data shown graphically in Fig. 81. Total thickness of bed, 7 ft. 1 in. Blue band is all pyrite here.

TABLE J
 FORMS OF SULPHUR IN SECTIONAL SAMPLES
 Middlefork Mine—Illinois Bed No. 6
 Face of Fifth South, off First West South

Values given in table in per cent, on a moisture-free basis

Sample No.	Vertical Thickness Ft. In.	Total Sulphur	Pyritic Sulphur		Total Sulphur Float 1.40	Organic Sulphur	Ash	Remarks
			Total	Fine				
39	11	5.13	3.58	0.97	2.88	1.55	10.1	
38	3 9	3.72	1.94	0.55	2.42	1.78	8.2	Two $\frac{1}{8}$ -in. bands and one $\frac{1}{2}$ -in. band pyrite; two $\frac{1}{4}$ -in. bands bone-coal.
37	1 5	3.05	1.36	0.57	2.46	1.69	15.6	$\frac{1}{8}$ -in. bone-coal and $\frac{1}{8}$ -in. pyrite.
36	1 0	7.15	5.68	0.80	2.68	1.47	20.2	$\frac{1}{2}$ -in. band pyrite.
Average		4.26	2.57	0.65	2.53	1.69	11.6	

Note: Location of section shown on mine map at (J); data shown graphically in Fig. 8J. Total thickness of bed, 7 ft. 1 in.

TABLE K
FORMS OF SULPHUR IN SECTIONAL SAMPLES
Middlefork Mine—Illinois Bed No. 6
Last Crosscut Between Seventh and Eighth South, off First West South Entry
Values given in table in per cent, on a moisture-free basis

Sample No.	Vertical Thickness Ft. In.	Total Sulphur	Pyritic Sulphur	Organic Sulphur	Ash	Remarks
30	7.5	2.29	1.40	0.89	6.5	One ½-in. band slate, pyrite lens 1 in. by 4 in. Two ¼-in. bands bone-coal and one ⅛-in. band mother-coal. 1 in. bone-coal, one ⅛-in. band pyrite.
29	8	3.85	2.97	0.88	10.1	
28	4 0	1.94	1.07	0.87	8.5	
27	11	2.66	1.66	1.00	10.2	
26	1 0	1.74	0.77	0.97	8.6	
Average		2.20	1.30	0.90	8.6	

Note: Location of section shown on mine map at (K); data shown graphically in Fig. 8K. Total thickness of bed, 7 ft. 2½ in.

TABLE L
 FORMS OF SULPHUR IN SECTIONAL SAMPLES
 Middlefork Mine—Illinois Bed No. 6
 Tenth North, off First West South Entry

Values given in table in per cent, on a moisture-free basis

Sample No.	Vertical Thickness Ft. In.	Total Sulphur	Pyritic Sulphur	Organic Sulphur	Ash	Remarks
35	0 5	4.28	3.03	1.25	7.5	
34	0 6	2.48	1.59	0.89	6.4	
33	5 3	1.57	0.75	0.82	8.0	1½ in. bone-coal and one ¼-in. band mother-coal.
32	0 6	1.41	0.80	0.61	11.0	
31	0 7	1.14	0.57	0.57	9.8	31 and 32 below the blue band.
Average		1.75	0.93	0.82	8.2	

Note: Location of section shown on mine map at (L); data shown graphically in Fig. 8L. Total thickness of bed, 7 ft. 3 in.

TABLE M
FORMS OF SULPHUR IN SECTIONAL SAMPLES
Middlefork Mine—Illinois Bed No. 6
Average of Bed for Various Locations

Values given in table in per cent, on a moisture-free basis

Section	Location	Total Sulphur	Pyritic Sulphur	Organic Sulphur	Ash
1	10 N., 1 W. N.	3.88	2.12	1.76	13.3
2	7 N., 1 W. N.	3.72	1.82	1.90	11.1
3	4 N., 1 W. N.	4.82	3.17	1.65	13.2
4	Main North	4.24	2.88	1.36	11.6
5	3 N., 1 E. N.	4.08	2.95	1.27	11.5
Average for	North Side.....	4.15	2.56	1.59	12.1
6	10 N., 1 E. S.	1.35	0.66	0.69	8.3
7	5 S., 1 E. S.	2.07	1.22	0.85	9.2
8	3 E. S.	4.01	2.15	1.86	11.4
9	Main South	3.28	1.43	1.85	10.4
10	5 S., 1 W. S.	4.26	2.57	1.69	11.6
11	7 S., 1 W. S.	2.20	1.30	0.90	8.6
12	10 N., 1 W. S.	1.75	0.93	0.82	8.2
Average for	South Side.....	2.70	1.46	1.24	9.7
Average for	Mine	3.30	1.92	1.38	10.7

West Kentucky Coal Co. No. 7 Mine, Kentucky No. 12 Bed.—Tables N to S inclusive give the figures for the forms of sulphur in the sectional samples taken at the No. 7 mine of the West Kentucky Coal Company. Corresponding letters are used to designate the graphs in the text, Fig. 11, N, O, P, Q, R, and S, which present the same data and designate on the mine map, Fig. 12, locations at which the samples were taken. Table V gives the weighted average of the forms of sulphur for the bed at the various locations in the No. 7 mine.

West Kentucky No. 8 Mine, Kentucky No. 9 Coal.—Tables T and U give the forms of sulphur in the sectional samples taken in the No. 8 mine of the West Kentucky Coal Co. Fig. 13, T and U, show the same data in graphical form.

TABLE N
 FORMS OF SULPHUR IN SECTIONAL SAMPLES
 West Kentucky Coal Co., Mine No. 7—Kentucky Bed No. 12
 First South, off Third Left Entry, off East Dip
 Values given in table in per cent, on a moisture-free basis

Sample No.	Vertical Thickness Ft. In.	Total Sulphur	Pyritic Sulphur	Organic Sulphur	Ash	Remarks
18	1 0	3.86	2.97	0.89	8.54	One ¼-in. band pyrite, 2-in. band pyrite adjacent, but not included.
17	1 5	1.47	0.64	0.83	6.19	One ¼-in. band bone-coal.
16	1 5	0.89	0.15	0.74	8.96	One ¼-in. band mother-coal, one ½-in. band bone-coal.
15	1 4	0.90	0.11	0.79	6.77	
14	1 4	1.04	0.23	0.81	10.57	One 2½-in. band mother-coal and shale.
Average		1.51	0.70	0.81	8.20	

Note: Total thickness of bed, 6 ft. 6 in. Top coal not sampled.

TABLE O
FORMS OF SULPHUR IN SECTIONAL SAMPLES
West Kentucky Coal Co., Mine No. 7—Kentucky Bed No. 12
Fourth Left, off East Dip Entry

Values given in table in per cent, on a moisture-free basis

Sample No.	Vertical Thickness Ft. In.	Total Sulphur	Pyritic Sulphur	Organic Sulphur	Ash	Remarks
13	1 5	0.69	0.05	0.64	5.32	One 1-in. band mother-coal.
12	1 5	0.73	0.10	0.63	7.76	One ½-in. band mother-coal
11	1 5	0.94	0.35	0.59	22.59	1 in. mother-coal, one ½-in. band and one 1½-in. band soft shale; these are continuous shale bands of varying thickness.
10	1 5	0.95	0.23	0.72	5.58	One 1-in. band bone-coal at top of section.
9	1 4	1.59	0.77	0.82	8.50	One ¼-in. band sulphur.
Average		0.98	0.30	0.68	9.95	

Note: Total thickness of seam 7 ft. 0 in. Two feet of top coal which is commonly left up was not sampled.

TABLE P
FORMS OF SULPHUR IN SECTIONAL SAMPLES
West Kentucky Coal Co., Mine No. 7—Kentucky Bed No. 12
Fifth Right Entry, off East Dip

Values given in table in per cent, on a moisture-free basis

Sample No.	Vertical Thickness Ft. In.	Total Sulphur	Pyritic Sulphur		Total Sulphur Float 1.40	Organic Sulphur	Ash	Remarks
			Total	Fine				
23	2 6	2.14	1.28	0.74	1.69	0.86	7.24	Sample of top coal ordinarily left up for a roof.
22	1 6	1.17	0.49	0.41	1.02	0.61	8.63	Car-face of sulphur 4 in. to one side of sample cut not included.
21	1 0	0.70	0.06	0.07	0.75	0.64	14.67	One $\frac{1}{8}$ -in. band, one $\frac{1}{4}$ -in. band, and one $\frac{3}{4}$ -in. band mother-coal and shale.
20	0 9.5	0.97	0.34	0.27	1.14	0.63	28.01	One 2-in. band bone-coal, one $\frac{3}{4}$ -in. band and one $\frac{1}{2}$ -in. band shale.
19	2 2	1.15	0.40	0.26	1.03	0.75	7.24	
Average		1.39	0.65	0.42	1.21	0.74	10.4	

Note: Total thickness 7 ft. 11½ in. Top coal sampled.

TABLE Q
FORMS OF SULPHUR IN SECTIONAL SAMPLES
West Kentucky Coal Co., Mine No. 7—Kentucky Bed No. 12
Cross-cut Between Rooms 8 and 9, Fifth South, off Third Left, off Main Rise
Values given in table in per cent, on a moisture-free basis

Sample No.	Vertical Thickness Ft. In.	Total Sulphur	Pyritic Sulphur	Organic Sulphur	Ash	Remarks
38	0 11	4.83	3.86	0.97	25.87	Sample of the top coal which is ordinarily left as a roof.
37	1 5	1.27	0.55	0.72	6.41	
36	1 1	0.91	0.19	0.72	9.75	
35	1 7	2.33	1.52	0.81	13.12	Two $\frac{1}{8}$ -in. bands pyrite; pyrite lens in edge of sample cut excluded. One $\frac{3}{4}$ -in. band bone-coal and one $\frac{3}{4}$ -in. band sulphur.
34	1 6	2.65	1.81	0.84	9.98	
Average		2.29	1.49	0.80	12.1	

Note: Total thickness of bed, 6 ft. 6 in. Top coal sampled.

TABLE R
FORMS OF SULPHUR IN SECTIONAL SAMPLES
West Kentucky Coal Co., Mine No. 7—Kentucky Bed No. 12
Face of Fourth Right Entry, off Main Rise
Values given in table in per cent, on a moisture-free basis

Sample No.	Vertical Thickness Ft. In.	Total Sulphur	Pyritic Sulphur	Organic Sulphur	Ash	Remarks
28	1 0	1.86	0.96	0.90	7.85	
27	1 0	1.12	0.36	0.76	7.18	
26	1 0	1.00	0.18	0.82	5.80	One 1-in. band bone-coal; one $\frac{1}{4}$ -in. band shale.
25	1 0	1.02	0.16	0.86	7.07	One $\frac{3}{8}$ -in. sulphur band.
24	0 10	1.57	0.62	0.95	9.16	One $\frac{1}{4}$ -in. band of mother-coal.
Average		1.30	0.45	0.85	7.35	

Note: Total thickness of bed, 4 ft. 10 in. 2 ft. 0 in. of top coal not sampled.

TABLE S
 FORMS OF SULPHUR IN SECTIONAL SAMPLES
 West Kentucky Coal Co., Mine No. 7—Kentucky Bed No. 12
 Room 23 Top, off Main Rise Entry

Values given in table in per cent, on a moisture-free basis

Sample No.	Vertical Thickness Ft. In.	Total Sulphur	Pyritic Sulphur		Total Sulphur Float 1.40	Organic Sulphur	Ash	Remarks
			Total	Fine				
33	1 1	2.27	1.40	0.16	1.04	0.87	7.10	
32	1 2	1.28	0.48	0.35	1.17	0.80	7.16	
31	1 1	1.07	0.26	0.19	1.04	0.81	7.82	One $\frac{1}{4}$ -in. band mother-coal and one $\frac{1}{4}$ -in. band bone-coal.
30	1 2	1.01	0.24	0.23	1.00	0.76	4.32	One $\frac{3}{4}$ -in. band mother-coal and one $\frac{1}{4}$ -in. band bone-coal.
29	1 5	1.52	0.63	0.37	1.39	0.89	7.13	
Average		1.43	0.60	0.27	1.14	0.83	6.71	

Note: Total thickness of bed, 5 ft. 11 in. One foot of top coal not sampled.

TABLE T
 FORMS OF SULPHUR IN SECTIONAL SAMPLES
 West Kentucky Coal Co., Mine No. 8—Kentucky Bed No. 9
 Face of Tenth East Entry, off the Main Dip

Values given in table in per cent, on a moisture-free basis

Sample No.	Vertical Thickness Ft. In.	Total Sulphur	Pyritic Sulphur		Total Sulphur Float 1.40	Organic Sulphur	Ash	Remarks
			Total	Fine				
4	1 3	2.46	0.72	0.41	2.18	1.74	11.59	One $\frac{1}{2}$ -in. and one $\frac{1}{4}$ -in. band of sulphur.
3	1 3	3.02	0.98	0.39	2.45	2.04	6.20	Two $\frac{1}{8}$ -in. bands, and one $\frac{1}{4}$ -in. band of mother-coal and one $\frac{3}{8}$ -in. band sulphur, one $\frac{1}{2}$ -in. band sulphur discarded.
2	1 3	3.45	1.73	0.94	2.74	1.72	11.77	One $\frac{1}{8}$ -in. band sulphur.
1	1 3	5.13	3.59	1.76	3.70	1.54	15.93	Two $\frac{1}{8}$ -in. bands pyrite.
Average		3.51	1.75	0.88	2.77	1.76	11.37	

Note: Total thickness of bed, 5 ft. 0 in.

TABLE U
 FORMS OF SULPHUR IN SECTIONAL SAMPLES
 West Kentucky Coal Co., Mine No. 8—Kentucky Bed No. 9
 Face of Tenth West Blind Entry, off the Main Dip
 Values given in table in per cent, on a moisture-free basis

Sample No.	Vertical Thickness Ft. In.	Total Sulphur	Pyritic Sulphur		Total Sulphur Float 1.40	Organic Sulphur	Ash	Remarks
			Total	Fine				
8	1 3	2.33	0.50	0.25	2.19	1.83	8.90	
7	1 3	4.11	2.15	0.73	2.74	1.96	8.72	Two $\frac{1}{4}$ -in. bands sulphur and one $\frac{1}{8}$ -in. band mother-coal included.
6	1 3	2.83	0.86	0.45	2.83	1.97	8.03	
5	1 5	4.28	2.57	0.82	2.64	1.71	13.05	One $\frac{1}{4}$ -in. band sulphur and one $\frac{1}{4}$ -in. band mother-coal included.
Average		3.42	1.55	0.57	2.61	1.86	9.85	

Note: Total thickness of bed, 5 ft. 2 in.

TABLE V
FORMS OF SULPHUR IN SECTIONAL SAMPLES
West Kentucky Coal Co., Mines Nos. 7 and 8—Kentucky Beds Nos. 12 and 9
Average of Beds for Various Locations

Values given in table in per cent, on a moisture-free basis

Location	Total Sulphur	Pyritic Sulphur		Total Sulphur Float on 1.40	Organic Sulphur	Ash
		Total	Fine			
Mine No. 7:						
R.02: 4th L: E. Dip.....	0.98	0.30			0.68	9.95
R.2: 1st S: 3d L: E. Dip.....	1.51	0.70			0.81	8.20
R.20: 5th R: E. Dip.....	1.39	0.65	0.42	1.21	0.74	10.4
4th R. Main Rise.....	1.30	0.45			0.85	7.35
23 Top Rm. Main Rise.....	1.43	0.60	0.27	1.14	0.83	6.71
5th S: 3d L: Main Rise.....	2.29	1.49			0.80	12.1
Average.....	1.48	0.70	0.34	1.17	0.78	9.12
Mine No. 8:						
10th East Main Dip.....	3.51	1.75	0.88	2.77	1.76	11.37
10th West Main Dip.....	3.42	1.55	0.57	2.61	1.86	9.85
Average.....	3.46	1.65	0.73	2.69	1.81	10.61

APPENDIX II

SINK AND FLOAT TESTS

Sink and Float Tests on Bench Samples.—Tables 11, 12, and 13 give the results of sink and float tests on the samples taken at two places in the No. 7 mine, two places in the No. 8 mine, and four places in the Middlefork mine. A zinc chloride solution of a specific gravity of 1.40 was used and the samples were crushed to $\frac{1}{4}$ -in. maximum size.

This test is often used as a preliminary washing test. The proportion of float coal and its ash and sulphur content give a general indication of what may be recovered as washed coal. The sink and float test gives a more perfect gravity separation than the commercial coal washing methods. For that reason washing can not be expected to result in both as high recovery of washed coal and as great a reduction in impurities as the sink and float test. It approaches these results more or less closely, depending upon the efficiency and suitability of the process used.

A solution of 1.35 specific gravity is more commonly used for this test, but this is varied to suit the characteristics of the coal. In this case a solution of 1.35 specific gravity gave an unwarrantably high proportion of sink. The specific gravity of two representative 100-gm. samples of this coal from the No. 7 mine, determined by a volumetric method, were 1.319 and 1.313. The specific gravity of a composite sample of the Middlefork coal including all samples on which sink and float tests were made was 1.325. A solution of 1.35 specific gravity was considered too close to this figure to be suitable for the sink and float test.

The float coal of sample 20 from the No. 7 mine was considerably higher in sulphur content than the raw coal; this is explained by the fact that the sample contained a large amount of ash in the form of coarse shale bands. The sink and float test therefore resulted in the removal of a large amount of heavy inorganic matter which was low in sulphur, leaving the sulphur concentrated in the float coal. The sink amounted to 46.6 per cent of the sample and carried only 0.66 per cent sulphur, while the sulphur content of raw coal was 0.97 per cent. Similar results are sometimes secured in coal washing practice for the same reason.

TABLE 11
SINK AND FLOAT TESTS ON SAMPLES FROM MINE No. 7, WEST KENTUCKY COAL CO.
At 0-in. to $\frac{1}{4}$ -in. size, on solution of 1.40 sp. gr.
Values given in table in per cent, on a moisture-free basis

No.	Weight Gms.	Original Sample				Float Coal		Location in Mine from Which Sample Was Taken
		Sulphur	Ash	Sink	Float	Sulphur	Ash	
23	270	2.14	7.24	10.4	89.6	1.69	6.95	Room 20, 5th R., East Dip.
22	333	1.17	8.68	11.9	88.1	1.02	6.36	
21	342	0.70	14.67	27.0	73.0	0.75	7.72	
20	333	0.97	28.01	46.6	53.4	1.14	8.44	
19	338	1.15	7.24	8.6	91.4	1.03	5.35	
Weighted average		1.39	10.40	15.8	84.2	1.21	7.4	
33	342	2.27	7.10	3.8	96.2	1.04	5.50	23 top room, Main Rise.
32	363	1.28	7.16	3.9	96.1	1.17	6.36	
31	349	1.07	7.82	15.9	83.1	1.04	5.30	
30	299	1.01	4.32	5.4	94.6	1.00	3.28	
29	325	1.52	7.13	6.8	93.2	1.39	5.26	
Weighted average		1.43	6.71	5.8	94.2	1.14	5.06	

TABLE 12
SINK AND FLOAT TESTS ON SAMPLES FROM MINE No. 8, WEST KENTUCKY COAL Co.
At 0-in. to $\frac{1}{8}$ -in. size, on solution of 1.40 sp. gr.
Values given in table in per cent, on a moisture-free basis

No.	Weight Gns.	Original Sample				Float Coal		Location in Mine from Which Sample Was Taken
		Sulphur	Ash	Sink	Float	Sulphur	Ash	
4		2.46	11.59	6.6	93.4	2.18	8.10	10 E., Main Dip.
3		3.02	6.20	4.0	96.0	2.45	4.00	
2		3.45	11.77	7.9	92.1	2.74	8.05	
1		5.13	15.93	18.4	81.6	3.70	12.00	
Weighted average		3.51	11.37	9.2	90.8	2.77	8.06	
8		2.33	8.90	3.0	97.0	2.19	8.49	10 W. Blind, Main Dip.
7		4.11	8.72	6.3	93.7	2.74	4.30	
6		2.83	8.03	3.8	96.2	2.83	5.15	
5		4.28	13.05	11.9	88.1	2.64	8.95	
Weighted average		3.42	9.85	6.4	93.6	2.61	6.78	

TABLE 13
SINK AND FLOAT TESTS ON SAMPLES FROM THE MIDDLEFORK MINE

At 0-in. to $\frac{1}{4}$ -in. size, on solution of 1.40 sp. gr.

Values given in table in per cent, on a moisture-free basis

No.	Weight Gms.	Original Sample				Float Coal		Location in Mine from Which Sample Was Taken
		Sulphur	Ash	Sink	Float	Sulphur	Ash	
5	468	5.47	13.3	18.0	82.0	3.42	5.13	Face of Fourth North off the First West North Entry.
4	524	4.60	9.5	9.2	90.8	2.99	4.09	
3	417	3.70	11.2	17.3	82.7	2.44	5.17	
2	516	4.32	17.2	26.8	73.2	2.87	10.11	
1	431	7.55	18.0	39.0	61.0	3.17	8.01	Face of First South off the First West South Entry.
39	541	5.13	10.1	25.5	74.5	2.88	5.40	
38	471	3.72	8.2	13.6	86.4	2.42	3.75	
37	445	3.05	15.6	34.8	65.2	2.46	8.08	
36	403	7.15	20.2	33.8	66.2	2.68	8.15	Tenth North, First West North Entry.
15	526	6.36	19.0	17.4	82.6	2.67	5.10	
14	453	3.15	7.6	8.8	91.2	2.60	3.25	
13	462	3.16	10.8	12.5	87.5	2.36	5.53	
12	491	3.97	17.9	30.6	69.4	2.64	10.07	Fifth South, First East South Entry.
11	532	4.90	15.4	26.0	74.0	3.03	7.78	
25	442	3.13	7.7	6.6	93.4	1.81	4.90	
24	455	1.40	8.0	7.9	92.1	1.44	3.15	
23	469	3.09	6.5	7.5	92.5	1.50	3.40	
22	468	1.51	11.1	16.0	84.0	1.41	6.46	
21	446	2.59	10.5	12.7	87.3	1.95	6.30	

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A STUDY OF THE EFFECT OF MOISTURE CONTENT UPON THE EXPANSION AND CONTRACTION OF PLAIN AND REINFORCED CONCRETE

BY
TORATA MATSUMOTO



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A STUDY OF THE EFFECT OF MOISTURE CONTENT UPON THE EXPANSION AND CONTRACTION OF PLAIN AND REINFORCED CONCRETE*

I. INTRODUCTION

1. *Object of Investigation.*—While the properties of concrete have been investigated for many years, attention has largely been given to considerations of strength alone. Studies of the less important properties, however, are also needed to explain phenomena that are observed in reinforced concrete structures, and to give information on questions relating to the appearance and durability of the material.

Aside from the action of direct load, deformations are produced in concrete by changes in temperature and in moisture content. With reference to temperature changes in reinforced concrete, it is well known that, regardless of differences in the mixture, concrete has practically the same coefficient of expansion as steel, so that the two materials contract or expand together. Moisture content, on the other hand, has the undesirable property of affecting concrete alone. Concrete, like wood, clay, and some other materials, expands when it absorbs moisture and contracts when it is dried; steel has no such action. After the concrete is poured the steel remains unchanged with changes in moisture conditions, while concrete ordinarily shrinks a considerable amount. Aside from the stresses set up in steel and concrete by the shrinkage of the latter, the resulting formation of cracks large or small will produce a condition which may be favorable to the corrosion of the steel or the disintegration of the concrete after repeated changes from dry to wet condition.

*This bulletin contains some of the results of experiments made by T. Matsumoto, a graduate student in Theoretical and Applied Mechanics at the University of Illinois, in 1918. Owing to his years of experience as an engineer on harbor works on the Island of Formosa, he was particularly interested in the question of the durability of concrete exposed to sea air and to the conditions found in a tropical climate. It is felt, however, that the results obtained apply to our ordinary conditions and may have many practical applications. The paper has therefore been revised by members of the Department of Theoretical and Applied Mechanics and prepared in form for publication as a bulletin of the Engineering Experiment Station.

The tests which are described were made to investigate the amount of shrinkage which may be expected in a mortar or a concrete, the relation between the change of moisture content and the change of length of these materials, the difference in shrinkage of plain and reinforced concrete, and the internal stresses set up in the latter. For purposes of comparison with the results obtained with concrete, a few tests were made on the effect of the absorption of water by sandstone and limestone.

2. *Acknowledgments.*—The work reported in this bulletin was done in the Laboratory of Applied Mechanics of the University of Illinois. Acknowledgment is made to PROFESSOR ARTHUR N. TALBOT, PROFESSOR H. F. GONNERMAN, and others, for helpful suggestions and aid.

II. MATERIALS USED IN TEST SPECIMENS

3. *Materials*.—The materials used in making the concrete test pieces were all of good quality. Universal portland cement, which passed the specifications of the American Society for Testing Materials as to fineness, soundness, tensile strength, and other qualities, was used. The fine aggregate was a well graded sand from Attica, Indiana. An average mechanical analysis from ten samples of the material is given in Table 1. The specific gravity of the sand was 2.69. The weight per cu. ft. when tamped into the mold with a rod was 111 lb., from which the voids in the dry material were estimated to be 34 per cent.

The coarse aggregate was gravel from Attica, Indiana. It was all screened to a size between $\frac{1}{4}$ and $\frac{1}{2}$ in.; larger sizes were not used because of the small dimensions of some of the test pieces. The gravel consisted of well rounded pebbles having a specific gravity of 2.71; its weight per cu. ft. was about 97 lb., making the voids in the coarse aggregate about 42 per cent. The steel used in the reinforced concrete test pieces consisted of plain round mild steel bars. In none of the tests did the measured stress in the steel approach the elastic limit of the material.

TABLE 1
MECHANICAL ANALYSIS OF SAND

SIEVE NO.	SIEVE OPENING IN INCHES	PER CENT PASSING THE GIVEN SIEVE
3	0.263	98.6
4	0.185	95.3
8	0.092	71.4
14	0.046	42.3
28	0.0232	18.0
48	0.0116	7.1
100	0.0058	2.6

All concrete was hand mixed, and each test piece was made from a separate batch of material. Since the size of the test pieces and the conditions of their storage varied greatly in the different tests, these details will be noted in the descriptions of the several sets of tests. In the various mortars and concretes used, the proportions of the ingredients are given by volume.

4. *The Coefficient of Expansion of Mortar and Concrete.*—The coefficient of expansion of mortar and concrete was measured for use in making a temperature correction of measurements. It was obtained from perfectly dried specimens only, so that there was no error due to the change in length with a change in moisture content of a specimen. The measurement of length was made by the use of a Berry strain gage, and the temperature was measured by a thermometer which was so placed among the test pieces that radiation of heat did not affect the reading.

The results obtained indicate that the coefficient of expansion of mortar and concrete is practically the same for different proportions and ages, provided that the same materials are used in all mixtures. The average values of the coefficient of expansion of 1:1, 1:2, and 1:3 mortar, and 1:1:2, 1:2:4, and 1:3:6 concrete, varied from 0.0000050 to 0.0000058 for one degree Fahrenheit, with no particular variation with the age or with the richness of the mixture.

III. EFFECT OF MOISTURE CONTENT ON LENGTH OF SPECIMEN

5. *Cement Mortar and Plain Concrete.*—Thirty test pieces, 2 by 3 by 24 in. in size, containing 2 steel plugs which formed a 20-in. gage line, were made of 1:1, 1:2, and 1:3 mortar, and 1:1:2, 1:2:4, and 1:3:6 concrete, five test pieces for each mixture. One day after the test pieces were made the molds were taken off, and initial measurements of the length and weight were made. All test pieces were then stored in a damp room and covered with double sheets of burlap which were kept wet. The measurements of length were made with a Berry strain gage of 20-in. gage length, using as a standard a steel bar immersed in water of known temperature. All measurements were corrected for the effect of temperature change. Changes in weight were taken to mean gains or losses in moisture content.

After 60 days of damp storage all test pieces were stored in air in a room where the relative humidity varied from 40 to 60 per cent when the room was heated by steam. As soon as the steam heating was discontinued with the advance of the season the relative humidity of the room increased, and it reached 90 per cent on cloudy days; whereupon the mortar and concrete absorbed moisture from the air and began expanding instead of contracting. After being kept 90 days in the air of this room the specimens were dried for 14 days in an oven at a temperature of 150 deg. F., and then for 15 days at 200 deg. F. The amounts of change of moisture and length during the test are shown in Table 2 and are indicated in Figs. 1 and 2.

As may be seen from Fig. 1, 1:1 mortar at seventy days contracted 0.025 per cent, notwithstanding the fact that it contained more moisture than its initial amount. From this phenomenon it can be concluded that the water which goes into the concrete may exist in two different ways, either combined with cement in a chemical compound or remaining in a state which affects the volume of the concrete. It also appears that concrete in which the total moisture content is kept constant for some length of time is liable to contract slightly. This slight contraction was observed on several occasions during the test. It follows that the absolute amount of contraction of concrete

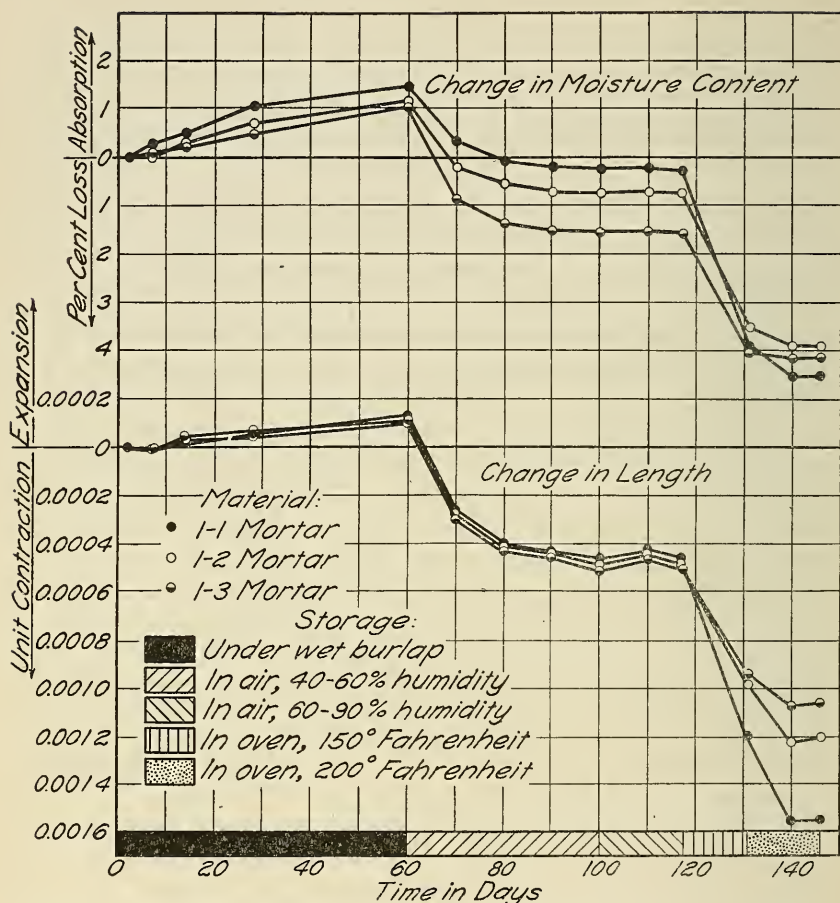


FIG. 1. CHANGE IN LENGTH WITH CHANGE IN MOISTURE CONTENT OF MORTAR

due to drying is indeterminate, because it depends upon the age of the concrete and the duration of the drying period. Figs. 1 and 2 show that in general these specimens of mortar and concrete expanded when they were kept wet. The richer mixtures absorb more water and consequently expand more than the leaner ones. Mortar and concrete lose moisture rapidly when placed in dry air and the loss is more rapid in lean mixtures than in rich ones. In air storage the shrinkage is greater for mortar than for concrete, and with artificial drying the shrinkage is greater for the richer mixtures.

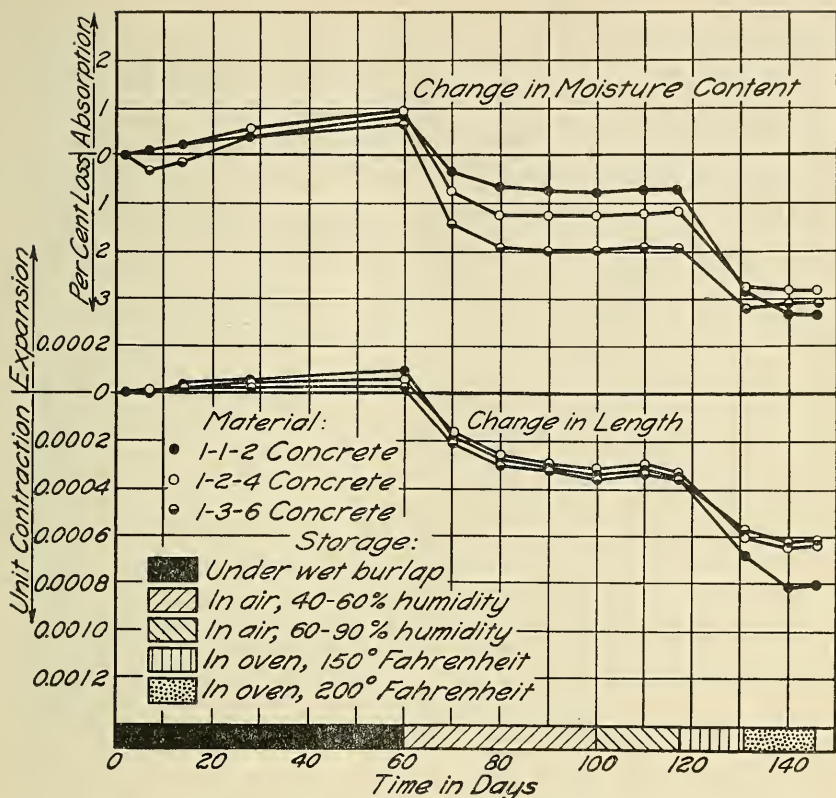


FIG. 2. CHANGE IN LENGTH WITH CHANGE IN MOISTURE CONTENT OF CONCRETE

For the specimens tested, the contraction of mortar and concrete at a temperature of 150 deg. F., which is somewhat higher than the highest outdoor temperature in summer in a tropical climate, was 0.12 per cent for 1:1 mortar, 0.10 per cent for 1:2 mortar, 0.09 per cent for 1:3 mortar, 0.07 per cent for 1:1:2 concrete, 0.06 per cent for 1:2:4 concrete, and 0.06 per cent for 1:3:6 concrete. The amount of expansion when specimens were kept wet reached 0.012 per cent in 1:1 mortar, and 0.002 per cent in 1:3:6 concrete, at the end of 60 days. Judging from these tests, shrinkage of concrete in a structure freely exposed to the air may be as much as 0.05 per cent in a 1:2:4

TABLE 2
CHANGE IN MOISTURE CONTENT AND CORRESPONDING CHANGES IN LENGTH
Expansion and Contraction, are given in Per Cent of Gage Length

PRO- PORTION	Kept in Wet Condition 60 Days		Kept in Air of Room 58 Days		Dried in Oven at 150 Deg. F. 14 Days		Dried in Oven at 200 Deg. F. 15 Days	
	Gain of Moisture Per Cent	Expansion Per Cent	Loss of Moisture Per Cent	Contraction Per Cent	Loss of Moisture Per Cent	Contraction Per Cent	Loss of Moisture Per Cent	Contraction Per Cent
1:1	1.46	0.0124	0.26	0.0460	3.92	0.1204	4.54	0.1570
1:2	1.14	0.0100	0.71	0.0488	3.51	0.0991	3.97	0.1237
1:3	1.05	0.0101	1.57	0.0503	4.04	0.0944	4.18	0.1077
1:1:2	0.85	0.0091	0.68	0.0358	2.86	0.0673	3.33	0.0807
1:2:4	0.89	0.0055	1.25	0.0333	2.80	0.0605	2.82	0.0636
1:3:6	0.64	0.0019	1.91	0.0362	3.21	0.0576	3.17	0.0622

concrete.* In a structure such as a retaining wall or a road pavement, where one side of the structure is against a mass of earth, the shrinkage should be considerably less.

In a second set of tests twenty-four test pieces, having the same size and form as those in the tests previously described, were prepared from the same materials, half of mortar with the proportions of 1:2, and half of concrete with the proportions of 1:2:4. The test pieces

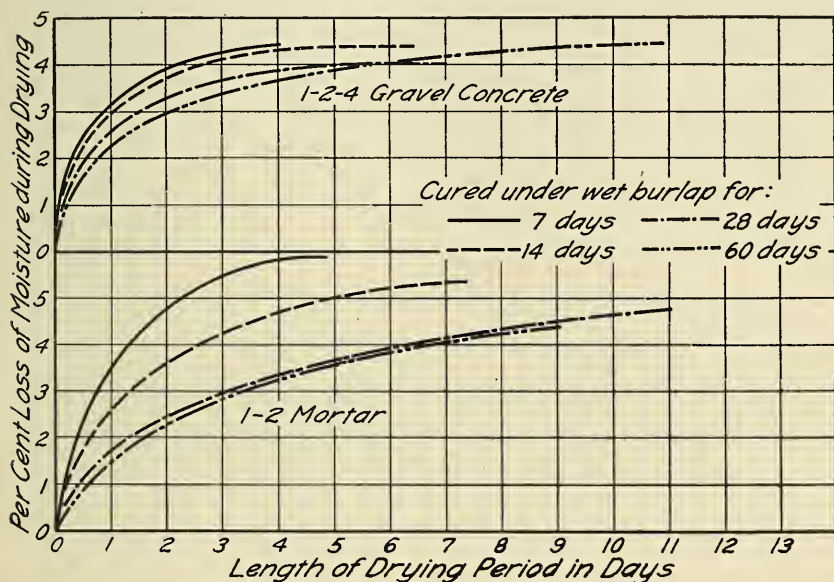


FIG. 3. LOSS OF MOISTURE FROM MORTAR AND CONCRETE WHEN DRIED AT 150 DEG. F. AFTER PRELIMINARY STORAGE NOTED IN DIAGRAM

* Quantitative information regarding the shrinkage of concrete over varying periods of time has been reported by a number of investigators, among whom are the following:

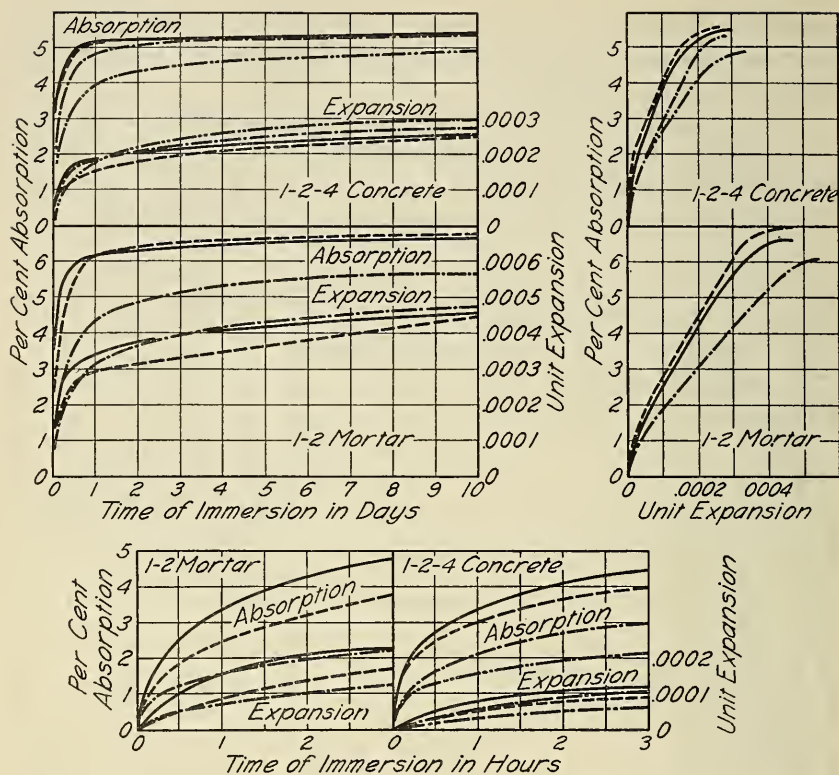
McMillan, F. R. "Shrinkage and Time Effects in Reinforced Concrete." Studies in Engineering, No. 3, University of Minnesota, 1915.

White, A. H. "Destruction of Cement, Mortars, and Concrete Through Expansion and Contraction." Proc. A. S. T. M., Vol. XI, 1911.

Nichols, C. S. and McCullough, C. B. "The Determination of Internal Temperature Range in Concrete Arch Bridges." Engineering Experiment Station, Ames, Iowa, Bul. No. 30, 1913.

Goldbeck, A. T. "The Expansion and Contraction of Concrete While Hardening." Proc. A. S. T. M., Vol. XI, 1911.

Abrams, D. A. "Effect of Fineness of Cement." Structural Materials Research Laboratory, Lewis Institute, Chicago, Bul. No. 4, 1919.



Treatment of Specimens

(1.) Cured under wet burlap for:

———— 7 days, - - - - - 14 days, - · - · - 28 days, · · · · · 60 days

(2.) Dried in oven at 150° F. as shown in Fig. 3.

(3.) Immersed in water

FIG. 4. ABSORPTION AND CORRESPONDING EXPANSION OF MORTAR AND CONCRETE IMMERSSED IN WATER

were stored in a damp room and covered with burlap which was kept wet. Three specimens of each kind were taken out of the storeroom when they had hardened 7, 14, 28, and 60 days, respectively, in the damp condition, and were then dried in an oven at a temperature of 150 deg. F. until the loss of weight did not exceed 0.20 per cent during 24 hours. The test pieces were then immersed in clean water in a shallow tank. The lengths of the specimens were measured by a 20-in.

Berry strain gage and all measurements were corrected for the effect of temperature changes in the instrument. The results obtained are shown in Figs. 3 and 4, the loss in moisture being given in terms of weight of the mortar or concrete. The mortar weighed from 140 to 145 lb. per cu. ft., and the concrete weighed from 145 to 155 lb. per cu. ft.

From Fig. 3 it is seen that the rate of loss of moisture in mortar and concrete by drying decreases with the age of the specimen. In the mortar, the rate of loss in specimens 28 days and 60 days old, respectively, becomes nearly the same. The total loss in moisture in mortar does not seem to be greatly different from that in concrete at the four ages used.

A number of significant features of the second set of tests are brought out in Fig. 4:

(a) Dry mortar and concrete expand as soon as they absorb water.

(b) The rate of absorption upon immersion is much less for the older specimens during the first two or three days of immersion, although the total amount of absorption does not change very greatly for specimens of different ages.

(c) The rate of expansion due to immersion is also less for the older specimens during the first two or three days of immersion. For longer periods of time the amount of expansion is nearly the same for all ages of each material; but in every case the expansion is considerably greater for mortar than it is for concrete.

(d) The amount of the expansion of mortar and concrete is small until an absorption of nearly 2 per cent is reached. After this point the expansion becomes nearly proportional to the additional absorption of moisture. This holds true until a sudden decrease in the absorption occurs after about one day's immersion, beyond which point additional absorption gives a greatly increased expansion.

(e) The absorption of moisture by mortar or concrete is much more rapid than the drying out of moisture from the same material. As a result, the expansion due to wetting is more rapid than the contraction produced by drying. For example, a mortar 7 days old gained as much moisture in 2 hours' immersion as could be dried out by heating to 150 deg. F. for 24 hours.

Another experiment on the relation between absorption of moisture and expansion of concrete was made upon two specimens of 1:2:4 gravel concrete taken from a floor of the Western Newspaper Union Building in Chicago. The concrete was 9 years old. The sizes of the test pieces were 3.5 by 5 by 21 in., and 3.5 by 5 by 26 in. The specimens were dried for 13 days in an oven at a temperature of 150 deg. F. In drying, the average contraction of the concrete was 0.016 per cent for the two specimens. After immersion in water for 15 days the average absorption was 5.09 per cent and the resulting expansion 0.042 per cent. The results of the test are shown in Fig. 5.

6. *Sandstone and Limestone.*—For comparison with the behavior of mortar and concrete, a few tests were made on the absorption of water by stone. The specimens used were sandstone and limestone which had nearly the same porosity as the mortar and concrete tested. If the expansion of such material is due to the separation of particles, the same amount of expansion might be expected with stones of the same porosity.

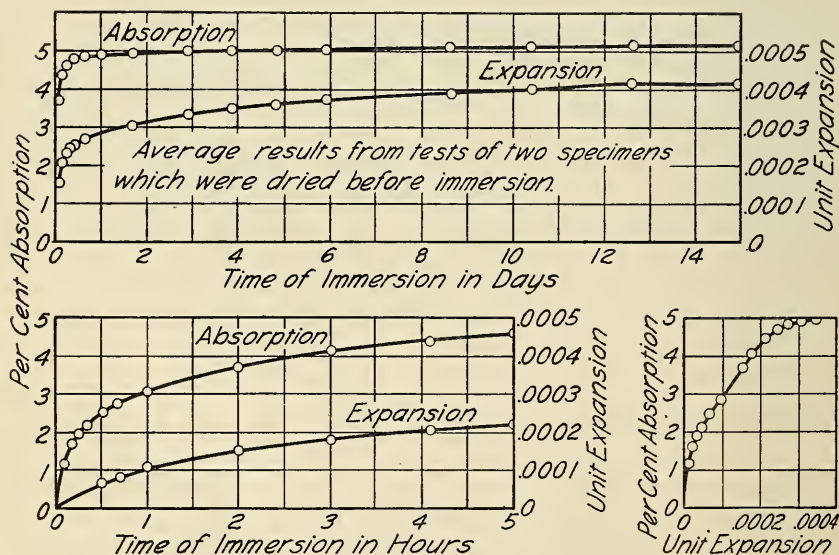


FIG. 5. ABSORPTION AND CORRESPONDING EXPANSION OF NINE-YEAR-OLD CONCRETE IMMERSSED IN WATER

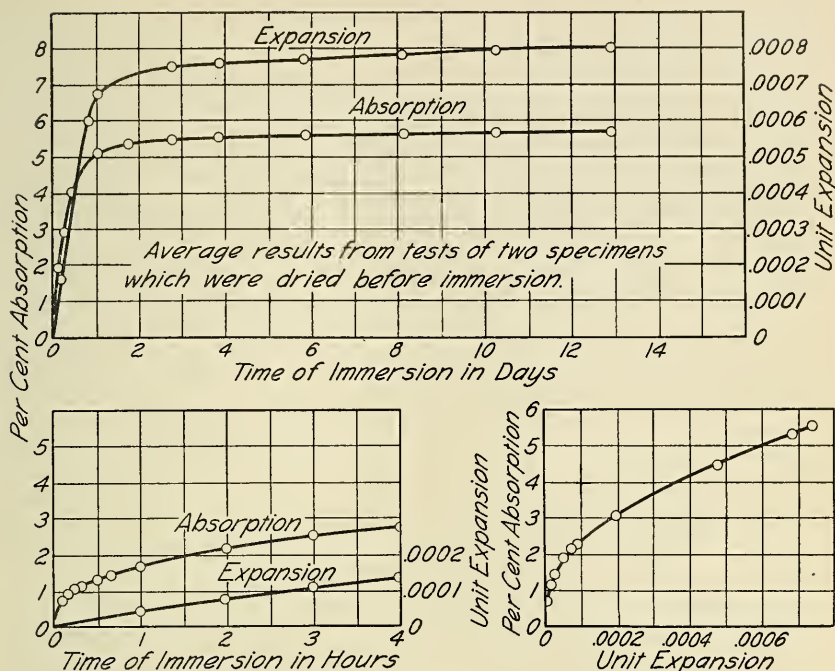


FIG. 6. ABSORPTION AND CORRESPONDING EXPANSION OF LAKE SUPERIOR RED SANDSTONE IMMERSSED IN WATER

Two specimens of a reddish brown sandstone of good quality, obtained from the Lake Superior region for use in the construction of a University building, were used in the test. The sizes of the specimens were 3 by 4 by 24 in. and 3.5 by 4 by 25 in. Both pieces were dried for 6 days in an oven at a temperature of 150 deg. F. After immersion in water for 13 days, the average absorption for the two specimens was 5.75 per cent, and the corresponding expansion was 0.080 per cent. By referring to Figs. 4 and 5 this amount of expansion is seen to be much greater than that of mortar or concrete after the same period of immersion. The results of the test are shown in Fig. 6.

The limestone specimens used in the test were a good quality of oölitic limestone of the kind used for building purposes, commonly known as Bedford Indiana stone. The stone has a porous structure. The two test pieces were 3.5 by 3.7 by 11.1 in., and 3.5 by 4.75 by 12.5

in. in size. The first specimen was dried for 7 days in an oven at 150 deg. F., with consequent loss of moisture, due to the drying, of 1.04 per cent, and no measurable contraction. The second specimen was dried for 11 days in an oven at 150 deg. F., with consequent loss of moisture of 1.42 per cent and no measurable contraction. They were then immersed in water. Fig. 7 shows that after an immersion of 6 days the average absorption was 6.1 per cent with practically no expansion.

The very rapid absorption and the lack of expansion of the limestone are in decided contrast to what occurred with all the other materials used. The behavior of the limestone throws doubt upon the theory that moisture entering a porous substance causes an expansion of material through separation of the particles.

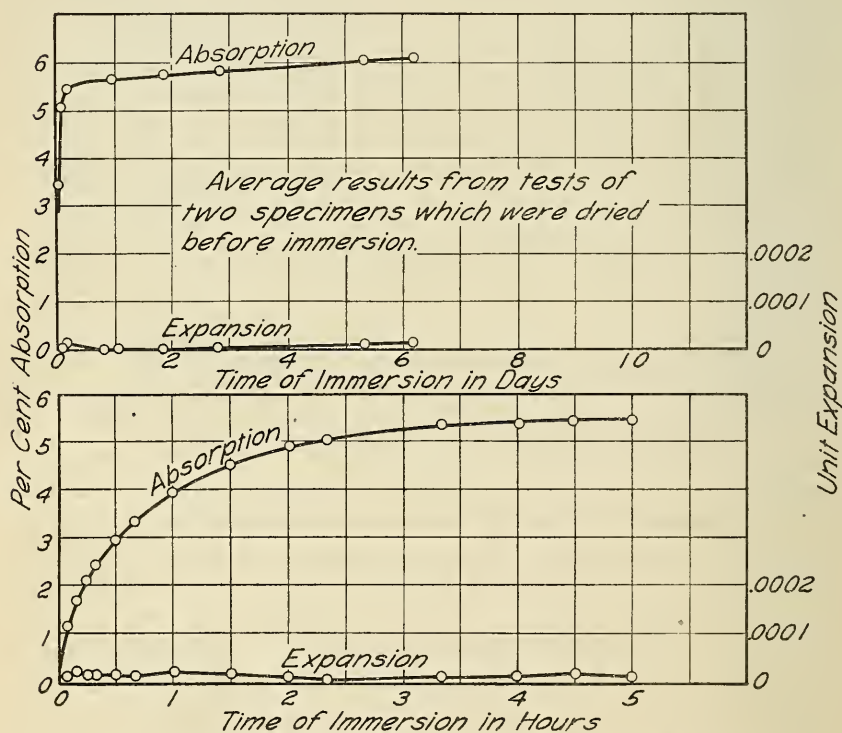


FIG. 7. ABSORPTION AND CORRESPONDING EXPANSION OF INDIANA OÖLITIC LIMESTONE IMMERSSED IN WATER

IV. SHRINKAGE STRESSES IN REINFORCED CONCRETE

7. *Analysis of Shrinkage Stresses.*—In the preceding tests on plain concrete, a considerable amount of information was obtained regarding the changes in length corresponding to changes in moisture content. It is desirable to know how such information can be applied to the action of reinforced concrete specimens. It is known that there is a well defined difference between the shrinkage in a set of plain concrete test pieces, and that in a set of reinforced concrete test pieces made from the same materials. This difference may be taken as due to the resistance of the reinforcement to the shrinkage of the concrete, which thus produces compressive stress in the steel and tensile stress in the concrete. It has been found possible to estimate the actual shrinkage stresses, both in steel and concrete, by comparison with the shrinkage of plain concrete under similar conditions.

When a plain concrete prism is dried out in air all particles have a tendency to move toward the middle of the prism, thus decreasing its length without causing cracks unless external forces resist this tendency. Consider one half of the length of a reinforced concrete prism as shown in Fig. 8. If there was no bond resistance between the concrete and the reinforcing bar, when shrinkage occurred the relative

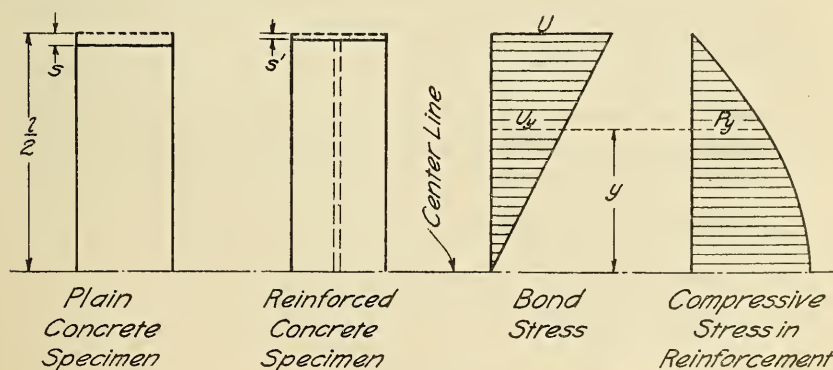


FIG. 8. DISTRIBUTION OF SHRINKAGE STRESSES IN REINFORCED CONCRETE SPECIMEN

The total deformation of the half-length of the bar is

$$s' = \int_0^{\frac{l}{2}} \frac{P_y dy}{aE_s} = \frac{uol^2}{12aE_s} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

Since the total stresses in steel and concrete are equal in amount at any section, the total deformation in the concrete is

$$s-s' = \frac{el}{2} - s' = \int_0^{\frac{l}{2}} \frac{P_u dy}{AE_c(1-p)} = \left[\frac{uol^2}{12 AE_c(1-p)} \right] \quad (3)$$

From equations (2) and (3), disregarding the factor $(1-p)$, the maximum bond stress is found to be

[illegible]

The total stress $P = \frac{1.5 \text{ } eaE_s}{1+np}$

whence the maximum compressive unit stress $f_s = \frac{1.5 e E_s}{1 + np}$. (5)

and the maximum tensile unit stress

$$f_c = \frac{af_s}{A} = pf_s \quad (6)$$

The shrinkage stresses in the steel and concrete are thus seen to be directly proportional to the amount of shrinkage in plain concrete. It is seen that the stress in the steel decreases and the stress in the concrete increases with an increase in the percentage of reinforcement.

It must be remembered that equations (5) and (6) give maximum stresses; the average stresses over the length of the specimen are two-thirds as great. In the following experiments gage lengths were used which were five-sixths and five-twelfths of the length of the specimen; in these cases the average measured stresses should be 77 per cent and 94 per cent, respectively, of the maximum stress at the middle of the bar.

8. *Measurement of Shrinkage Stresses in Reinforced Concrete Specimens.*—Two sets of tests were made for the purpose of measuring

the shrinkage stresses in reinforced concrete specimens. For the first set of experiments six test pieces, 2 by 3 by 24 in. in size, were prepared, two of them being of plain concrete, two having two $\frac{3}{8}$ -in. round steel bars ($p = 3.68$ per cent), and two having two $\frac{1}{2}$ -in. round steel bars ($p = 6.54$ per cent). The proportions of the concrete were 1:2:4. The specimens were dried in the air of a room for 22 days; then they

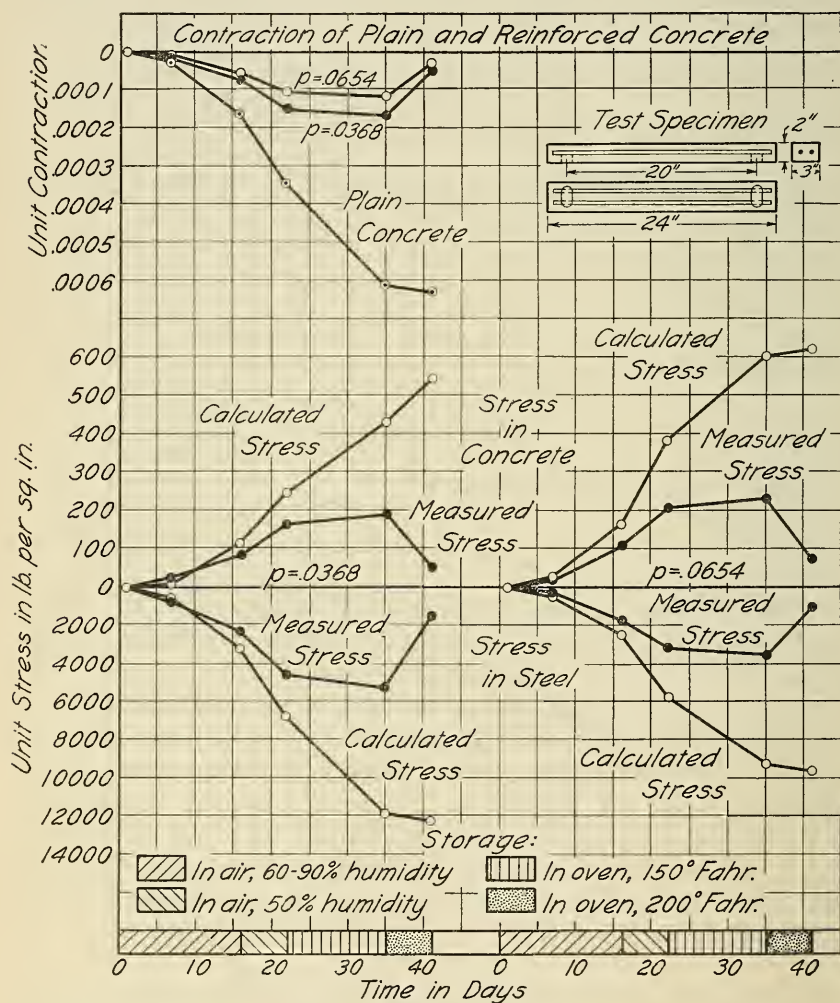


FIG. 9. SHRINKAGE STRESSES IN REINFORCED CONCRETE SPECIMENS

were dried in an oven for 13 days at a temperature of 150 deg. F.; and finally they were dried for 6 days at a temperature of 200 deg. F.

The change of length was measured on a 20-in. gage line between points on each embedded steel bar in the reinforced concrete test pieces, and between points on steel plugs embedded in the plain concrete test pieces. The results, which were very uniform in companion specimens, are shown in Fig. 9. The measured stresses in the steel were obtained by multiplying the measured deformation in the steel by the modulus of elasticity of steel; the curves marked "measured stress in concrete" were obtained by multiplying the measured stress in the steel by the percentage of reinforcement, see equation (6), and represent the average stress over the cross-section of the concrete. The calculated stress is the average calculated stress over the 20-in. gage length, or 77 per cent of the maximum stress given by equations (5) and (6).

As may be seen from Fig. 9, even reinforced concrete having a very high percentage of reinforcement contracts rapidly in air. When the specimens were 22 days old the shrinkage stress in the concrete reached 166 lb. per sq. in. in the specimens having 3.68 per cent reinforcement, and 204 lb. per sq. in. in the specimens having 6.54 per cent reinforcement.

The same specimens, after being dried 13 days longer at a temperature of 150 deg. F., developed stresses of 190 lb. per sq. in. and 240 lb. per sq. in., respectively, which nearly correspond to the ultimate tensile strength of the concrete. Furthermore, being dried at a temperature of 200 deg. F., the concrete failed in tension and showed two transverse cracks at points about 8 in. from each end.

This last case, however, may not be representative, because there may have been some injury to the texture of the concrete when it was heated beyond 200 deg. F. But the fact that the shrinkage stress in the concrete reached 170 to 200 lb. per sq. in. in 22 days, in the air of a room, may be sufficient reason for concluding that cracks will eventually be formed in concrete which has a high percentage of reinforcement.

The sudden drop of the shrinkage stress in the concrete when heated beyond 200 deg. F., as shown in Fig. 9, indicates injury to the concrete at this temperature; so that the use of reinforced concrete exposed to a temperature higher than 200 deg. F. may be dangerous unless proper precautions are taken.

A second set of tests was made in which smaller percentages of reinforcement were used. The size of the test pieces was 6 by 6 by 24 in. and they were made of 1:2:4 concrete. Deformations were measured on a gage length of 10 in. One piece was made without reinforcement, one with four $\frac{1}{4}$ -in., one with four $\frac{3}{8}$ -in., and one with four $\frac{1}{2}$ -in. round steel bars. A similar set of test pieces was made with the same amount of reinforcement, but which had in addition three anchor-lugs welded to each end of the reinforcing bars to guard against a slipping of the reinforcement during the setting of the concrete; a test piece of plain concrete was also included in this set. The size and form of the test pieces are shown in Figs. 10 and 11.

These specimens were left in the air of the room for 99 days. They were then dried in an oven at a temperature of 200 deg. F. The contraction which took place during the drying and the measured shrinkage stresses in both steel and concrete are shown in Figs. 10, 11, and 12. The measured stresses in steel and concrete were determined as in the preceding tests. The calculated stress is the average calculated stress over the 10-in. gage length, or 94 per cent of the stress given by equations (5) and (6).

In the case of the plain concrete, the results of the two sets of tests were quite uniform, both as regards the loss of moisture and the resulting contraction during setting and hardening. The difference between the contraction of plain concrete specimens and those having different amounts of reinforcement was quite regular at all conditions of storage, and is very instructive. The deformation in the reinforcing bars provided with lugs was greater than the deformation in the plain bar. This indicates that slipping of the bars took place with the latter during the shrinking of the concrete; the shrinkage stress is seen to be greater in reinforced concrete having good anchorage of the reinforcement. In general some slipping must be expected.

The measured stress in the steel reached 18 000 lb. per sq. in. in the specimen having reinforcement of 0.55 per cent, and the stress in the concrete reached 250 lb. per sq. in. in the specimen having reinforcement of 2.18 per cent. This stress in the steel exceeds the accepted working stress of soft steel and the stress in the concrete is nearly equal to the ultimate tensile strength of concrete of this kind.

When these specimens were heated above 200 deg. F. these stresses both in concrete and steel suddenly dropped down, probably due to injury of the concrete by the heat.

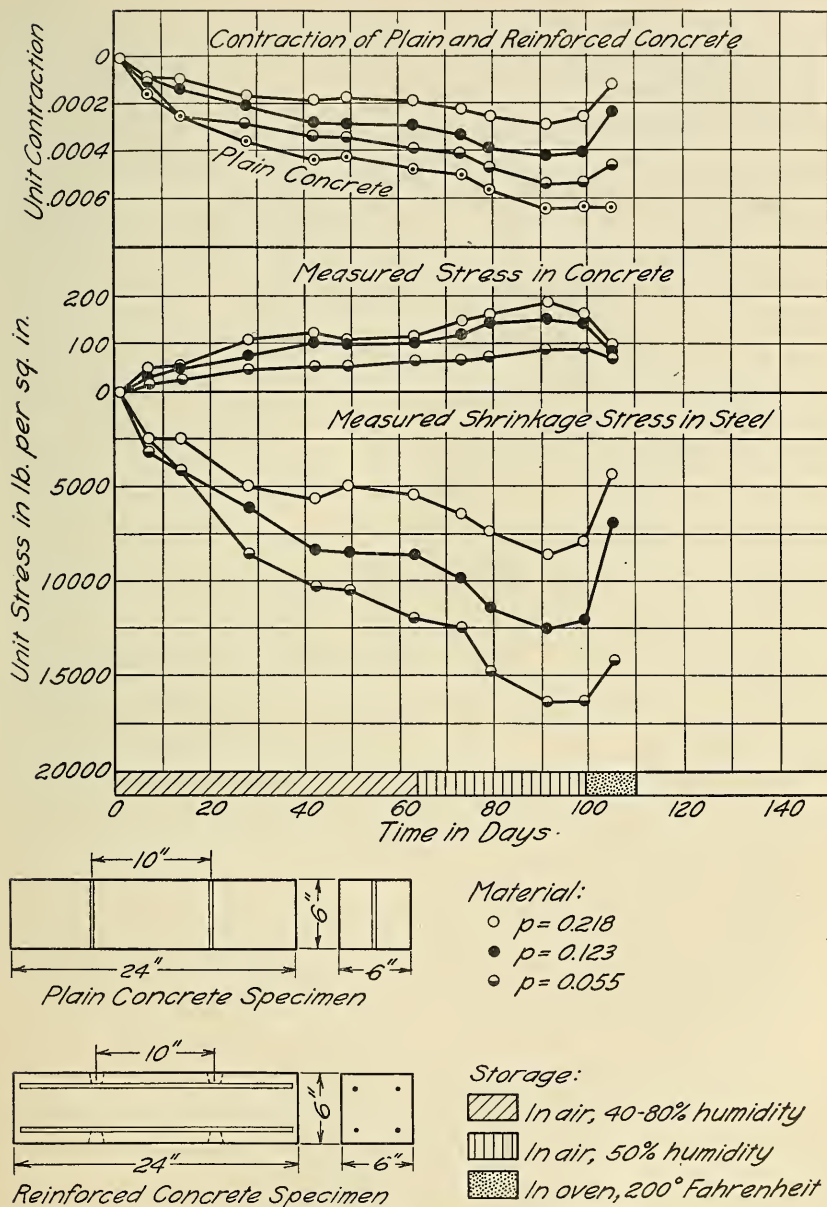


FIG. 10. SHRINKAGE STRESSES IN REINFORCED CONCRETE SPECIMENS

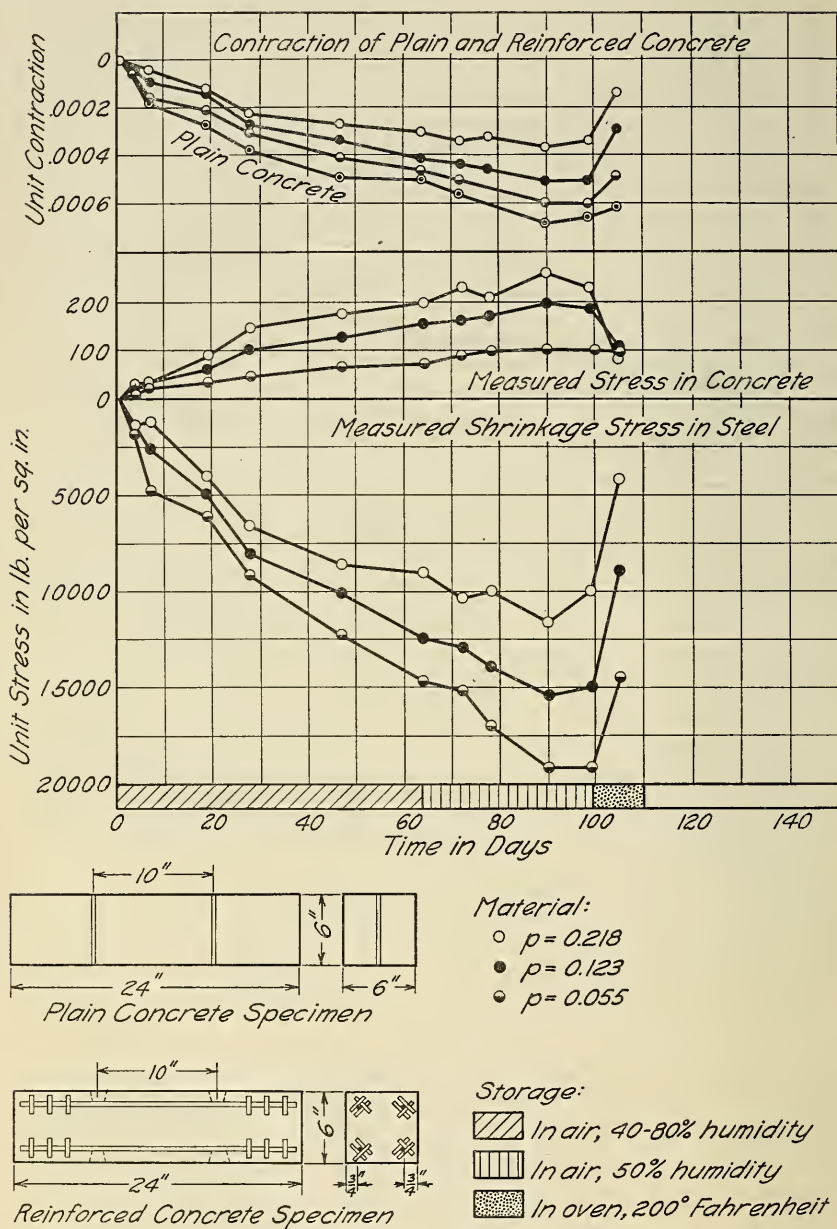


FIG. 11. SHRINKAGE STRESSES IN REINFORCED CONCRETE SPECIMENS

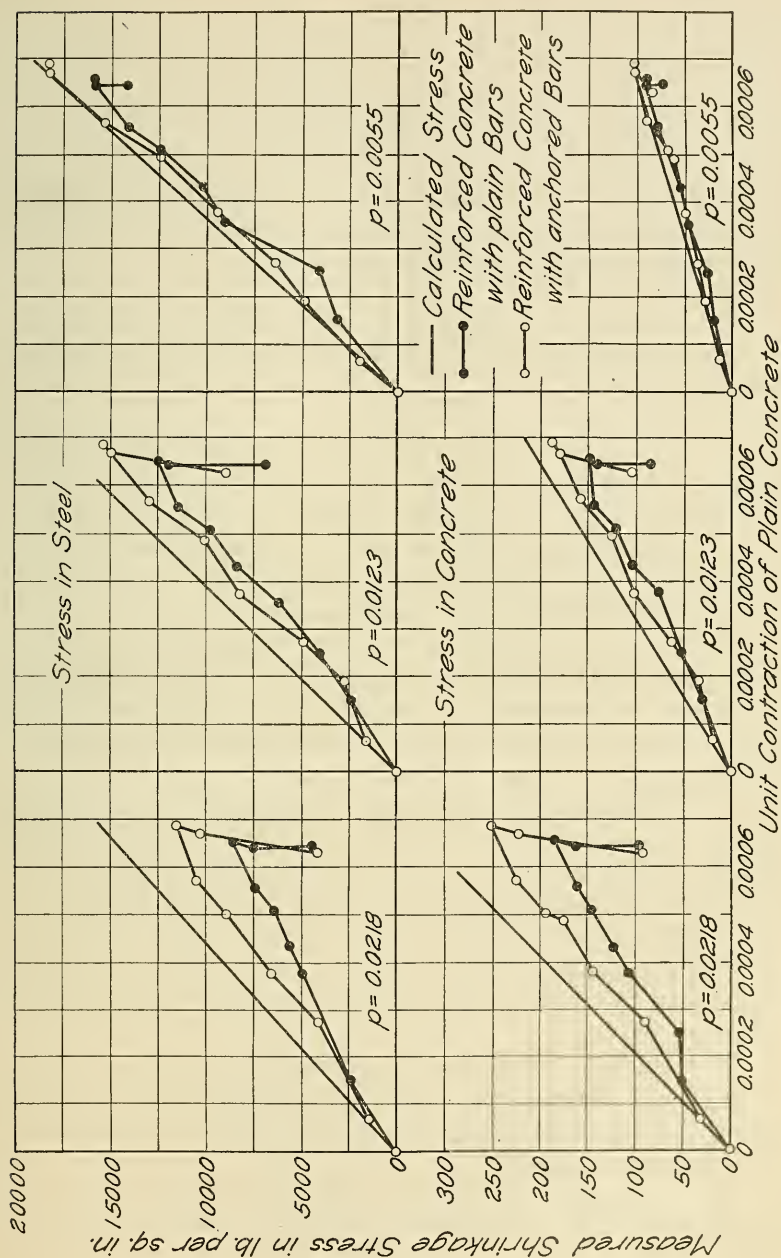


FIG. 12. COMPARISON OF MEASURED AND CALCULATED SHRINKAGE STRESSES

V. CONCLUSION

9. *Comments.*—The following comments are offered ; part of them relate to the concrete used in the tests and part are inferences based on the writer's general experience and observation :

1. Concrete expands when it absorbs moisture and contracts when it is dried. Concrete of a 1:2:4 mixture is likely to contract during hardening as much as 0.05 per cent in an ordinary structure.

2. Contraction of concrete by the loss of moisture causes stress in the concrete when it is restrained by an external force. The amount of this stress is not as small as is generally supposed.

3. The shrinkage stress caused in the steel in reinforced concrete may reach the usually accepted working stress of steel when the amount of reinforcement is less than 1.5 per cent.

4. The shrinkage stress developed in 1:2:4 concrete may reach the ultimate tensile strength of the concrete when the amount of reinforcement is greater than 1.5 per cent. With richer mixtures the increase in shrinkage stress may be relatively greater than the increase in ultimate strength.

5. The greater the percentage of reinforcement the greater the tensile stress that may develop in the concrete, and concrete having a higher percentage of reinforcement than 1.5 per cent is likely to have cracks formed unless proper provision is made.

6. In reinforced concrete out of doors, subject to alternate wet and dry conditions, cracks may readily be formed under the repeated stress which is nearly equal to the tensile strength of the concrete.

7. Reinforced concrete does not appear likely to be a durable material in a place where a corrosive influence on steel, such as sea air, is active, unless proper protection against the formation of shrinkage cracks is made.

8. It is suggested that the prevention of shrinkage stress in concrete might be accomplished in two ways, either by finding a

cement giving less expansion and contraction, or by the use of a perfect waterproofing treatment.

9. It may be expected that an integral waterproofing compound might lessen the change of volume for a short time, but it would not prevent the final diffusion of moisture with consequent change in volume.

LIST OF
PUBLICATIONS OF THE ENGINEERING EXPERIMENT STATION

Bulletin No. 1. Tests of Reinforced Concrete Beams, by Arthur N. Talbot. 1904. *None available.*

Circular No. 1. High-Speed Tool Steels, by L. P. Breckenridge. 1905. *None available.*

Bulletin No. 2. Tests of High-Speed Tool Steels on Cast Iron, by L. P. Breckenridge and Henry B. Dirks. 1905. *None available.*

Circular No. 2. Drainage of Earth Roads, by Ira O. Baker. 1906. *None available.*

Circular No. 3. Fuel Tests with Illinois Coal (Compiled from tests made by the Technological Branch of the U. S. G. S., at the St. Louis, Mo., Fuel Testing Plant, 1904-1907), by L. P. Breckenridge and Paul Diserens. 1908. *Thirty cents.*

Bulletin No. 3. The Engineering Experiment Station of the University of Illinois, by L. P. Breckenridge. 1906. *None available.*

Bulletin No. 4. Tests of Reinforced Concrete Beams, Series of 1905, by Arthur N. Talbot. 1906. *Forty-five cents.*

Bulletin No. 5. Resistance of Tubes to Collapse, by Albert P. Carman and M. L. Carr. 1906. *None available.*

Bulletin No. 6. Holding Power of Railroad Spikes, by Roy I. Webber. 1906. *None available.*

Bulletin No. 7. Fuel Tests with Illinois Coals, by L. P. Breckenridge, S. W. Parr, and Henry B. Dirks. 1906. *None available.*

Bulletin No. 8. Tests of Concrete: I, Shear; II, Bond, by Arthur N. Talbot. 1906. *None available.*

Bulletin No. 9. An Extension of the Dewey Decimal System of Classification Applied to the Engineering Industries, by L. P. Breckenridge and G. A. Goodenough. 1906. Revised Edition, 1912. *Fifty cents.*

Bulletin No. 10. Tests of Concrete and Reinforced Concrete Columns, Series of 1906, by Arthur N. Talbot. 1907. *None available.*

Bulletin No. 11. The Effect of Scale on the Transmission of Heat through Locomotive Boiler Tubes, by Edward C. Schmidt and John M. Snodgrass. 1907. *None available.*

Bulletin No. 12. Tests of Reinforced Concrete T-Beams, Series of 1906, by Arthur N. Talbot. 1907. *None available.*

Bulletin No. 13. An Extension of the Dewey Decimal System of Classification Applied to Architecture and Building, by N. Clifford Ricker. 1906. *None available.*

Bulletin No. 14. Tests of Reinforced Concrete Beams, Series of 1906, by Arthur N. Talbot. 1907. *None available.*

Bulletin No. 15. How to Burn Illinois Coal without Smoke, by L. P. Breck-enridge. 1907. *None available.*

Bulletin No. 16. A Study of Roof Trusses, by N. Clifford Ricker. 1907. *None available.*

Bulletin No. 17. The Weathering of Coal, by S. W. Parr, N. D. Hamilton, and W. F. Wheeler. 1907. *None available.*

Bulletin No. 18. The Strength of Chain Links, by G. A. Goodenough and L. E. Moore. 1907. *Forty cents.*

Bulletin No. 19. Comparative Tests of Carbon, Metallized Carbon, and Tan-talum Filament Lamps, by T. H. Amrine. 1907. *None available.*

Bulletin No. 20. Tests of Concrete and Reinforced Concrete Columns, Series of 1907, by Arthur N. Talbot. 1907. *None available.*

Bulletin No. 21. Tests of a Liquid Air Plant, by C. S. Hudson and C. M. Gar-land. 1908. *Fifteen cents.*

Bulletin No. 22. Tests of Cast-Iron and Reinforced Concrete Culvert Pipe, by Arthur N. Talbot. 1908. *None available.*

Bulletin No. 23. Voids, Settlement, and Weight of Crushed Stone, by Ira O. Baker. 1908. *Fifteen cents.*

**Bulletin No. 24.* The Modification of Illinois Coal by Low Temperature Dis-tillation, by S. W. Parr and C. K. Francis. 1908. *Thirty cents.*

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Bulletin No. 27. Tests of Brick Columns and Terra Cotta Block Columns, by Arthur N. Talbot and Duff A. Abrams. 1908. *Twenty-five cents.*

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Bulletin No. 38. The Weathering of Coal, by S. W. Parr and W. F. Wheeler. 1909. *Twenty-five cents.*

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SOUND-PROOF PARTITIONS

BY

F. R. WATSON



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BULLETIN No. 127

MARCH, 1922

SOUND-PROOF PARTITIONS

AN INVESTIGATION OF THE ACOUSTIC PROPERTIES
OF VARIOUS BUILDING MATERIALS, WITH
PRACTICAL APPLICATIONS

BY

F. R. WATSON

PROFESSOR OF EXPERIMENTAL PHYSICS

ENGINEERING EXPERIMENT STATION

PUBLISHED BY THE UNIVERSITY OF ILLINOIS, URBANA

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SOUND-PROOF PARTITIONS

I. INTRODUCTION

1. *Purpose of Investigation.*—The demand for quiet rooms in hospitals, hotels, and office buildings, the desirability of insulating music studios and other rooms where disturbing sounds are produced, and the necessity for solving other problems for the control of noise have led to repeated requests from architects and builders for reliable information on effective methods for insulating sound. Although present knowledge of the subject is incomplete, nevertheless, on account of the pressing need for guidance in such matters, it is thought desirable to collect and present the available information in a systematic way, giving the methods and results of various investigations relating to the action of materials on sound, describing practical installations of soundproofing, and setting forth in accordance with existing knowledge recommendations that may be applied where sound insulation is wanted.

Before 1915 little was known definitely about the subject and cut-and-try methods were used when soundproofing was desired. These cases were usually isolated and few published accounts are available, so that little progress was made. Since 1915 the problem has engaged the attention of scientists who have conducted investigations in the light of the known phenomena of sound and obtained results that are now being applied in practical constructions.

2. *Acknowledgments.*—In addition to the published accounts of investigations, to which proper reference will be made, acknowledgment is due to the ARMSTRONG CORK COMPANY, the UNITED STATES GYPSUM COMPANY, and the ASSOCIATED METAL LATH MANUFACTURERS for their financial support and coöperation in the conduct of special investigations.

II. ACTION OF SOUND WITH APPLICATIONS TO BUILDINGS

3. *Origin of Sound.*—Sound of a definite frequency consists of a regular series of alternate compressions and rarefactions generated by a vibrating body and progressing in spherical waves in the surrounding medium. Figure 1 pictures the waves in air sent out by the human voice through a megaphone.

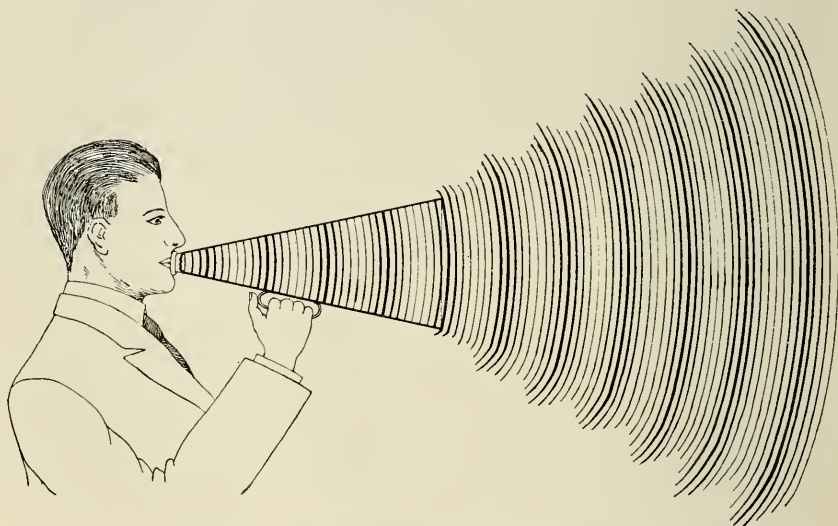


FIG. 1. DIAGRAMMATIC REPRESENTATION OF COMPRESSIONS AND RAREFACTIONS (SOUND WAVES) SENT OUT FROM A SOURCE OF SOUND

The vibrations in this case are due to the vocal cords which are set into rapid oscillations by a stream of air from the lungs. An unbalanced motor may cause its supporting base to shake, thus imparting to the surrounding air small motions that are propagated as sound waves. A case for consideration in buildings is the generation of sound waves when a wall or floor is set in vibration by a motor, an elevator, or other agency.

4. *Amplitude of Sound Vibrations.*—The amplitude of vibration in sound waves is small, according to estimates varying from 0.00000005 in. for a sound barely audible to 0.004 in. for a loud

sound.* A very small motion of a building partition† will therefore be sufficient to generate in air a sound that may be detected by the ear. Thus one of the difficult problems in sound insulation of buildings is to reduce the motions of walls as far as possible.

5. *Propagation of Sound*.—Sound waves set up by vibrating bodies are propagated through the surrounding medium—solid, liquid, or gaseous—with a considerable velocity, v , depending on the elasticity, E , and the density, d , of the medium, according to the formula: $v = \sqrt{E/d}$. The values of the velocities for a few media are given in Table 1.‡

TABLE 1
VELOCITY OF SOUND IN VARIOUS MEDIA

Medium	Velocity of Sound at 0 Deg. C.
Air.....	1088 ft. per sec.
Water.....	4728 ft. per sec.
Pine Wood.....	10900 ft. per sec.
Brick.....	11980 ft. per sec.
Steel.....	16360 ft. per sec.

An inspection of the data given in this table shows that sound travels very fast, about one-fifth of a mile per second in air, and about three miles per second in steel. A sound traveling in the steel structure of a building 260 feet high would require only $260 \div 16360$, or 0.0159 second to pass from the basement to the roof.

6. *Action of Materials on Sound*.—When sound waves in one medium encounter a second medium with a different elasticity or density, their regular progression is disturbed. Part of the energy is thrown back in the form of reflected waves, part is absorbed in the second medium, and part is transmitted—the relative amounts depending on the changes in elasticity and density of the second medium compared with the first, that is, in accordance with the change in the velocity of sound.¶ (See Fig. 2.)

* Shaw, P. E. "The Amplitude of Minimum Audible Impulsive Sounds." Proc. Roy. Soc., Vol. 76A, p. 360, 1905.

† Hall, E. E. "Graphical Analysis of Building Vibrations." Elec. Wld., Vol. 66, p. 1356, 1915.

‡ Smithsonian Tables, Tables 80 and 81.

¶ Rayleigh, Lord. "Theory of Sound." Vol. 2, Sections 270-272. Equations are given with discussions for different media.

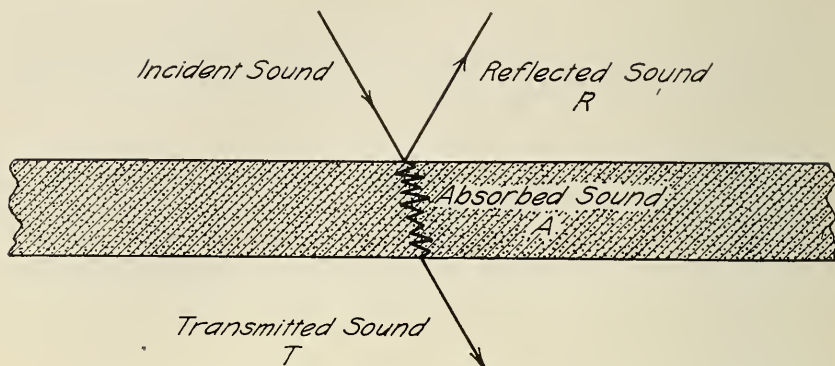


FIG. 2. REFLECTION, ABSORPTION, AND TRANSMISSION OF SOUND

A porous material like hairfelt presents little resistance to sound. The reflection is small, but the absorption in the porous channels may be quite large. What is not reflected and absorbed is transmitted. If the sound waves generated in a room meet solid plaster walls of sufficient rigidity they will suffer a reflection of over 99 per cent because of the large change in the elasticity and density between air and solids. If a ventilator opening is encountered instead of a wall there is no change in the medium and the waves progress with little hindrance through the continuous air passage, being confined in the ventilation duct by reflection from the metal walls. In a similar way sound vibrations generated in the solid matter of a building structure are confined to the structure by almost total reflection at the air boundaries and will pass with little interruption through the continuity of steel and concrete to distant parts of the building. These vibrations may be converted into sound waves in air where a wall or other structural member is set into sympathetic lateral vibration.

7. *Absorption of Sound.*—When air passages in which sound is passing become small in cross-section, friction that converts the wave energy into heat occurs between the sides of the passage and the oscillating air particles. Sound entering a small crack in a thick wall may be completely absorbed before emerging on the other side. The channels and interstices in carpets, hairfelt, and other porous materials, act in the same way in the absorption of sound energy.*

* Rayleigh, Lord. Behavior of Porous Bodies in Relation to Sound. "Theory of Sound," Vol. 2, Section 351.

The absorption and transmission of sound vary with the thickness of the absorbing material but not in direct proportion. For example, if 1 in. of hairfelt stops 10 per cent of the incident sound, 2 in. will stop 19 per cent, 3 in. 27 per cent, etc.; that is, the intensity of transmitted sound decreases according to the recognized exponential law: $i = i_0 a^{-x}$, where i_0 and i are the respective intensities of the sound that enters and is transmitted by the material, a is a constant, and x the thickness of the material.

The absorption of sound is an essential factor in the solution of sound insulation. It is not sufficient to reflect and scatter sound waves, for the energy cannot be destroyed in this manner; it must be absorbed, that is, converted by friction into heat energy.

8. *Transmission of Sound.*—Sound waves in air may be transmitted through an obstructing medium in three ways. First, they may pass through the air spaces of a porous material. Second, they may be transmitted by modified waves in the new medium. In this process sound compressions and rarefactions progress rapidly through the air, moving the molecules successively as they pass in somewhat the same way as a gust of wind blows the separate stalks in a wheat field. On reaching a solid partition the forward motion is hindered, particularly if the molecules of the new material are massive and resist compression. In this case most of the energy is reflected and only a small proportion progresses through the wall. On meeting a further discontinuity of material, such as wood or air, the waves are again affected, until finally a part of the energy emerges. Third, sound may be transmitted by setting a partition as a whole in vibration. The partition then acts as an independent source of waves, setting up compressions and rarefactions on the further side and giving a sort of fictitious transmission. If the partition is rigid and massive the vibrations are small and very little sound is transmitted; if the partition is thin and flexible a considerable amount of energy is transferred.* Usually in building constructions the partitions are complex, as for example plaster on wood lath and studding. In this case the plaster areas between the studding act in a manner similar to drum heads and transmit sound. Hard

* Rayleigh, Lord. "Theory of Sound." Loc. cit. See also:

Jäger, G. "Zur Theorie des Nachhalls." Sitzungsber. der Kaisl. Akad. der Wissenschaften in Wien. Math. Natur. Klasse, Bd. CXX, Abt. 2a, 1911.

plaster on wire lath presents a different surface with a modified action on the incident sound.

The transmission of sound involves a number of phenomena and is not a simple matter. It depends essentially on the character of the structure through which sound is transmitted and can be calculated only for simple cases of homogeneous materials of known constants.

III. INSULATION OF SOUND IN BUILDINGS

9. *Two Types of Sound in Buildings.*—Two types of sound should be considered in the problem of insulation in buildings. One type includes sounds that are generated in the air and that progress through the air to the boundaries of the room; the other is composed of compressions generated in the building structure by motors, elevators, and street traffic. The insulation of these disturbances is best accomplished by considering the actions of sound described in the preceding discussion. Several suggestions for soundproofing are given in the paragraphs immediately following.

10. *Insulation of Sounds in Air.*—Sounds of moderate intensity such as those generated by the human voice or a violin may be stopped with comparative ease if the walls of the room are continuous and fairly rigid. The more vigorous sounds of a cornet, trombone, etc. would require especially heavy walls or else double partitions. Any breaks in the walls for ventilators, pipes, or doors should be guarded by effective insulation.

11. *Insulation of Building Vibrations.*—Compressional waves generated in the building structure pass readily along the continuity of solid materials, and, as they have more paths for escape, are more difficult to insulate than sounds in air. Moreover, they may create trouble when they cause a wall or floor to vibrate. The insulation is based on the same procedure as that used for air sounds; namely, to interpose a new medium differing in elasticity and density. An air space in masonry would be effective if not bridged over by solid material, but since this is impossible for ordinary building constructions as the weight of bodies necessitates contact for support, an approximate insulation is sought by using air-filled substances like dry sand, ground cork, hairfelt, or flax, that possess but little rigidity but are capable of sustaining a floor or a partition that is not too heavy.

12. *Need for Experimental Data.*—The preceding discussion indicates from the standpoint of theory how sound waves will act under given conditions and how they may be controlled. Calculations may be made for a few simple cases of more or less homogeneous materials, but for the complex structures usually found in buildings

the present theory is inadequate to give trustworthy information. In view of this situation the procuring of data by direct experimental tests is quite desirable, particularly for the transmission of sound through materials both homogeneous and complex. Information about the absorption of sound is also needed. The following section gives in historical order descriptions of experimental investigations that have been completed or are now in progress.*

* See condensed account of experiments performed previous to 1909 in the article: "Schalleitung der verschiedenen Körper," in Winkelmann's "Handbuch der Physik," Vol. 2, pp. 554-562.

IV. EXPERIMENTAL INVESTIGATIONS OF ACTION OF MATERIALS ON SOUND

13. *Conduction of Sound by Solids.*—Tyndall describes an interesting and instructive experiment on the conduction of sound. He writes,* “In a room underneath this, and separated by two floors, is a piano. Through the two floors passes a tin tube $2\frac{1}{2}$ in. in diameter, and along the axis of this tube passes a rod of deal (wood), the end of which emerges from the floor in front of the lecture table. The rod is clasped by india-rubber bands which entirely close the tin tube. The lower end of the rod rests upon the sound board of the piano, its upper end being exposed before you. An artist is at this moment engaged at the instrument but you hear no sound. When, however, a violin is placed upon the end of the rod, the instrument becomes instantly musical, not, however, with the vibrations of its own strings, but with those of the piano. When the violin is removed, the sound ceases. . . . What a curious transference of action is here presented to the mind! At the command of the musician’s will, the fingers strike the keys; the hammers strike the strings, by which the rude mechanical shock is converted into tremors. The vibrations are communicated to the sound board of the piano. Upon that board rests the end of the deal rod, thinned off to a sharp edge to make it fit more easily between the wires. Through the edge, and afterwards along the rod, are poured with unflinching precision the entangled pulsations produced by the shocks of those ten agile fingers. To the sound board (of the violin) before you the rod faithfully delivers up the vibrations of which it is the vehicle. This second sound board transfers the motion to the air, carving it and chasing it into forms so transcendently complicated that confusion alone could be anticipated from the shock and jostle of the sonorous waves. But the marvelous human ear accepts every feature of the motion, and all the strife and struggle and confusion melt finally into music on the brain.”

Hesehus† investigated the sound conductivity of various solids. The samples were shaped so as to be geometrically similar and placed

* Tyndall, J. “Sound.” Pp. 78-79.

† Hesehus, N. “Ueber das Schalleitungsvermögen der Körper.” Jour. Russian Phys. Chem. Soc., Vol. 17, pp. 326-30, 1893. See also abstract in Fortsch. der Physik, Vol. 43, p. 542, 1893.

successively with the lower end resting on a resonating support. A vibrating tuning fork was then placed against the upper end and the relative conduction of sound was made manifest by the intensity of the resonance tone heard. He found that the conductivity for cylindrical bodies was directly proportional to the cross-section and inversely proportional to the length—the same relation that holds for conduction of heat and electricity. He found also that wood conducts sound much better parallel to the grain of the wood than at right angles. This fact suggests a construction for hindering the progress of sound waves, namely, a series of wooden slabs cut alternately parallel and perpendicular to the grain of the wood. If the slabs were separated by layers of felt and not nailed, the obstruction would appear quite efficient.

14. *Experiments by Tufts.*—The transmission of sound by porous materials was tested in 1901 by F. L. Tufts.* For the initial experiment the porous material consisted of lead shot piled in a tube. This was exposed to the action of sound through a metal gauze that supported the shot. A thin rubber membrane closed the bottom of the tube below the gauze and by the amplitude of its vibration gave a measure of the sound waves transmitted. Cotton batting, felting, and other porous materials were tested in the same manner. Tufts concluded that the resistance of a porous material to sound was in the same proportion as the resistance to air currents.

In a later experiment he measured the transmission of sound by materials impervious to air.† Two thin discs of such material closed the ends of a bifurcated tube. Sound pulses, caused by a metal ball dropped on a pine board, impinged on the discs so that the transmitted waves passed on through the tubes to a junction point, where an observer could compare the loudness of the two sounds by alternately closing one tube and then the other. The materials were tested in pairs, each sample being in the form of a disc, 10.5 cm. in diameter. Table 2 gives the results obtained.

* Tufts, F. L. "The Transmission of Sound Through Porous Materials." Am. Jour. of Sc., Vol. 11, p. 357, 1901.

† Tufts, F. L. "Transmission of Sound Through Solid Walls." Am. Jour. of Sc., Vol. 13, pp. 449-454, 1902.

TABLE 2
TRANSMISSION OF SOUND BY MATERIALS IMPERVIOUS TO AIR

Material	Thickness in cm.	Rigidity*	Transmitted Sound
1. Lead	0.012	0.000106	Louder
Glass	0.012	0.000053	
2. Pine	0.65	0.000013	Louder
Leather	0.65	0.000212	
3. Reinforced brass	0.015	0.00008	{ Equal loudness
Two thicknesses of cardboard	0.44	0.00008	
4. 10 sheets cardboard	0.70	0.0002	{ Equal loudness
1 sheet cardboard	0.22	0.0002	
5. Reinforced brass	0.015	0.00008	Much louder
Brass with mass at center	0.015	0.0022	
6. Cardboard	0.22	0.0004	Louder
Cardboard with 34 gm. mass at center	0.22	0.0004	
7. 3 cardboards separated by air space	cardboards all cut from same sample		Louder
3 cardboards in contact			

*The numbers in the rigidity column represent the displacement of the discs for a pressure of 1 gm. per sq. cm., the smaller values thus indicating the greater rigidities.

A study of this table leads to the following conclusions:

1. The more rigid glass is the better insulator;
2. The more rigid pine is the better insulator;
3. For the same rigidity the cardboard, 30 times thicker than the brass, transmits the same amount of sound;
4. For the same rigidity 10 sheets of cardboard transmit the same sound as one sheet;
5. The more massive disc with smaller rigidity transmits more sound;
6. For equal rigidity the more massive disc is the better insulator;
7. The cardboards in contact gave a greater rigidity and cut off more sound. The air spaces are thus not as effective as was supposed.

Inspection of these results, therefore, shows that rigidity is the deciding factor, the amount of sound transmitted as an elastic wave being negligible compared with the to-and-fro vibration of the material. For instance in case 4, where the two discs possess the same rigidity, one would expect the transmission to be greater through the single thickness of cardboard than through the built up disc with many reflecting surfaces; but the transmitted sound is the same for the two constructions.

A few additional experiments were conducted using an organ pipe for the source of sound, thus giving a sustained series of alternate compressions and rarefactions instead of a single pulse. It was found that the results might be influenced by the pitch of the note used, so that, if a disc happened to be set into resonant vibration, it would transmit sound to a greater extent than without such resonance.

15. *Investigation by Weisbach.*—A tuning fork giving 236 vibrations per second was used as the source of sound and placed on one side of a wall. A telephone connected to a galvanometer was placed on the other side of the wall and served as a receiver for the sounds transmitted through materials installed over a small hole in the wall.* Weisbach found that metal, wood, etc. vibrated, whereas cloth and porous materials did not. When the disc was free from resonant vibrations the transmitted sound was proportional to the mass of the disc per unit area. When resonance occurred, the energy transmitted by the lateral vibrations was so great that it obscured the other effects, the results depending on the size, structure, and method of fastening the material. He concluded that sound is transmitted through the pores of materials, also by longitudinal vibrations (elastic waves) and by transverse vibrations of the materials.

16. *Tests by McGinnis and Harkins.*—In this investigation, the source of sound was an organ pipe of 768 vibrations per second, the measuring instrument being a telephone receiver in series with a crystal rectifier and a galvanometer.† The materials tested were clamped over an aperture $\frac{1}{2}$ in. in diameter, through which the sound was transmitted. For porous materials such as calico and felt the investigators concluded that the amount of sound transmitted depended on the size and nature of the pores of the material, that is, on the true absorption. The non-porous materials like paper, oilcloth, and tin-foil transmitted less than one per cent of the incident sound when the lateral vibrations were eliminated. All the materials tested were quite thin and of small area.

17. *Work of Wallace C. Sabine.*—An investigation,‡‡ fundamental and far-reaching in its scope, was inaugurated and partly

* Weisbach, F. "Versuche über Schalldurchlässigkeit, Schallreflexion und Schallabsorption," *Annalen der Physik*, Vol. 83, p. 763, 1910.

† McGinnis, C. S. and Harkins, M. R. "Transmission of Sound Through Porous and Non-Porous Materials." *Phys. Rev.*, Vol. 33, pp. 128-136, 1911.

‡‡ Sabine, W. C. "The Insulation of Sound." *The Brickbuilder*, Vol. 24, pp. 31-36, 1915.

carried out by the late Professor W. C. Sabine of Harvard University. A sound of measured intensity was generated in a room so that transmission took place through a partition under test. The sound was then reduced in intensity until an observer outside the partition could barely hear it. The test was repeated for other samples and comparative values deduced. The test rooms were especially adapted to confine the transmission of sound to the test material.

Table 3 gives results obtained in the preliminary investigation. Sheet iron and hairfelt were tested separately and in combination. The sheet iron because of its great rigidity and mass reflected much sound and transmitted but little. The reduction of transmission by the hairfelt was due largely to its absorbing power. The combination of the two in alternate layers gave the more effective insulation, the hairfelt serving to reduce further the small amount of sound transmitted by the iron.

TABLE 3
INTENSITY OF SOUND TRANSMITTED BY SHEET IRON AND HAIRFELT

Layers	Hairfelt	Sheet Iron	Sheet Iron and Hairfelt Combined
0	1 000 000	1 000 000	1 000 000
1	270 000	22 700	23 000
2	128 000	8 700	3 300
3	65 000	4 880	700
4	33 000	3 150	220
5	21 500	2 060	150
6	11 400	1 520	88

18. *Investigations by P. E. Sabine.*—Later experiments on sound transmission through windows and doors have been conducted by P. E. Sabine,* using methods similar to those of W. C. Sabine. He found that flexural vibrations play an important part in transmission. A plate of glass $\frac{3}{16}$ in. thick transmitted less sound than a $\frac{3}{16}$ -in. composite structure of two $\frac{1}{16}$ -in. plates sealed together with a sheet of celluloid. Dead air spaces between double windows were not as effective as commonly supposed. The air space should not be bridged over by solid materials even at the edges of the glass. Inserting sound absorber between the glass plates increased the insulation. A steel door $\frac{1}{4}$ in. thick proved more effective than a solid oak door $1\frac{3}{4}$ in. thick or a refrigerator door 4 in. thick filled with heat-insulation material.

* Sabine, Paul E. "Architectural Acoustics." Am. Arch., Vol. CXVIII, pp. 102-108, 1920.

Additional experiments were conducted to test the transmission of sound through flexible materials.* Results were obtained to determine to what extent the damping of sound by such materials as hairfelt, sea-weed, and asbestos is an effective compensation for lack of mass and stiffness. Numerous curves are given showing the transmission of sound for a range of pitches. Further investigation was planned to test more rigid materials, including plaster walls.

19. *Sound Deadening in Hospitals.*—Richard E. Schmidt gives a discussion† of sound transmission in buildings, paying particular attention to the character of building materials with acoustic properties suitable for use in hospital construction. The article gives sketches of rooms and photographs of buildings in Chicago that have sound-proofing features.

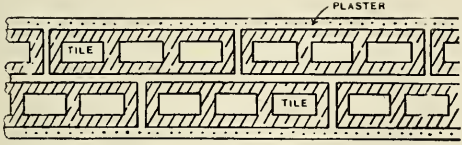
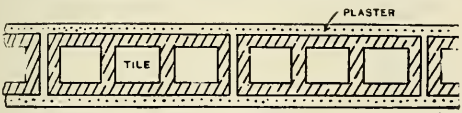
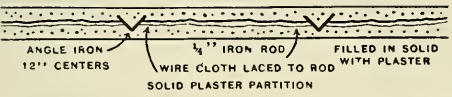
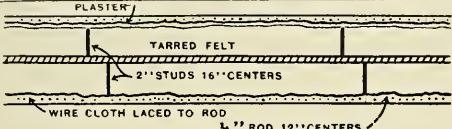
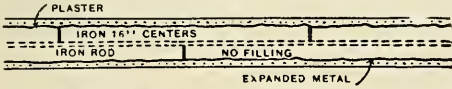
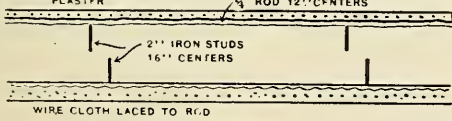
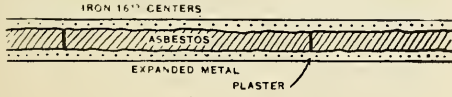
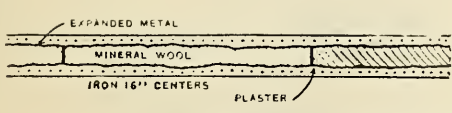
20. *Sound-Proofing Tests Made in Chicago Music Building.*—Table 4 pictures the construction and relative sound-proof efficiency of various types of building constructions. This table appears in the 1911 catalogue of the Corrugated Bar Company. It has been impossible to locate any description of the tests other than the information given in the table, but the results set forth are so thoroughly in accord with the conclusions reached in other investigations that the data are included as additional information.

* Sabine, P. E. "Architectural Acoustics. The Transmission of Sound Through Flexible Materials." *Am. Arch.*, Vol. 120, pp. 215, 266, 1921.

† "Sound Deadening in Hospitals." *Surgery, Gynecology, and Obstetrics*, Vol. XXXI, pp. 105-110, July, 1920.

TABLE 4

COMPARATIVE SOUND-PROOF TESTS MADE AT MUSIC BUILDING,
CHICAGO, IN 1895.

TYPE OF PARTITION	Rank in Sound Proof Efficiency	
	1.	COMMON MORTAR (CEMENT)
	2.	COMMON MORTAR (CEMENT)
	3.	COMMON MORTAR (CEMENT)
	4.	ROCK PLASTER COMMON MORTAR (CEMENT)
	5. *	ACME PLASTER COMMON MORTAR (CEMENT)
	5. *	COMMON MORTAR (CEMENT)
	5. *	ACME PLASTER COMMON MORTAR (CEMENT)
	5. *	ACME PLASTER COMMON MORTAR (CEMENT)

* Sound carried through due to metal connections.

V. APPARATUS AND METHODS USED IN PRESENT INVESTIGATION

21. *General Methods and Apparatus.*—The experiments on which this bulletin is based have extended over a period of seven years, but have been confined largely to three periods. The method used remained essentially the same throughout, but the apparatus and conditions were improved greatly for the later tests. Materials tested had considerable area—at least 3 ft. by 5 ft.—and varied in structure

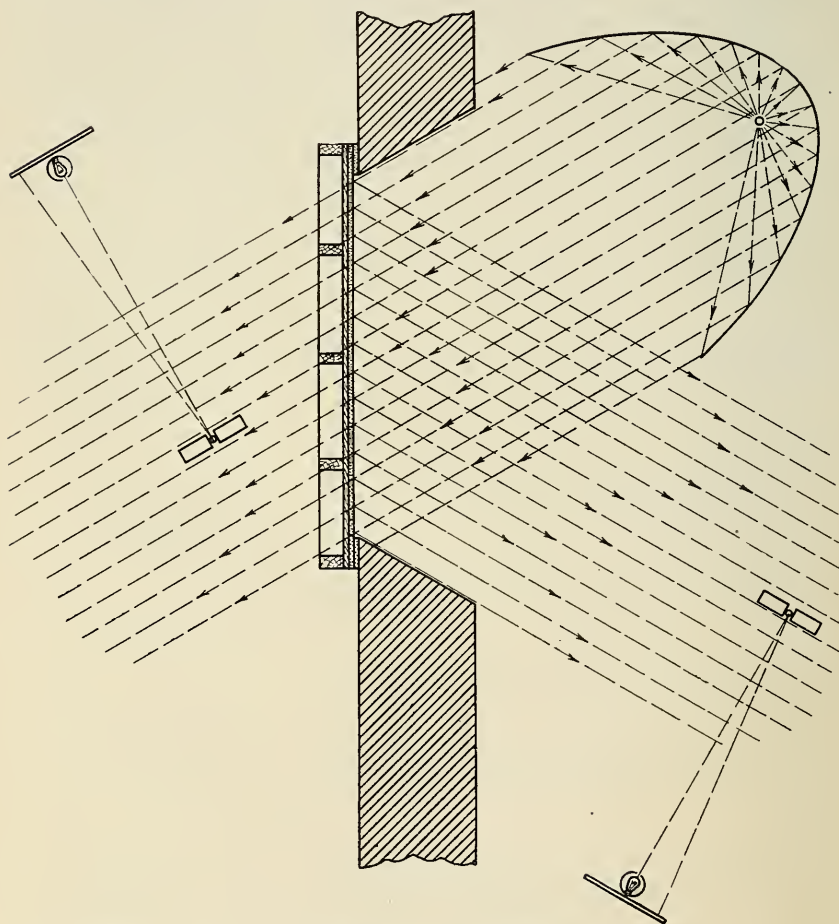


FIG. 3. DIAGRAM OF APPARATUS FOR TESTING TRANSMISSION AND REFLECTION OF SOUND

from thin flax and hairfelt to solid plaster partitions. An adjustable metal organ pipe blown by air from a constant pressure tank served as the source of sound. This was mounted at the focus of a parabolic reflector 5 ft. in diameter (see Fig. 3), so that a large percentage of the generated sound was directed in a parallel bundle against the partition under test. The sound was partly reflected and partly transmitted, the amounts of transmission and reflection being measured simultaneously by Rayleigh disc resonators placed in the paths of the sound.

22. *The Rayleigh Disc Resonator.*—The simplest form of this instrument consists of a tube closed at one end and open at the other. (See Fig. 4.) Sound waves of suitable frequency, coming

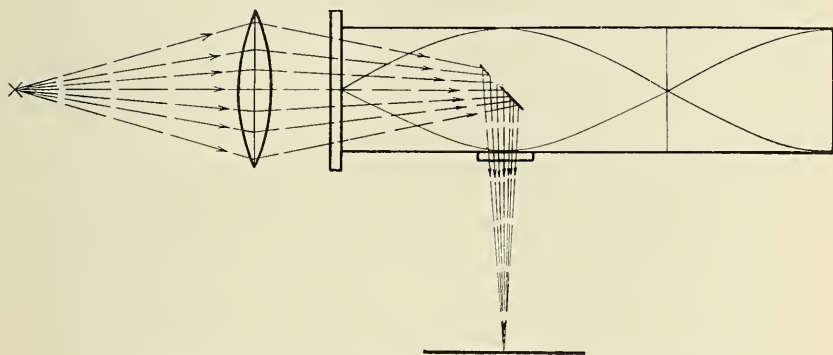


FIG. 4. DIAGRAM OF SIMPLE RAYLEIGH RESONATOR

to the instrument, set up standing waves in the tube with a considerable back and forth surging of the air at a "loop." A thin circular mica (or glass) disc suspended at this loop at an angle to the axis of the tube tends to turn flatwise to the alternating air currents. If a fine quartz thread is used to suspend the disc and the latter is placed at an angle of 45 deg. to the axis, a delicate means is provided for measuring sound, the amount of rotation of the disc varying from 0 deg. to 45 deg. depending on the intensity of the incident sound. This turning action is in accordance with the general principle that a flat object in a current tends to set itself flatwise to the current. Any turning of the disc is made visible by the movement of a beam of light reflected from its surface to a scale. Theory shows that

the comparative intensities of two sounds acting separately on the instrument are given by the equation:*

$$\frac{I_1}{I_2} = \frac{\phi_1 \sin 2 (\theta - \phi_2)}{\phi_2 \sin 2 (\theta - \phi_1)}$$

where I_1 and I_2 are the intensities of the sounds, ϕ_1 and ϕ_2 the corresponding deflections of the disc, and θ (usually 45 deg.) is the angle between the normal to the disc and the axis of the tube.

Since the intensities of the transmitted sounds varied over a wide range during the tests, several resonators of different sensitivity were required to obtain the measurements. For the faint sounds transmitted by solid plaster partitions, it was necessary to develop a sensitive double resonator† of the proper frequency to obtain measurements. (See Fig. 5.) This latter instrument measured sounds of "threshold audibility," that is, sounds so faint that they could barely be detected by the ear. The resonators are much more sensitive than the ear in measuring sounds of different intensities. This is evidenced by the need of several resonators of different sensitivity to measure the same range detected by the ear. The hearing sensation is proportional to the logarithm of the intensity and is therefore quite insensitive.

* Watson, F. R. "An Investigation of the Transmission, Reflection, and Absorption of Sound by Different Materials." *Phys. Rev.*, Vol. 7, pp. 125-132, 1916.

† Rayleigh, Lord. "Note on the Theory of the Double Resonator." *Phil. Mag.*, Vol. 36, pp. 231-234, 1918.

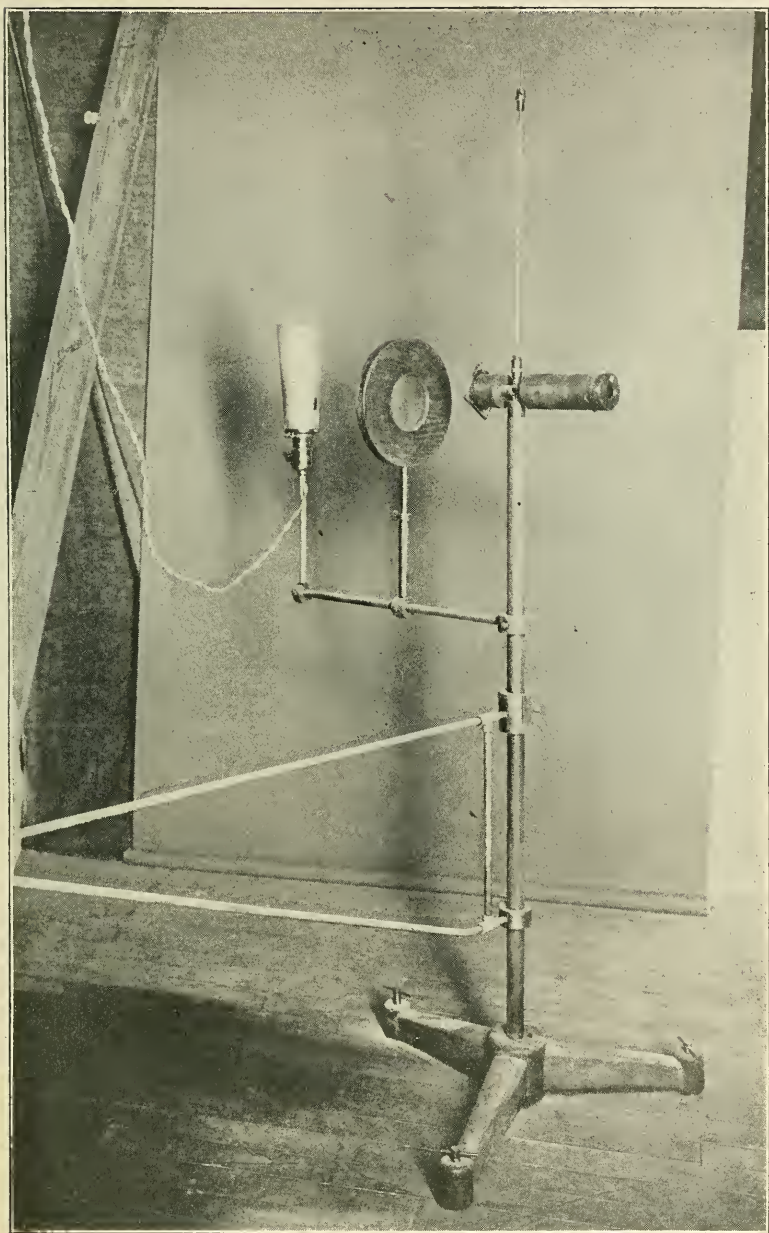


FIG. 5. SENSITIVE RAYLEIGH RESONATOR FOR MEASURING FAINT SOUNDS

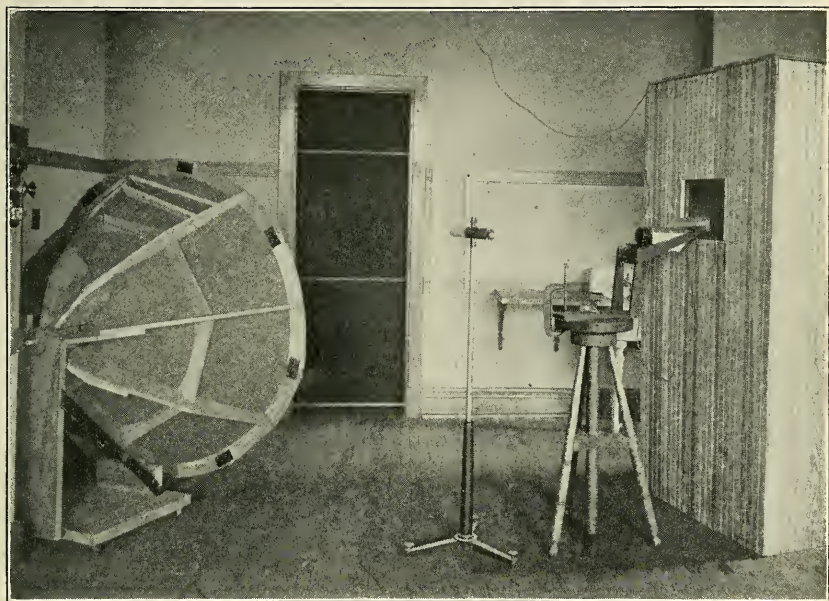


FIG. 6. APPARATUS USED IN INVESTIGATION

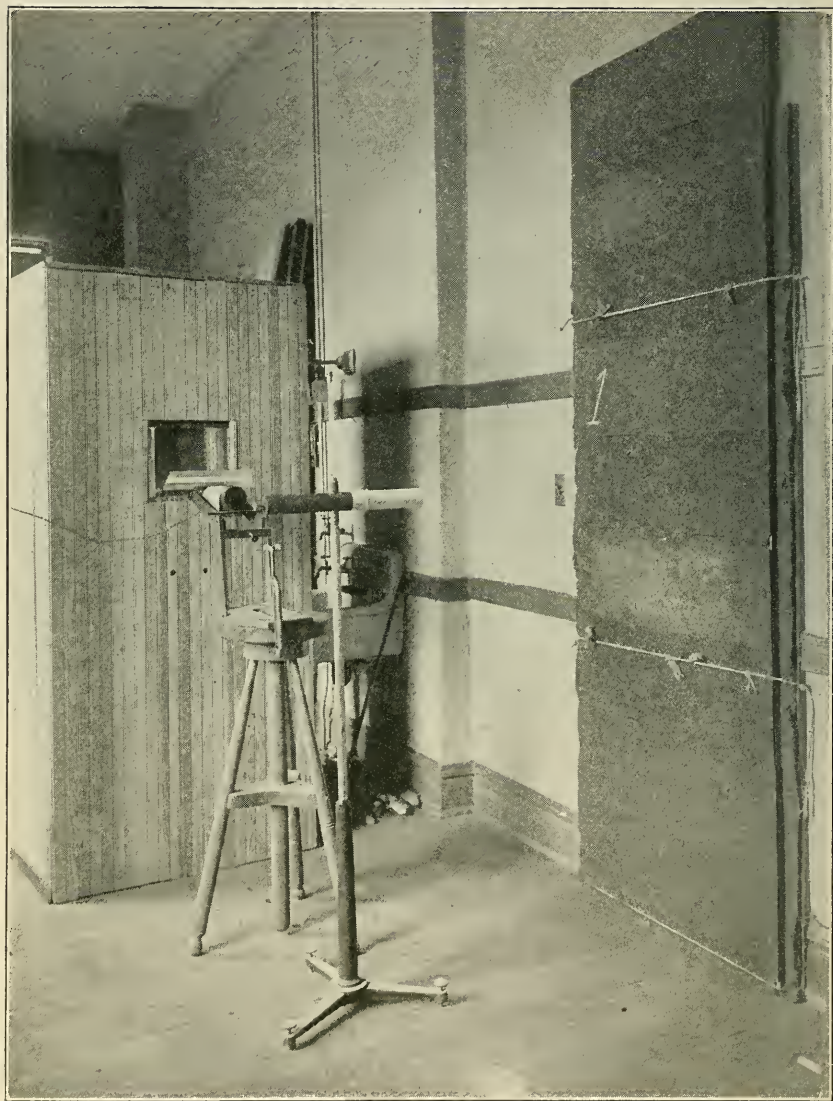


FIG. 7. APPARATUS FOR MEASURING TRANSMITTED SOUND

VI. PRELIMINARY INVESTIGATION

23. *Arrangement of Apparatus.*—Figures 6 and 7 illustrate the conditions for the first test. An organ pipe blown steadily by constant air pressure generated a sound that was transmitted through a doorway into an adjacent room where a Rayleigh resonator measured the energy. Measurements were taken, first through the open doorway, then with one panel of the material over the doorway, next with two panels, and finally with three panels; the deflection of the resonator being noted for each case. Since the tests were comparative, all conditions were maintained as constant as possible. Every article in the room was left undisturbed so that the interference pattern due to reflection of sound from various surfaces would remain unchanged and not affect the readings of the resonator. To eliminate the effect of the observer, who necessarily must have some freedom of motion, a small booth was built with a glass window. The observer could shut himself in this compartment and take readings through the window. The samples to be tested were fastened on similar frames of 1-in. cypress and mounted over the doorway by two ropes. A strip of hairfelt was installed around the door frame to prevent leakage of sound at the edges.

24. *Experimental Results Obtained.*—Table 5 gives results for the reflection and transmission of sound.

TABLE 5
TRANSMISSION AND REFLECTION OF SOUND

Material	Deflection of Resonator in cm.							
	Reflection				Transmission			
Thickness in layers	0	1	2	3	0	1	2	3
Open Doorway	3.9				39.4			
$\frac{1}{2}$ -in. Hairfelt		4.9	6.6	10.5		22.6	15.4	10.4
$\frac{1}{4}$ -in. Cork Board		15.7	22.0	22.6		7.9	3.75	2.9
$\frac{1}{4}$ -in. Cork Board		25.9	21.2	22.1		1.15	2.05	0.85
$\frac{1}{4}$ -in. Paper-lined Hairfelt		20.7	5.9	10.0		5.0	21.7	3.8
$\frac{3}{4}$ -in. Paper-lined Hairfelt		10.4	6.6	9.3		6.5	1.95	0.4
$\frac{3}{4}$ -in. Flax Board		22.5	20.0	20.0		2.25	0.55	0.1
$\frac{1}{4}$ -in. Pressed Fiber		23.2				0.32		
$\frac{3}{4}$ -in. Pressed Fiber						0.2		

25. *Discussion of Results.*—The results show the effects of different materials on sound waves. Porous hairfelt transmits con-

siderable sound, and the reflection is small. Other samples, impervious to air, reflect more and transmit less. A better conception of the results is obtained by considering the relative values of the transmission and reflection together with the absorption. Curves for these values may be plotted on the same scale by assuming that the open doorway transmits 100 per cent of the incident sound and reflects 0 per cent; also that the maximum reflection value (25.9 for the $\frac{3}{4}$ -in. cork board) may be taken as 100 per cent. These assumptions are not rigorously correct, but they allow comparisons that are near the truth.*

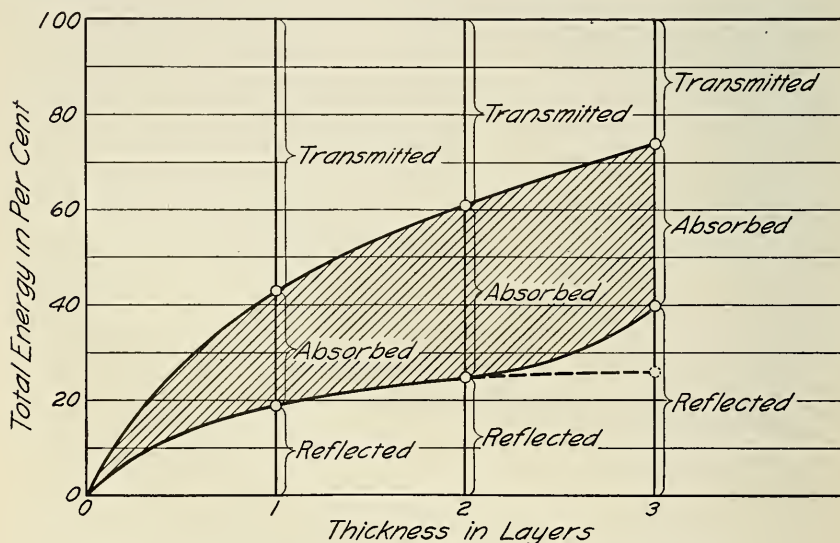


FIG. 8. RELATIVE AMOUNTS OF SOUND REFLECTED, ABSORBED, AND TRANSMITTED BY HAIRFELT

26. *Action of Porous Materials.*—Fig. 8 shows the results for $\frac{1}{2}$ -in. hairfelt. Due to absorption of energy in the pores of the material the transmission decreases with increasing thickness, according to the law stated in paragraph 7. The reflection increases with increasing thickness but tends toward a constant value, indicating that the reflection does not take place entirely at the surface of a porous material but requires a certain thickness to give the maximum value. The absorption also appears in this diagram because the

* Watson, F. R. Loc. cit.

incident sound ($I = 100$ per cent) is equal to the sum of the percentages reflected, absorbed, and transmitted.

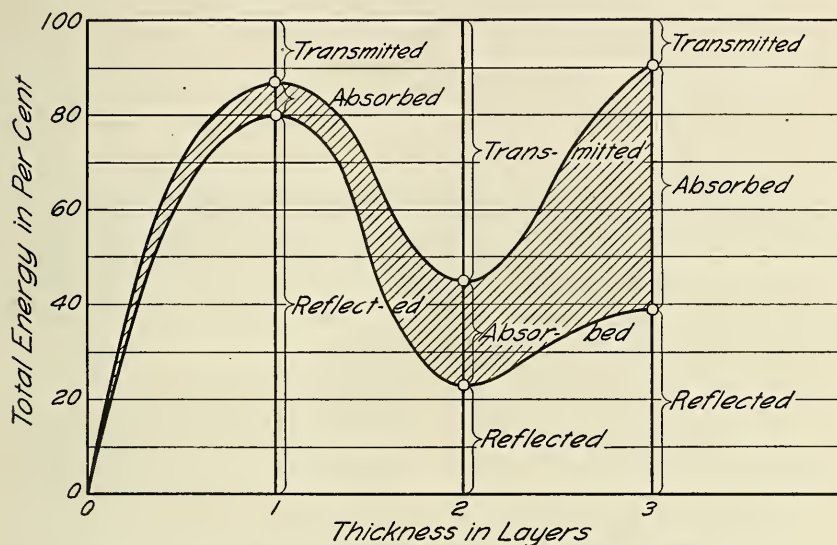


FIG. 9. TRANSMISSION, REFLECTION, AND ABSORPTION OF SOUND BY VIBRATING PARTITION

27. *Action of Materials Impervious to Air.*—Fig. 9 shows results that at first were quite puzzling. Two layers of paper-lined felt transmitted *more* sound than one layer; also, the reflection was *less* for two layers than for one. The explanation is easily seen when the transmission and reflection are plotted together. The two layers *vibrated* under the action of the sound so that the transmission was disproportionately large. An extreme case of this kind would exist if a material vibrated exactly as the air would if the material were not present. There would then be no resistance to the sound waves, hence no reflection, and all the sound would be transmitted.

28. *Conclusions.*—The information given in Figs. 8 and 9 is quite vital in the consideration of sound-proof structures. Fig. 8 shows the normal reflection, absorption, and transmission to be expected for compressional waves passing from one medium through another. Fig. 9 shows the anomalous effects set up when a layer of material vibrates as a whole. This is the usual case in building constructions where partitions vibrate more or less under the action of sound and give unexpected results. The obvious procedure to avoid this complication is to use rigid partitions and minimize the vibration.

VII. INVESTIGATION OF THIN PLASTER PARTITIONS

29. *Arrangement of Apparatus.*—This set of tests was conducted in two basement rooms separated by a double wall consisting of two 9-in. brick partitions with separating air space. The walls enclosing the rooms were of heavy brick construction, the ceilings consisted of 4-in. concrete slabs, and the wood floors rested on concrete. A window cut through the double wall between the rooms was lined with cement plaster and allowed the sound generated in one room to pass directly to the partition under test. (See Figs. 3 and 10.) The reflected sound and the sound transmitted into the neighboring room were measured by Rayleigh resonators. The source of sound was an adjustable Edelmann pipe giving about 630 vibrations per second. The double doors connecting the rooms were padded with felt and arranged to shut as tightly as possible. The arrangements proved to be quite satisfactory for the experiments. Air currents were eliminated and disturbing outside sounds were reduced to a minimum. Any transmission of sound from one room to the other was thus confined to the sample under test.

30. *Materials and Method of Measurement.*—The materials tested varied in structure from porous hairfelt to plaster coating on wood lath. Since the measurements were to be comparative all conditions were maintained as constant as possible, the only changing factors being the samples under test. When taking measurements, the observer entered a small booth built for the purpose and closed the door. Each panel had the same size, 3 ft. by 5 ft., and was clamped in place over the window in the same manner by a wooden frame with iron bars and tightening nuts. Collapsible samples were held in place by wire netting. The range of intensities of sound transmitted was so great that two Rayleigh resonators of different sensitivity were used, intermediate values being taken by both resonators to give continuity of results. The reflected sound was measured simultaneously with the transmitted sound, thus giving check measurements, since any variation in conditions affected both the transmission and the reflection.

31. *Results Obtained.*—The plaster samples were constructed by an expert workman and allowed to dry before testing. They were

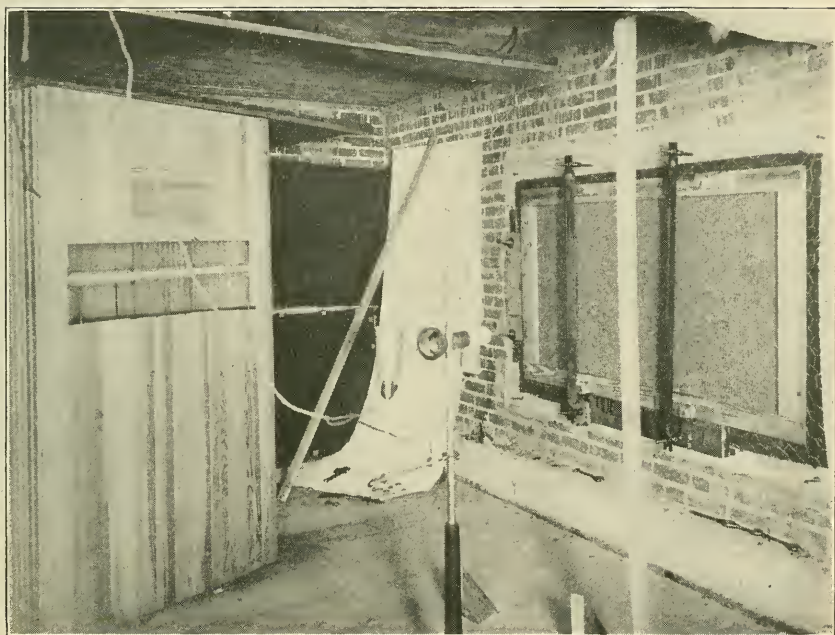


FIG. 10. VIEW OF TEST ROOM, SHOWING SAMPLE PARTITION
CLAMPED OVER WINDOW

then tested one after the other under conditions kept as uniform as possible. The measurements were repeated five times, thus giving six observations for each result noted. In some cases of doubt, further measurements were taken. Table 6 indicates the results obtained. The first column of figures gives the actual deflections observed for transmitted sound, the three Sackett boards being tested by both resonators to establish the connecting ratio between the two sets of readings. The second column gives relative transmission values for the entire set of materials. These values were obtained by reducing proportionately the numbers in the second set so that the readings for the Sackett boards were the same in both sets. The third column gives deflections for the reflected sound, the same resonator being used throughout for these measurements.

32. *Discussion of Results.*—The results show that the porous, burlap-lined flax and the hairfelt transmit considerable sound and reflect little. Paper-lined materials—the Keystone Hair Insulator and Cabot Quilt—transmit less and reflect more. The Sackett boards are the most efficient sound insulators of the thinner samples, while in the plaster panels those containing gypsum plaster appear to be more effective in stopping sound. This is probably due to the fact that gypsum produces a stiffer, more rigid structure. Some estimation of the absorption of sound may be formed by remembering that the sum of the transmitted, reflected, and absorbed sound equals the constant incident sound. If the transmission and reflection are both small, as with the mineral wool for example, the absorption is correspondingly large. Thus with the plaster panels the reflection is extremely large, and the transmission and absorption correspondingly small.

TABLE 6
DEFLECTIONS OF RESONATOR FOR TRANSMITTED AND REFLECTED SOUND

Material	Transmission		Reflection
$\frac{1}{8}$ -in. Flax, burlap-lined	69.4	69.4	5.2
$\frac{1}{8}$ -in. Hairfelt	59.7	59.7	6.0
$\frac{1}{8}$ -in. Paper-lined hairfelt	35.2	35.2	14.7
$\frac{1}{4}$ -in. Cabot Quilt single ply	34.3	34.3	15.6
$\frac{1}{8}$ -in. Building paper	12.7	12.7	32.3
$\frac{1}{2}$ -in. Flax board	6.5	6.5	26.2
$\frac{1}{4}$ -in. Sackett board	0.36	0.36	41.9
$\frac{3}{8}$ -in. Sackett board	0.26	0.26	42.2
$\frac{1}{2}$ -in. Sackett board	0.15	0.15	42.7

TABLE 6 (CONTINUED)

TRANSMISSION OBTAINED WITH MORE SENSITIVE RESONATOR

Material	Transmission		Reflection
1/4-in. Sackett board	28.3	0.36	
3/8-in. Sackett board	23.4	0.26	
1/2-in. Sackett board	9.4	0.15	
2-in. Mineral wool	18.5	0.22	16.9
2-in. Gypsum furring strips	6.1	0.073	45.8
PLASTER PANELS			
Lime base, gypsum finish	5.2	0.062	45.2
Sanded gypsum base, lime finish	6.5	0.078	45.1
Sanded gypsum base, gypsum finish	3.5	0.042	45.8
Wood fiber base, gypsum finish	3.0	0.036	46.0
Wood fiber base, lime finish	1.8	0.022	46.1

Since the materials in this series of tests were compared under circumstances maintained as uniform as possible, the results obtained are valuable for guidance when selecting material and constructions for sound-proofing purposes.

VIII. MISCELLANEOUS TRANSMISSION TESTS

33. *Test Rooms and Apparatus.*—The apparatus and arrangements for these tests were quite similar to those used in the previous ones. (See Fig. 11.) The apparatus was rearranged for convenience

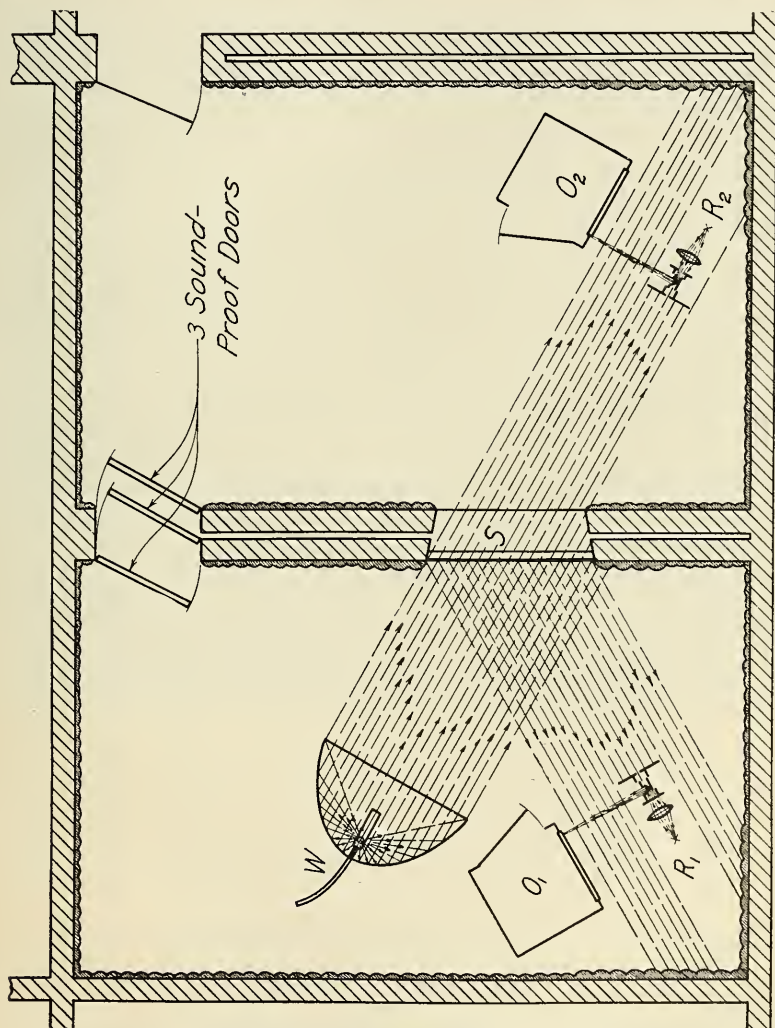


FIG. 11. ARRANGEMENT OF TEST ROOMS AND APPARATUS

and to guard the sensitive resonators from disturbance when workmen were about. The source of sound was an adjustable metal organ pipe designed for the tests and arranged to give 512 vibrations per second, this being considered an average pitch for sounds ordinarily met with in buildings. The organ pipe was blown by air from a constant pressure tank so that throughout the tests the pitch was maintained at 512 vibrations per second, with a maximum variation of 3 vibrations per second as shown by comparison with a standard tuning fork at different times. A double Rayleigh resonator developed to a high degree of sensitivity served to measure the faint sounds transmitted by the thicker wall. (See Fig. 5.) It gave responses for sounds that could barely be detected by the ear, thus furnishing a desirable instrumental substitute for the hearing, because the ear is untrustworthy in its quantitative comparison of sounds of different intensity.

34. *Further Details of Arrangements.*—The amount of sound transmitted was small, so special care was taken to minimize the effect of any leakage of sound. As already explained, the bulk of the sound was directed by means of a reflector against the partition under test. The reflected sound was then largely absorbed by padding, especially by additional layers hung on the walls first struck by the reflected waves. Figs. 12 and 13 show the details.

The doorway between the rooms received special attention. Two sound-proof doors were installed in the wall of the inner room, the door casing cracks being stopped by hairfelt and plaster of Paris. These proved sufficient for the preliminary tests, but when the thicker plaster partitions were tested a small transmission of sound became noticeable. A third sound-proof door was therefore mounted in the second brick wall, after which no leakage of sound through the door could be detected by the ear.

The sound transmitted by the test partition passed directly to the Rayleigh resonator, where it produced its effect, and after this it was absorbed by padding as in the other room. Investigation with the ear verified the actions just described. It showed that an intense sound was directed to the partition and that it was then reflected strongly, the paths of the waves being as shown in Fig. 11.

Since the measurements were to be comparative, the effort was made, as in the previous tests, to maintain constant conditions. Articles of furniture and draperies in the rooms were kept fixed in position so that the interference pattern would remain unchanged. The observer placed himself in a small booth where he could take

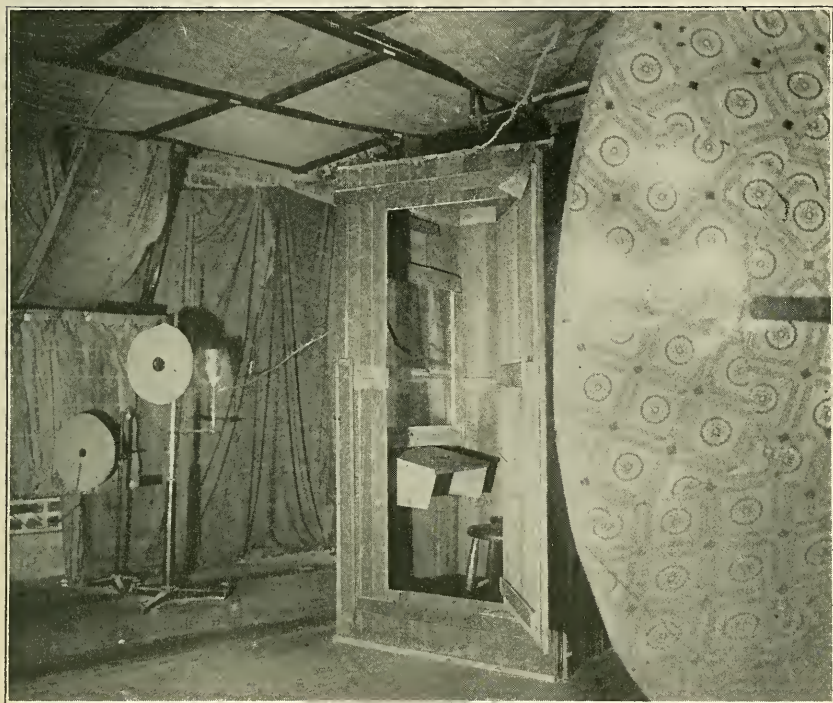


FIG. 12. VIEW OF TEST ROOM, SHOWING RAYLEIGH RESONATOR AND
BOOTH FOR OBSERVER

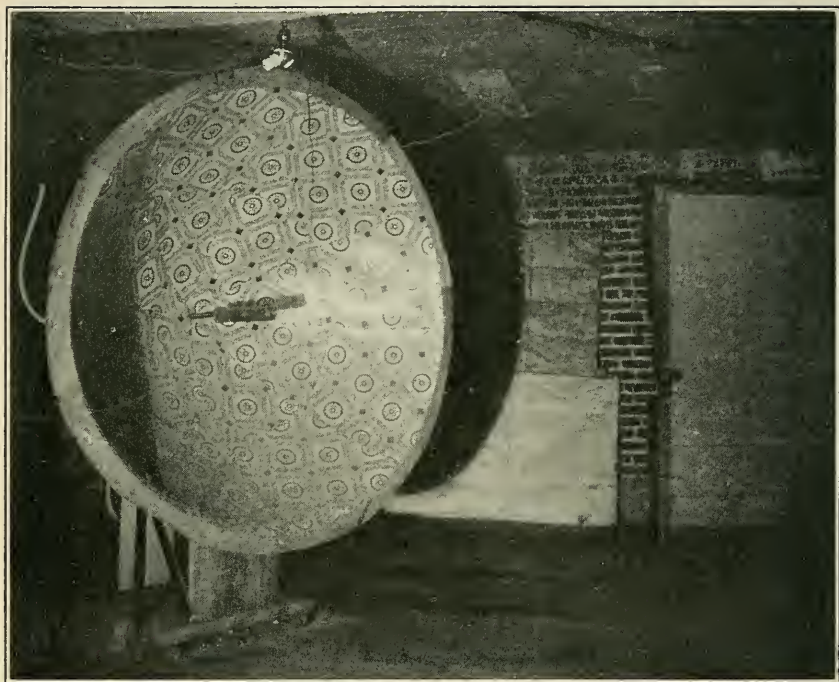


FIG. 13. VIEW OF TEST ROOM, SHOWING PARABOLIC REFLECTOR
LINED WITH LINOLEUM

readings through a glass window without disturbing the action of the sound. After the initial adjustments the organ pipe and resonator were not modified. The pressure tank and the pipe connections were set up permanently at the beginning of the experiments. Changes were made only when one partition was taken out and its successor built in the wall in the same position. Where the partitions were of equal thickness the measurements were strictly comparable. The tests were conducted as rapidly as was consistent with accuracy in order to secure fairly uniform conditions of temperature and humidity, the chief delaying factor being the time required for the plaster coats to dry. The tests on the plaster walls were made during the months of June and July, 1919.

Additional measurements were taken throughout by methods other than the one described. While these extra measurements are not considered as reliable as those given by the Rayleigh resonator, they are in the same order and thus furnish an independent check on the results.

35. *Preliminary Test on Transmission of Sound Through Outing Flannel.*—It was the intention in this test to make certain that the apparatus and method would be applicable to plaster partitions, particularly those of some thickness. It was planned also to establish if possible the law connecting the intensity of the transmitted sound with the thickness of the partition. For this latter purpose a preliminary experiment was performed on the transmission of sound through outing flannel. One, two, three, four, five, and six layers of this material were mounted in succession over a metal lath core installed in the opening between the two rooms, the transmitted sound for each case being measured by a Rayleigh resonator. The results are shown graphically in Fig. 14, where the intensities of the transmitted sound are plotted against the respective thicknesses. If this were a case of pure absorption, expressible by the equation $i = i_0 a^{-x}$, as in paragraph 7, the logarithms of the intensities plotted against the thicknesses would give a straight line. The logarithms plotted for this case are shown in Fig. 14, the small departure from a straight line being explained simply by the fact that the reflected sound was not constant, but became greater with increasing thicknesses, so that the intensity of the transmitted sound fell off more rapidly than it would by absorption alone. The success of this experiment verified the various assumptions concerning the method and apparatus and gave confidence for further tests with denser walls.

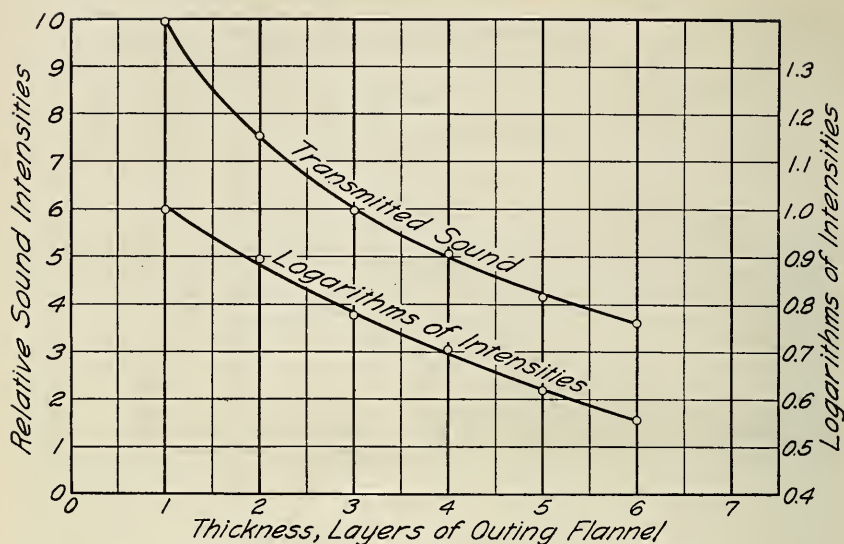


FIG. 14. TRANSMISSION OF SOUND BY OUTING FLANNEL

36. *Transmission Through Thin Plaster Partition.*—Plaster was then applied in place of the outing flannel (see Fig. 15), and it proved to be so much more effective in stopping sound that the resonator used with outing flannel gave no readable deflection, making it necessary to install one much more sensitive—the resonator pictured in Fig. 5—before a reading could be obtained. Measurements were taken on this panel for a week, until the plaster dried. More plaster was then applied with a brush to the back of the panel in order to fill the holes between the keys, thus producing a fairly uniform thickness of about $\frac{1}{2}$ in. For this increased thickness the resonator gave a deflection about one-sixth as great as for the thinner layer. Further measurements were then taken for nine successive days, the results being shown graphically in Fig. 16, which displays some interesting features. The transmission increased during the second and third days after the first coat of plaster was applied, due probably to some change in the plaster as it set and dried. This phenomenon is shown for the second coat of plaster also.

37. *Effect on Transmission of Sound When Pressure is Applied to a Thin Partition.*—Finally, the transmission of sound was measured when this panel was subjected to successively increasing pressures until it broke. The pressures were applied by a 2-in. board and two

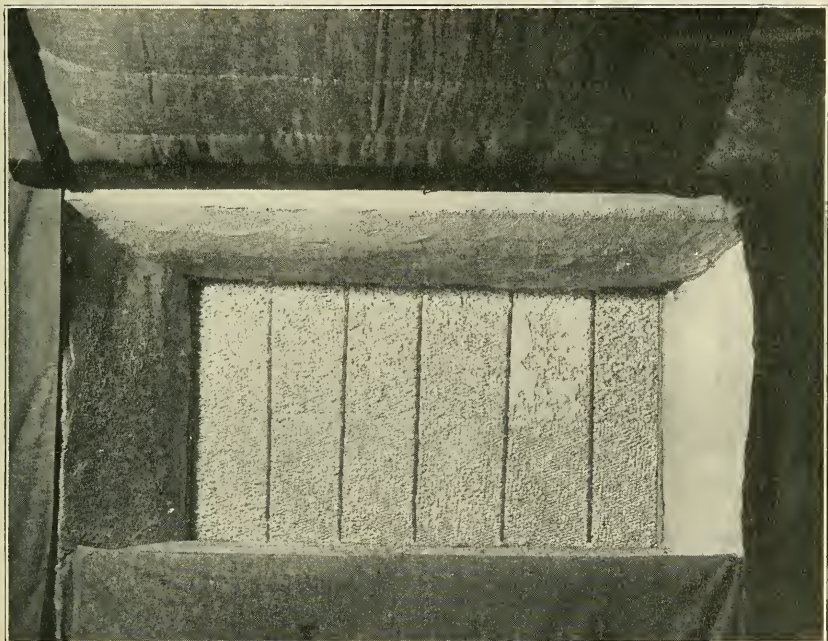


FIG. 15. EXPERIMENTAL PARTITION FORMED BY A COAT
OF GYPSUM PLASTER ON METAL LATH

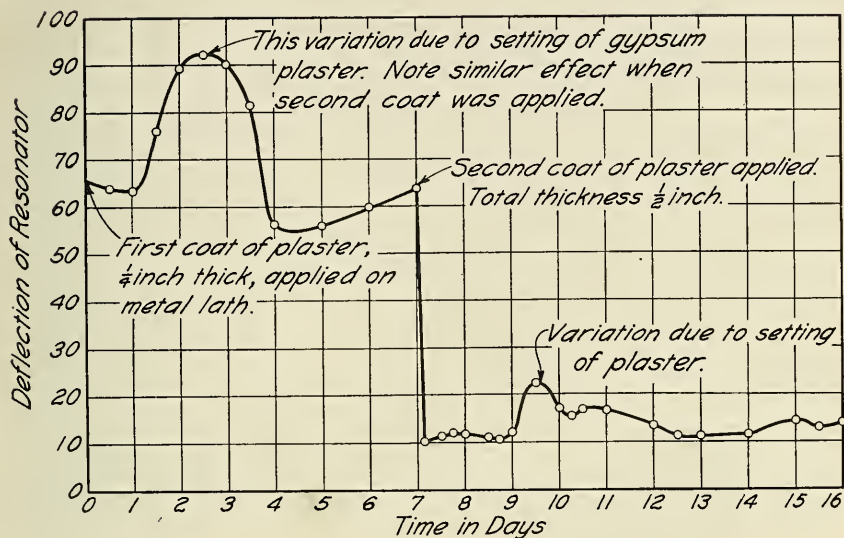


FIG. 16. TRANSMISSION OF SOUND BY PLASTER PARTITIONS OF TWO DIFFERENT THICKNESSES

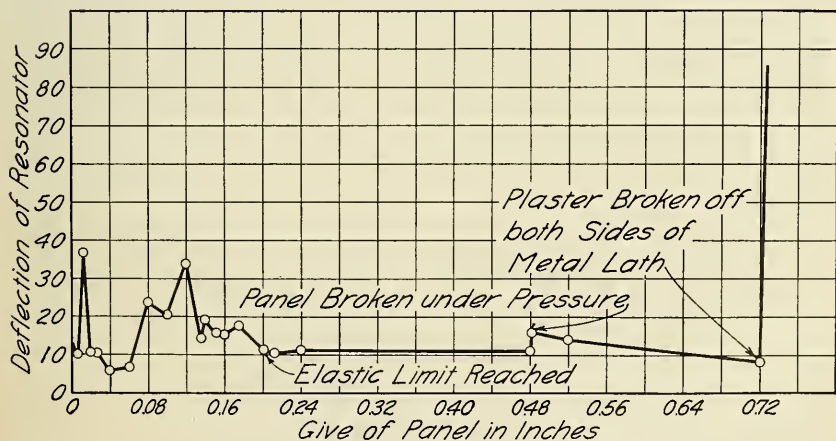


FIG. 17. TRANSMISSION OF SOUND BY PLASTER PARTITION UNDER PRESSURE

iron bars. Turning the nuts on the bolts supporting the iron bars gave successive increases in pressure. Under this treatment the panel acted very much like a drumhead, transmitting more sound when tuned up and less when something in the panel gave way and relieved the tension. Additional pressure tightened it again and the transmitted sound was increased, etc., until finally it passed the elastic limit and gave way like putty under the increases of pressure. No great amount of sound was transmitted in any case until plaster was knocked off both sides of the metal lath, thus leaving an open passage about 6 in. long. Fig. 17 gives the details.

38. *Effect of Pressure on a Thick Plaster Partition.*—The foregoing experiment was repeated with a 2-in. solid plaster board and plaster partition. A 4-in. wooden block was pushed with successively increasing pressures against the center of the partition and measurements of transmission taken at successive stages until cracks of some size were developed. The amounts of sound transmitted were not large, probably because of the friction between the rough walls of the cracks and the vibrating air particles. Figs. 18 and 19 show photographs of the partition, while Fig. 20 gives the transmission measurements.

39. *Transmission of Sound Through Threshold Apertures.*—In order to test the transmission through threshold apertures, a door was built into a 2-in. solid metal lath and plaster partition as shown in Fig. 21. The door was constructed of 2-in. wooden planks and was carefully fitted into the opening. Door stops were used on the side opposite the source of sound except at the threshold aperture. The relative intensities of sound calculated from the resonator readings are as follows:

2-in. metal lath partition, before insertion of door	0.93
With door installed with $\frac{3}{16}$ -in. threshold opening	7.3
With door installed with $\frac{1}{2}$ -in. threshold opening	11.7

While the results probably do not express exactly the intensities of the transmitted sound because of the diffraction effects by the narrow opening, they give relative values of the effect of threshold openings. These openings are equivalent to wide cracks.

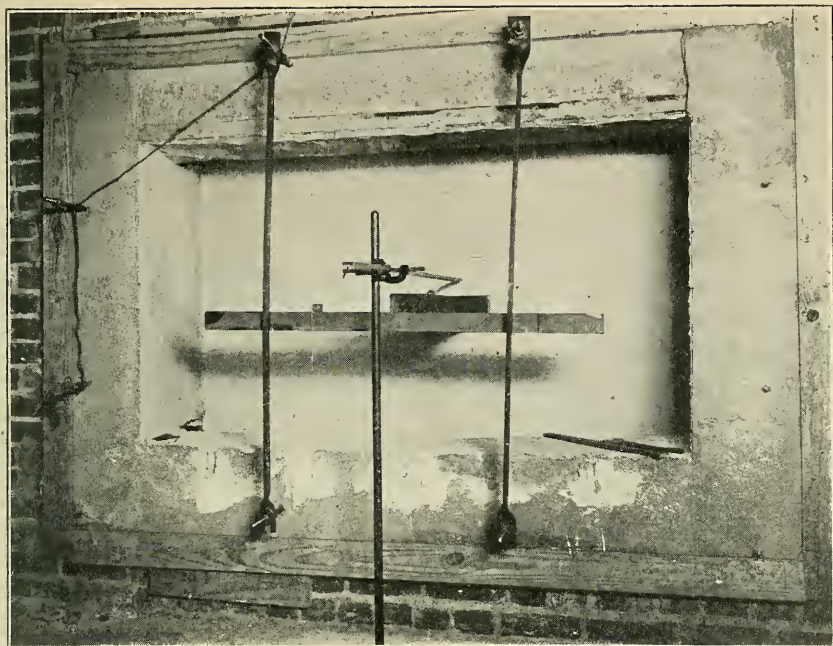


FIG. 18. METHOD OF APPLYING PRESSURE TO PARTITIONS



FIG. 19. CRACKS DEVELOPED IN SOLID PLASTER BOARD AND PLASTER PARTITION

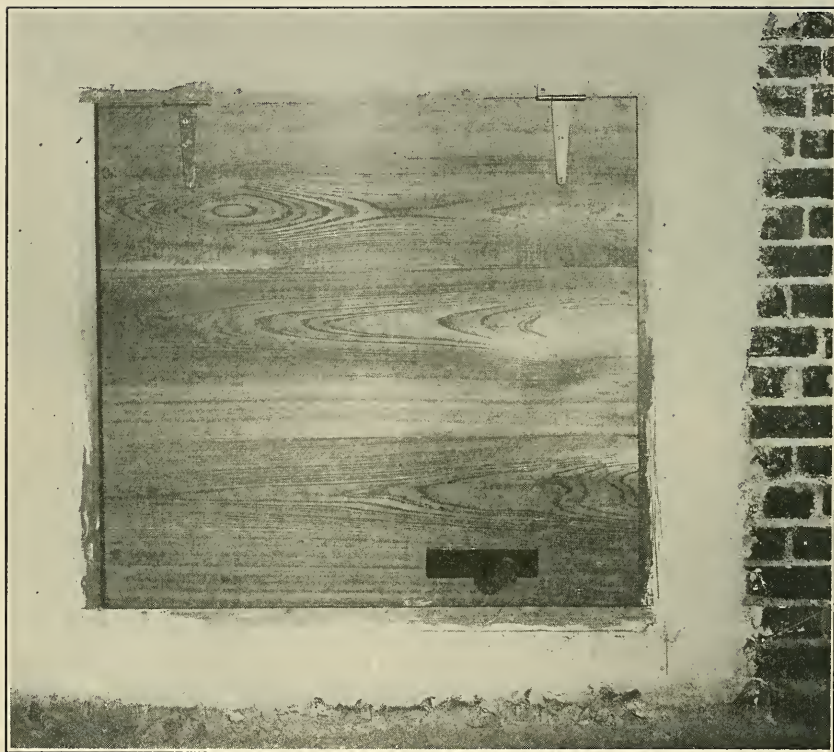


FIG. 21. DOOR INSTALLED IN 2-IN. METAL LATH AND PLASTER PARTITION

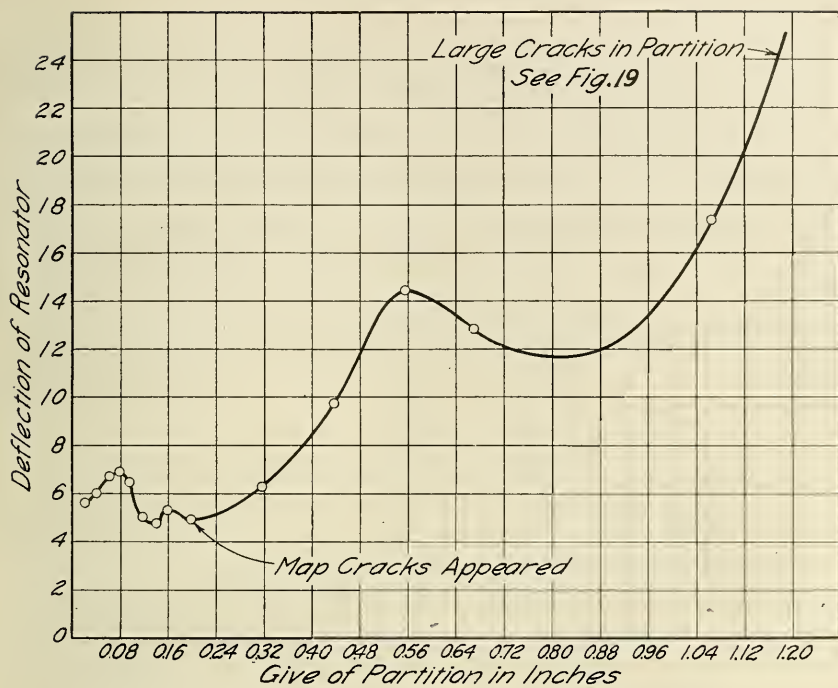


FIG. 20. TRANSMISSION OF SOUND BY PLASTER PARTITION
WHEN CRACKS DEVELOP

IX. TRANSMISSION OF SOUND BY SOLID PLASTER PARTITIONS

40. *Introductory Statement.*—The preceding tests showed that the method could be applied to solid plaster partitions as well as to outing flannel and thin partitions, and that the instruments were sensitive enough to measure the fainter sounds transmitted by thick plaster. The way was now open to test thicker plaster partitions. According to the method described comparative measurements on the intensity of transmitted sound were made on the following constructions:

- 2-in. solid metal lath and plaster partition;
- 2-in. plaster board and plaster partition;
- 3-in. plaster block partition plastered on both sides;
- 3-in. plaster block partition plastered on both sides with the air holes in the plaster block filled with plaster.

The partitions, 49 in. by 64 in. in area, were erected by an expert journeyman plasterer according to the usual practical methods. All materials were purchased by a local contractor in the open market. The same kind of plaster was used on all partitions.

41. *Case for 2-in. Solid Metal Lath and Plaster Partition.*—The partitions were built solidly into the wall nearest the source of sound, the plaster coats being applied successively according to standard specifications. Instead of waiting until the partition was completed, measurements on transmission of sound were taken for each thickness, thus giving some idea of the effect of increasing thicknesses of plaster in stopping sound. Table 7 gives details of results for the metal lath partition and indicates the procedure followed for the other partitions tested for which only the essential records are shown. Each measurement recorded is the average of two measurements in most cases, in others of three.

Fig. 23, Curve 1, shows the deflections of Fig. 22 plotted against the increasing thicknesses of the partition. The transmitted sound decreases rapidly until the thickness of the wall increases to about $\frac{3}{4}$ in., after which the decrease is slower. The thicknesses for the various layers were measured when the partition was torn down, average values of several broken pieces being taken.

TABLE 7

READINGS OF RAYLEIGH RESONATOR FOR SOUND TRANSMITTED BY A 2-IN. METAL LATH AND PLASTER PARTITION*

Date	Hour	Conditions
June 24	A.M.	First coat of plaster applied on metal lath
	8.00	
	P.M.	
	2.00	
June 25	4.30	72.5 cm. deflection of resonator
	A.M.	68.0 cm.
	8.30	87.7 cm.
	11.30	89.7 cm.
June 26	P.M.	second coat of plaster applied
	1.00	
	3.00	
	4.00	
June 27	A.M.	third coat of plaster applied
	8.30	
	11.30	
	P.M.	
June 28	1.00	14.6 cm.
	2.30	16.2 cm.
	8.30	7.3 cm.
	A.M.	5.4 cm.
June 29	8.30	4.3 cm., drying fans put on
	P.M.	
	1.00	
	3.00	
June 30	8.00	both finish coats applied
	A.M.	
	8.30	
	P.M.	
July 1	1.00	3.8 cm., transmitted sound very faint
	3.00	
	8.00	
	A.M.	
July 2	8.30	3.0 cm.
	P.M.	
	1.00	
	4.30	
July 3	1.30	2.8 cm.
	A.M.	
	8.30	
	P.M.	
July 4	1.00	1.2 cm.
	3.00	
	8.00	
	A.M.	
July 5	8.30	3.4 cm., Sunday, quiet
	P.M.	
	1.00	
	5.00	
July 6	5.00	2.6 cm.
	A.M.	
	8.30	
	P.M.	
July 7	1.00	2.6 cm.
	3.00	
	8.00	
	A.M.	
July 8	8.30	3.6 cm.
	P.M.	
	1.00	
	7.30	
July 9	A.M.	3.8 cm.
	8.30	
	P.M.	
	5.00	
July 10	5.00	3.7 cm.
	A.M.	
	8.30	
	P.M.	
July 11	1.00	3.4 cm.
	3.00	
	8.00	
	A.M.	
July 12	8.30	3.6 cm., plaster wet on surface
	P.M.	
	1.00	
	4.30	
July 13	4.30	3.2 cm., drying fans on
	A.M.	
	8.30	
	P.M.	
July 14	1.00	3.9 cm.
	3.00	
	8.00	
	A.M.	
July 15	8.30	3.4 cm.
	P.M.	
	1.00	
	5.00	
July 16	5.00	3.5 cm.
	A.M.	
	8.30	
	P.M.	
July 17	1.00	3.4 cm., partition dry except one spot
	3.00	
	8.00	
	A.M.	
July 18	8.30	3.6 cm.
	P.M.	
	1.00	
	4.30	

*The data for the metal lath partition are shown graphically in Figure 22, where the deflections of the resonator due to the transmitted sound are given for successive days as additional coats of plaster were applied.

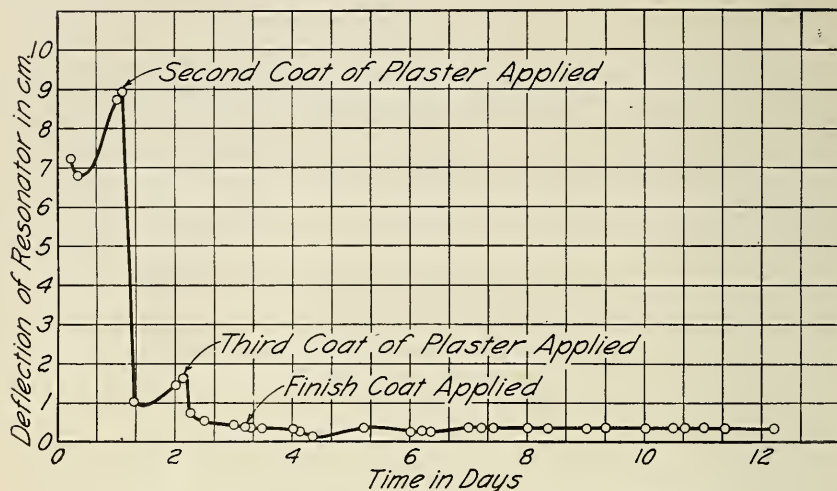


FIG. 22. TRANSMISSION OF SOUND BY PLASTER PARTITION
AS SUCCESSIVE COATS OF PLASTER ARE APPLIED

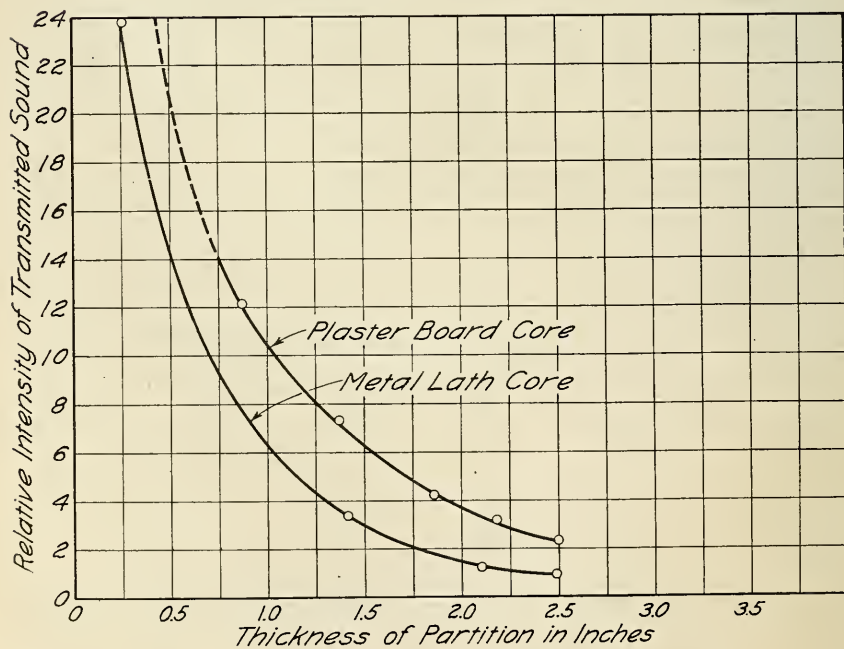


FIG. 23. EFFECT OF INCREASING THICKNESS OF PARTITION
ON TRANSMISSION OF SOUND

42. *Comparative Tests of 2-in. Solid Plaster Partitions.*—A similar set of readings was taken for a 2-in. plaster partition with plaster board core. (See Fig. 23). The comparative efficiencies of the finished partitions are given in Table 8, the deflections of the resonator noted being the average of all the measurements taken after the finish coats were applied, and the intensities of the transmitted sound being calculated by using the relation given in paragraph 22.

TABLE 8
TRANSMISSION OF SOUND BY SOLID PLASTER PARTITIONS

	Average Deflection	Relative Transmission
2-in. solid metal-lath and plaster partition	3.35 cm	0.93
2-in. plaster-board and plaster partition	8.52 cm.	2.35

43. *Comparative Transmission of Sound by 3-in. Plaster Block Partitions.*—The partitions tested are described as follows:

(a) 3-in. plaster block partition plastered on both sides, giving a total thickness of 4 in.

(b) 3-in. plaster block partition plastered on both sides with the air holes in the plaster blocks filled with plaster, giving a total thickness of 4 in.

The measurements were taken in the same manner as for the 2-in. partitions. The plaster blocks for test (b) were filled with plaster and allowed to dry before erection into the partition. The results obtained are as follows:

	Average Deflection	Relative Transmission
Partition (a)	13.9 cm.	3.85
Partition (b)	4.23 cm.	1.16

44. *Effect of a Rigid Plaster Partition on Sound.*—The transmission of sound through a plaster partition appears to depend on its rigidity and mass. Thin partitions transmit considerably more sound than thick ones, largely because they are less rigid, and vibrate more easily. Vibrations are set up which may become quite large when the natural frequency of the partition is in tune with the incident sound. Thick partitions on the other hand are more rigid and vibrate less, so that they stop sound largely in proportion to their mass.

Sound in a room may be transmitted through the partitions with more or less difficulty, depending on the qualities of the structure. Thus, the *inertia* of a partition plays an important part. On striking the heavy partition particles, the air particles of small weight are thrown back in much the same way that a tennis ball would be, on striking a cannon ball. Fig. 24 pictures a conception of this idea.

The partition particles are moved back and forth slightly by the incident pressures and rarefactions, so that sound waves greatly diminished progress through the partition. Since the latter is thin compared with the wave length of the sound, all the particles in it may be thought of as moving simultaneously in the same direction; that is, the partition may be considered to move back and forth as a unit. This motion is carried to the air on the further side where diminished waves, constituting a faint sound, are set up.

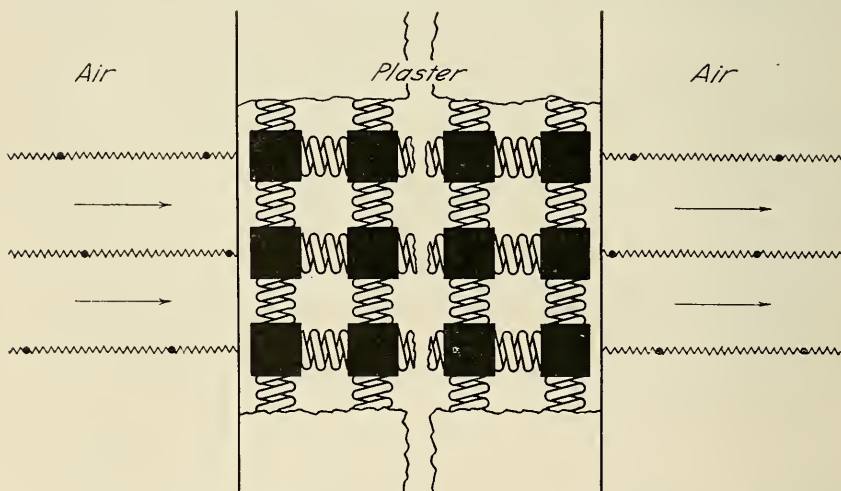


FIG. 24. DIAGRAM ILLUSTRATING TRANSMISSION AND REFLECTION OF SOUND WAVES BY A SOLID PARTITION

Theory shows that the sound in air reflected from a rigid material is given by the expression :*

$$\frac{\pi \rho_1 l / p \lambda}{\sqrt{1 + \pi^2 (\rho_1 l / p \lambda)^2}}$$

where ρ is the density of air, ρ_1 that of the material, λ is the wave length of sound, and l the thickness of the material. Calculations

* Rayleigh, Lord. "Theory of Sound." Vol. 2, Sec. 271, Equation 11.

from this expression show that the sound reflected from a rigid gypsum wall 2.5 in. thick is 99.99968 per cent where the incident sound is taken equal to 100 per cent. The transmitted sound is the difference between these quantities, or 0.00032 per cent, giving approximately a transmission of 3 parts in 1 000 000 assuming that no sound is absorbed in passing through the wall or lost in any other manner. The determining factor in the reflection is the ratio of the density of air compared to that of plaster; that is, the wall behaves like a rigid body and acts mainly because of its inertia. It is quite unlikely, however, that such a partition will be absolutely rigid, particularly if it has some area; therefore, some sound will be transmitted because of vibrations.

45. *Effect of a Flexible Plaster Partition on Sound.*—In case the partition is limited in extent and fastened at the edges, as it must be in any form of partition construction, the sound pressures must overcome not only the inertia of the plaster but also its *rigidity* or resistance to being distorted. A sound pressure applied perpendicularly to the surface causes a small displacement, greatest at the center and zero at the edges, so that the partition becomes slightly bulged, with a small increase in area. The less a wall gives under such a force, the more efficiently will it stop sound. An inspection of Figs. 24 and 25 in this connection give some idea of the strains set up by a

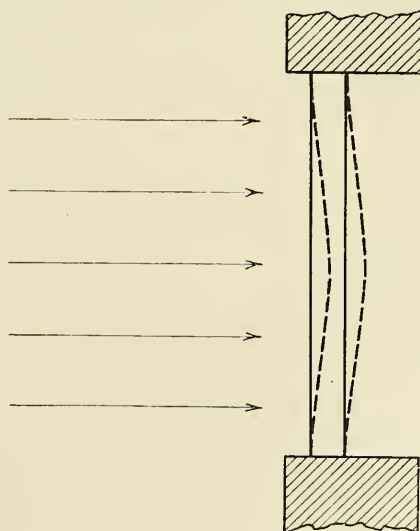


FIG. 25. HOW A PARTITION IS BULGED BY PRESSURE OF SOUND

bulging force. It should be remembered that the pressures due to sound are not static but are rapidly alternating pressures and rarefactions of very small amplitude.

The transmission of sound depends on the vibration of the partition, which acts in this regard like a thick elastic plate. The rapidly alternating pressures and rarefactions of the sound waves set the partition in minute motion and thus create corresponding pressures and rarefactions (sound waves) on the farther side of the partition. The effect is intensified and the transmitted sound increased in volume if the natural period of the partition is in tune with the sound waves. The vibrations for ordinary sounds and partitions are usually small, probably not exceeding 1/100 in. in amplitude. Other factors being the same, they decrease in intensity in walls of greater rigidity.

For a rectangular partition fixed at the edges the period of vibration, N , is deduced from the equation:*

$$N = \frac{\pi h}{2} \left(\frac{m^2}{a^2} + \frac{n^2}{b^2} \right) \sqrt{\frac{q}{3^3 (1 - \sigma^2)}}$$

where q is Young's modulus of elasticity, ρ the density of the partition, σ is Poisson's ratio of lateral contraction to longitudinal extension, a , b , and $2h$ the length, breadth, and thickness of the partition, and m and n whole numbers depending on the mode of vibration. This equation shows that thick elastic partitions vibrate quickly, but massive partitions of some dimensions vibrate more slowly.

46. *Effect of the Structure of Plaster Partition.*—Another factor affecting the transmission of sound through a partition is the character of the structure. Compared with a thin partition a thick, homogeneous structure has the advantages of greater inertia and rigidity. The use of an air space completely separating two members of a rigid, non-vibrating double partition would have a marked action on sound, and, according to theory, a partition of this construction would stop many times more sound than a similar single partition whose thickness equals the sum of the thicknesses of the two members of the double partition. This is due to the abrupt change in elasticity and density

* Barton, E. H. "Text Book of Sound." Sec. 226, Equation 11.
 Rayleigh, Lord. "Theory of Sound." Vol. 1, Sec. 225.

from plaster to air and from air to plaster as the sound strikes the second member. In case the air space is bridged over, as in practical constructions is usually the case at the ceiling, floor, and other points, this theoretical efficiency is greatly diminished because the vibrations travel easily along the paths afforded by the continuity of solid materials. Thus the bridged over partition should be considered as a unit instead of two separate members, and its efficiency in stopping sound judged mainly on its weight and rigidity.

The core of a partition is another feature of structure that affects the transmission. It may be of such a nature as to increase the strength of the partition, it may be simply the central part of a homogeneous medium, or it may so separate the partition into two parts that the structure is weaker than a homogeneous unit. A partition with increased strength due to the core, such as a steel reinforcement, would be more rigid than an equally thick homogeneous partition and would stop more sound. The homogeneous partition in turn would be more efficient in stopping sound than the double partition weakened by the core. The latter, however, has some possible advantage in reflecting sound because of the change in elasticity and density in the core. An extreme illustration of this kind would be the case where the core consists of hairfelt, so that its action in stopping sound would be analogous to that of an air space. If the core consists of a sheet of thick paper, making, without air space, a continuous contact with plaster on both sides, this efficiency is largely lost and the small gain due to reflected sound would appear to be overcome by the loss in rigidity of the structure.

47. *Discussion of Experimental Results Obtained with Solid Plaster Partitions.*—The experimental results obtained in the tests appear to be in accord with these theoretical considerations. Thin partitions vibrated under the action of sound waves, as could be ascertained by touching the surface of the plaster. The vibrations (and transmitted sound) were more vigorous for the proper co-ordination of the elasticity of the partition and the frequency of the sound as shown by the data pictured in Fig. 17. Thick partitions transmitted but little sound. They were quite rigid because of their small area and considerable thickness, and also because they were mounted solidly in a double brick wall. Their action in stopping sound must have been due largely to their rigidity and mass.

It should not be concluded from these tests that partitions of similar construction in buildings will all have exactly the same sound-proof qualities; larger area of building partitions, resulting in lessened rigidity, will allow a greater transmission of sound. The value of the results obtained in the tests lies in the fact that they were obtained by direct comparison under identical surroundings, rather than by tests on different types of partitions in different buildings, with varying floor and ceiling constructions, unequal sizes of rooms, uncontrolled extraneous sounds, etc. The results thus give information for guidance in the choice of materials and constructions where sound insulation is contemplated.

X. EXAMPLES OF SOUND-PROOF ROOMS

48. *Preliminary Statement.*—In order to supplement the information given in the previously described experiments examples of soundproofing should be included in the discussion. These examples are mostly practical constructions in buildings with walls of greater size than could be tested conveniently in laboratory experiments, and with additional complexities such as a ventilation system, pipes for steam, water, and wires, etc. An account of such constructions, together with the results of the previous tests, allows more definite conclusions to be drawn concerning the general procedure in sound insulation.

The illustrations chosen for discussion concern two types of installations; first, single rooms that are soundproofed for experimental purposes, and second, entire buildings where sound insulation was needed. A detailed description of all the examples available would involve too long an account; therefore, several typical cases are selected for discussion and references are given for the others.

49. *Sound-Proof Rooms for Acoustic Experiments.*—A small laboratory building containing a number of rooms for acoustical experiments was designed by W. C. Sabine according to his extensive experience in architectural acoustics.* The essential construction was simple yet effective. Heavy brick walls served to give sufficient rigidity to reduce vibrations to a minimum. The weight of these walls together with an air space between them presented a decided hindrance to the transmission of sound waves. (See Fig. 26.)

* Sabine, P. E. "Wallace Clement Sabine Laboratory of Acoustics." *Am. Arch.*, Vol. 116, pp. 133-138, 1919.

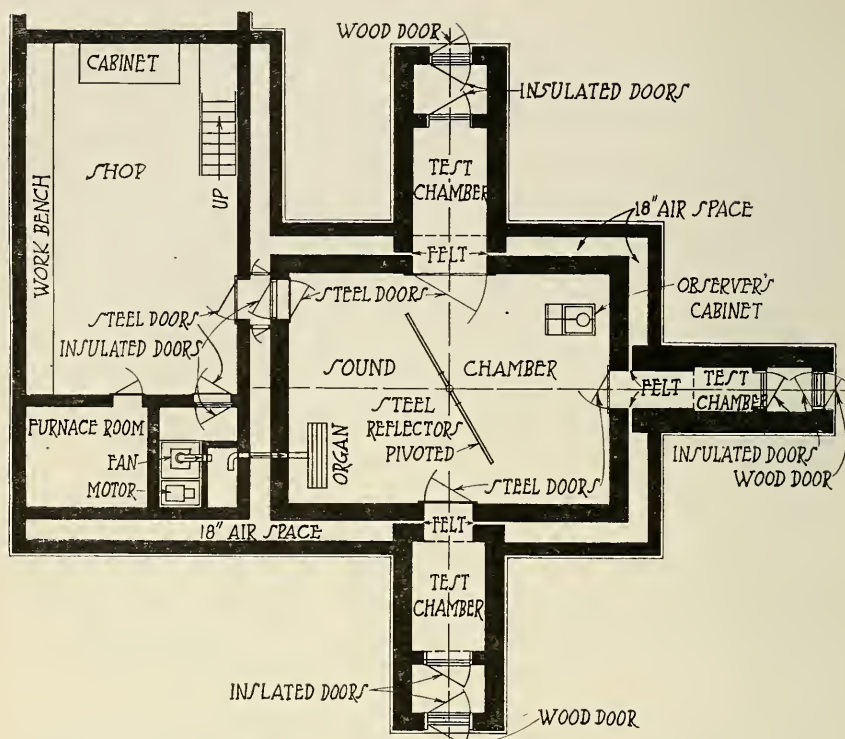


FIG. 26. PLAN OF WALLACE CLEMENT SABINE ACOUSTICAL LABORATORY

(This laboratory was built and is maintained by Colonel George Fabyan at Geneva, Illinois, because of his interest in scientific work and his personal regard for Professor Sabine. Since the death of the latter the investigational work has been conducted by Dr. Paul E. Sabine, a cousin of Professor Sabine.)

50. *Sound-Proof Constant-Temperature Room.*—The structure illustrated in Fig. 27 is a small building constructed primarily for securing constant temperature. It appears also to be an excellent sound-proof structure. The description* is as follows:

* "Results of Observations made at the Coast and Geodetic Survey Magnetic Observatory at Cheltenham, Maryland, 1901-1904." Dept. of Commerce and Labor, U. S. C. and G. S., pp. 11-13, 1909.

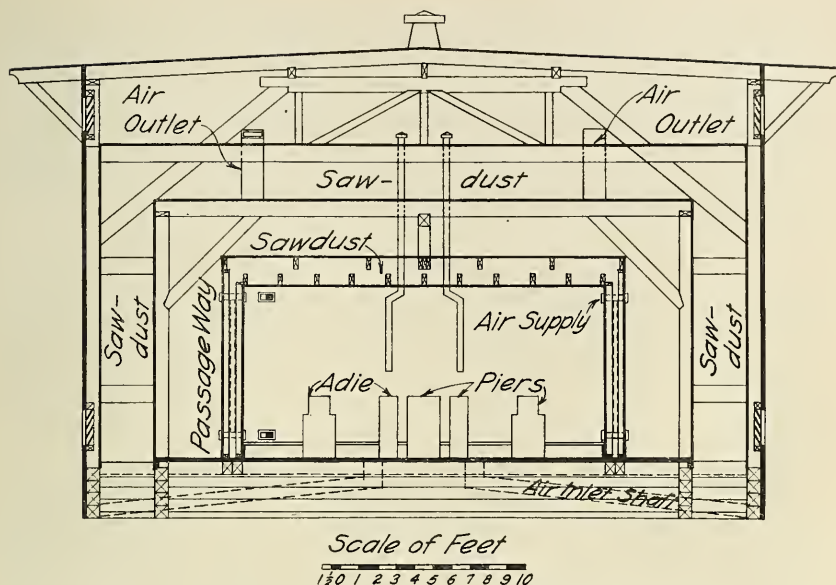


FIG. 27. VERTICAL CROSS-SECTION OF SOUND-PROOF CONSTANT-TEMPERATURE ROOM

“The variation observatory is essentially two small buildings inclosed by a third larger one. The outside dimensions of the variation observatory are 36 by 56 by 24 ft., with a vestibule 10 by 13 ft. The inner rooms are each 16 by 19½ ft. inside measure, separated from each other by a passageway 5½ ft. wide. The wall insulation is as follows (see Fig. 27); beginning at the outside, pine weatherboarding, 8-ply building paper, 1-in. pine sheathing, 8-in. air space, 1-in. pine sheathing, 8-ply paper, 3 ft. pine sawdust, 8-ply paper, ⅞-in. pine sheathing, 3 1/16 ft. air space (passageway around inner rooms), ⅞-in. pine sheathing, 8-ply paper, 1 ft. pine sawdust, 8-ply paper, ⅞-in. pine sheathing. Beginning at the roof and going down: gravel and asphalt-pitch roof, 1-in. pine sheathing, 3 2/3 ft. air space, 1-in. rough pine floor, 3 ft. pine sawdust, 8-ply paper, ⅞-in. pine sheathing, 3 ft. air space above inner rooms, 1-in. rough pine floor, 1½ ft. pine sawdust, 9-ply paper, ⅞-in. ceiling. Insulation from bottom of foundation is 2 2/3 ft. earth, 6 to 8 in. layer of gravel, 3 ft. sawdust, 1-in. pine floor, ⅞-in. matched pine floor. The 8-in. air space next to the outside of the building is provided with slat ventilators at top and bottom, which when open permit a free circulation of air up the sides of the building and out through the ventilators in the roof. By the use of tight-fitting shutters it can be converted into a practically air-tight space. By this arrangement of alternating air spaces and sawdust packing the variation in the temperature is kept within the desired limits without the aid of heating apparatus. The daily range is kept down to almost nothing and the annual range to only a few degrees Centigrade.”

Mr. W. W. Merrymon, one of the magnetic observers, stated that the room was so sound-proof that he thought he would be unable to make outsiders hear in case he were in distress—from lightning or other causes. Trial by loud shouting within the room gave almost no sound outside, and this faint sound appeared due to the passage of sound through the air ducts in the floor. By closing these the room would doubtless have been made “sound-proof.”

51. *Sound-Proof Room for Psychology Tests.*—Concerning this room, Dean C. E. Seashore of the State University of Iowa writes: “Our experience in building the sound-proof room in psychology would not be of any great help in the soundproofing of a music building, since all the precautions that we have taken are such as could be taken for a small cage room.” The description* of the room is as follows:

“The construction of the observing room deserves especial mention. To make a dark room impervious to external light is a matter presenting no serious difficulty. To make a room impervious to external sound or wholly free from the jarring from surrounding rooms or adjacent streets is a problem which has not yet been solved and of course never will be. We made the attempt to approach a little nearer to this end than has hitherto been done. The result is a room as free from external disturbances as is needed in any experiments in which it is necessary to control visual, auditory and tactual stimuli. So far as this has been accomplished, the credit is largely due to the architects, Messrs. Proudfoot and Bird, who worked out many of the details of construction. The position of the observing room is central, occupying a place not otherwise desirable from lack of light. The room rests on an independent foundation, having no solid connection with the rest of the building either below, above, or on the sides. The superstructure which supports the room rests upon a sand bed and a second sand bed at a higher level still further assists in eliminating possible jarring or sound which might be communicated from the ground. The walls of the room itself, inside the main partitions, which separate the whole space from the surrounding apartments, are made of two four-inch walls of hollow tiles separated by an air space and each covered with a thick insulating material made of sea-weed. Inside of all, the walls are plastered and then lined throughout with black broadcloth. The inside room is divided into the main observing room 12 ft. 2 in. by 12 ft. 7 in., and a vestibule, 4 ft. by 12 ft. 7 in. The room is entered through five doors, the outer one being an ordinary oak door and the other four especially constructed tight fitting cedar doors, covered with black cloth on the sides and edges. The doors close with strong springs and are held open by automatic catches. The floor is made of Tennessee red cedar and covered with linoleum painted black. The room is heated and ventilated by means of hot air introduced not directly

* Studies in Psychology, Vol. III, pp. 132-143; State Univ. of Ia., Bul., New Series, No. 49, May, 1902.

but from the attic through cedar shafts provided with overlapping cloth partitions which admit the air but help to exclude sound. For very fine experiments the ventilating shaft may be closed and the room ventilated during intermissions. The room may be lighted by gas or electricity, the former being introduced through rubber tubes. The furnishing consists of black tables and chairs and a telephone connected with the measuring room. In the exclusion of sound and vibration, as well as in other respects, the room has proved to be a complete success. The loudest stentorian shouting just outside the doors is absolutely unheard within.''

52. *A Noiseless Room.*—A noiseless room for sound experiments was built in Utrecht, Holland.* Figs. 28 and 29 show the complicated details. The room was about 7 ft. by 7 ft. by 7 ft., and the walls were 11 in. thick. A person in this room could hear his heart beat.

53. *A Group of Sound-Proof Rooms.*—Norton† tested five small rooms whose walls were specially constructed of different materials. The tests were not strictly comparative because three of the rooms had one thin side that vibrated under resonance and transmitted sound to adjacent sides; also there was some leakage of sound through the doors and ceiling. Observations were taken by the ear, and different partitions rated by estimation. The results show that increasing the thickness of the walls promoted sound insulation; also, that double partitions with air space and enclosed sound absorbing materials are more effective than single partitions.

54. *Room Soundproofed for Machine Gun.*—During the war a room about 12 by 14 by 12 ft. was designed to insulate the explosive sounds of machine guns when these were being tested. The room occupied one corner of a larger room. The walls of the insulating room were brick on two sides—those of the larger room—and wood on the other two sides. Four inches of ground cork lined the wood walls, being held in place by metal netting nailed to 2 in. by 4 in. wood studding. A double layer of flax boards lined the ceiling. A pile of sand into which the bullets were fired assisted in the absorption of sound. People across the street from the room were unaware of the machine gun operation until the observer forgot to close the double windows after opening them to let the powder smoke out.

* Franz, S. I. "A Noiseless Room for Sound Experiments." *Sci.*, Vol. 26, pp. 878-881, 1907.

† Norton, C. L. "Sound-Proof Partitions." *Insur. Eng.*, Vol. 4, p. 181, 1902.

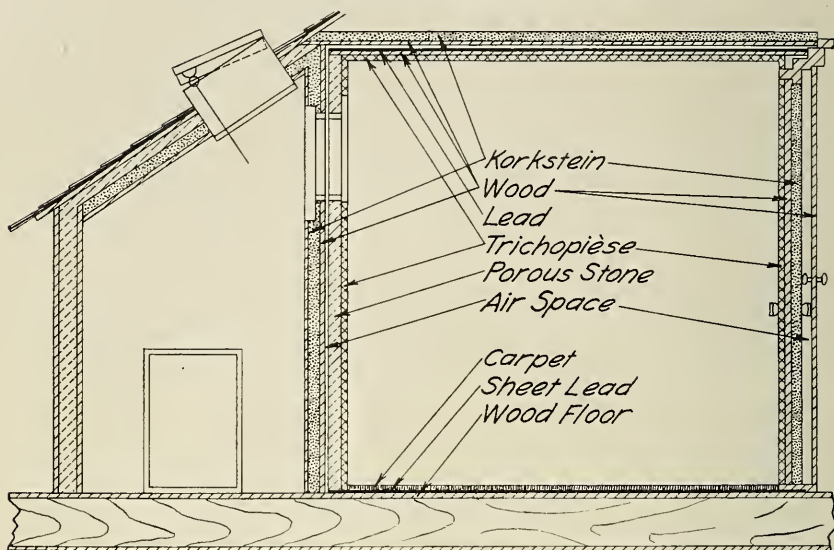


FIG. 28. VERTICAL CROSS-SECTION OF NOISELESS ROOM

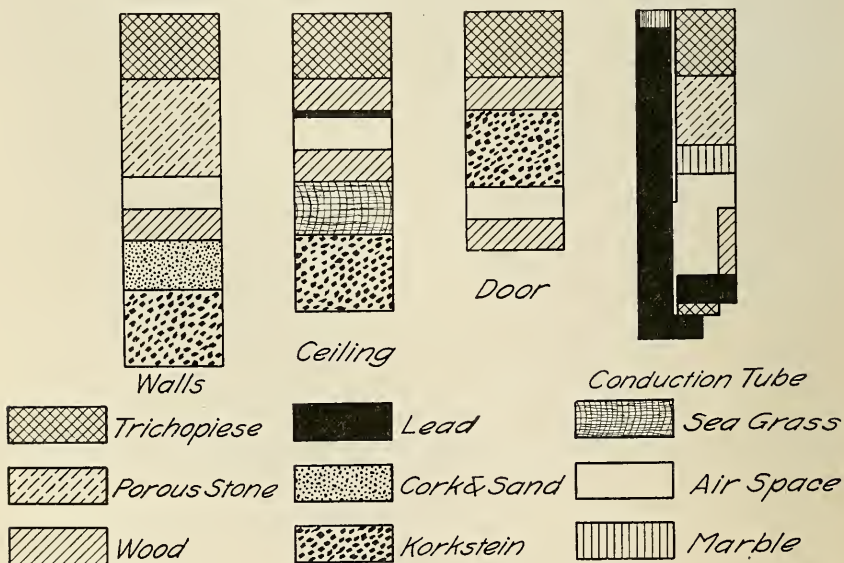


FIG. 29. DETAILS OF CONSTRUCTION OF NOISELESS ROOM



FIG. 30. SOUND-PROOF BOX TO REDUCE NOISE OF BRICK RATTLER

55. *Soundproofing a Noisy Machine.*—A brick rattling machine, used by the Highway Engineering Department at the University of Illinois, was extremely noisy, so that suggestions for a method of minimizing the disturbance were requested. The machine was mounted on a platform and enclosed first in a sheet iron covering which served to prevent the escape of dust as well as sound. (See Fig. 30.) A larger, double walled wooden box, having the interspace filled with saw-dust, and being equipped with a hinged cover, was built to give further insulation. Without the covers, the machine is so noisy that one person cannot be heard by another without shouting loudly into his ear. With the covers on, the noise is greatly reduced and conversation is possible. This matter has its economic side, because, with noisy machines, the hearing and therefore the efficiency of employees becomes affected.

XI. SOUND-PROOF BUILDINGS

56. *The Smith Music Building.*—An increasing number of buildings with soundproofing features have been constructed in the past few years. F. R. Watson, in collaboration with Architect James M. White, incorporated a number of soundproofing constructions in the Smith Memorial Music Building at the University of Illinois.* This problem was more complex than soundproofing a single room. It involved the sound insulation of some fifty small practice rooms, twelve studios, and the larger concert hall, besides the acoustic control of sounds of motors, fans, and elevators.

Since the possibility of transmission of sound was greatest between adjacent rooms, each dividing wall, ceiling, or floor was made double, with air space containing absorbing material, and was left entirely unbroken. All pipes, conduits, ventilator ducts, doors, and windows were specially placed in outside or corridor walls where the leakage of sound would be less harmful. This systematic construction *throughout* the building meant that a sound generated in a room must penetrate the insulation to escape. To enter another room, it must pass a second time through a special insulation. When traversing the building structure, a sound would continually meet hindrances that would either stop or absorb it.

Fig. 31 pictures some of the features that were adopted to control sound. The concrete floor, 12 in. thick, was broken in its continuity by the form planks that were purposely left in place. Walls between rooms were constructed of two 3-in. gypsum partitions insulated at the bottom by machinery cork and at the top and sides by hairfelt. "Insulite" was installed in the air space between the gypsum partitions, to absorb sound and also to present a barrier in case cracks developed in the gypsum. The finish floors were floated on a 1-in. layer of dry sand in order to break the continuity of material and thus stop the progress of vibrations.

* Watson, F. R. "Sound-Proofing a Building." Arch. For., Vol. 35, pp. 178-182, Nov., 1921.

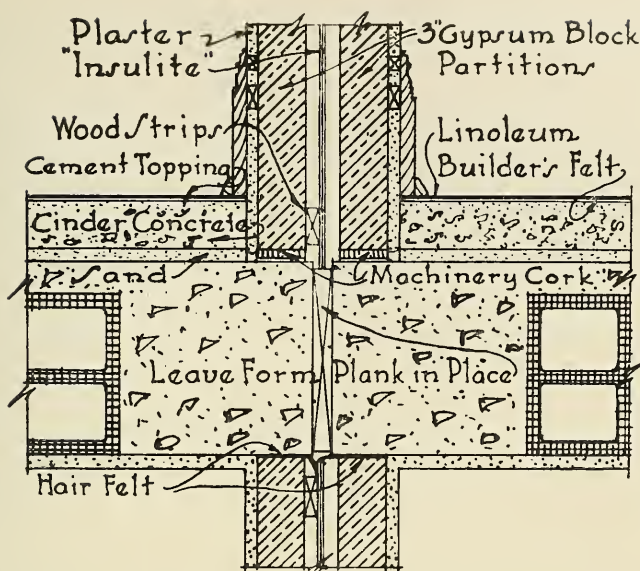


FIG. 31. DETAIL OF FLOOR AND PARTITION CONSTRUCTION
IN SOUND-PROOF BUILDING

Experiments conducted in the building after its completion show that a measure of success attended the design and construction. Loud speaking and shouting in the practice rooms can hardly be heard outside. Music, however, penetrates the insulation more easily, so that sound, largely diminished, may be heard in adjacent rooms. This leakage of sound, however, does not appear of great disadvantage. Students use adjacent rooms for singing practice, and for piano, violin, and other instrumental drill, without serious disturbance to each other.

The ventilation system is not as sound-proof as desired, and appears to be the greatest drawback in the control of sound. Each room was equipped with a separate inlet and outlet duct. Four independent ventilating systems furnished air to four groups of rooms in order to lessen the chance for transfer of sound from one group to others. The ventilation system is now under investigation with a view to improving the insulation.

The building is not absolutely sound-proof nor does this appear necessary for practical purposes. The sounds that leak through the insulation are greatly diminished in intensity and therefore of minor

importance compared with the sounds generated in the room containing the observer.

57. *Other Buildings with Soundproofing.*—Music buildings with soundproofing features have been erected at other institutions, and, from the reports received, appear to give satisfaction. The constructions involve double walls with absorbing material, and more or less attention has been given to the design of the ventilation.

XII. CONCLUSIONS

58. *Summary of Conclusions and Recommendations.*—The information in this bulletin was drawn from three sources; the theory of the behavior of sound waves, experimental investigations of the effect of materials on sound, and examples of sound-proof installations. The details of this information, while drawn from different sources and apparently unrelated, coördinate in a satisfactory way in setting forth similar conclusions.

Some of the more general principles and recommendations are stated in the following paragraphs, but the details and comments necessary for a more comprehensive conception of the problem of soundproofing are to be found in the descriptions throughout the bulletin.

Sound may be transmitted from one side of a partition to the other in three ways; it may progress through continuous air passages, it may pass as an elastic wave through the solid structure of the partition, or, by setting the partition in vibration, it may originate sound waves on the further side.

These actions are quite readily understood by remembering that sound consists of a series of compressions and rarefactions that progress rapidly through a medium without interruption unless they meet a new medium with a different elasticity or density. For instance, sound waves in air proceed without hindrance through air passages, such as ventilation openings in a partition. If, however, the passages are small in cross-section, as in the case of a porous material, the progress is hindered and a certain amount of absorption of the energy takes place, due to the friction set up between the vibrating air column and the sides of the pores.

In case the partition is impervious to air, the direct progress of the waves is interrupted. A thin partition is set in vibration and thus originates new waves on the side opposite the incident sound. For a thicker, more rigid partition, the vibrations are smaller and a very considerable part of the energy is reflected. The transmission in this case takes place by compressional waves communicated to the solid material of the partition. The amount of energy thus transmitted is usually quite small.

In view of these considerations a sound-proof partition should be as rigid and free from air passages as possible. For effective sound-proofing of a group of rooms, the partitions, floors, and ceilings between adjacent rooms should be made continuous and rigid. Any necessary openings for pipes, ventilators, doors, and windows should be placed in outside or corridor walls where a leakage of sound will be less objectionable.

In case the sound is generated in the building structure, as the vibrations set up by a motor fastened to the floor, the compressional waves proceed through the continuity of solid materials. In order to stop them, it is necessary to make a break in the structure so as to interpose a new medium differing in elasticity and density. For instance, the vibrations of a motor may be minimized by placing a layer of hairfelt, or similar air-filled material, between the supporting base and the floor. Where the machine is quite heavy, footings may be made of alternate layers of asbestos, lead, and leather. Bolting through this material will reduce the insulation, because the vibrations in this case will pass easily through the bolts to the floor. The insulation should thus be left without any bridging over of the discontinuities. Air gaps in masonry will be effective if the air space is not bridged over at any point. A floor floated on sand, sawdust, or hairfelt would approximate this condition. The edges of the floor should be insulated from the walls by felt or similar material.

Especially attention should be paid to the ventilation system. All effective sound-proof constructions either omit entirely a ventilation system or else construct it in some special manner to avoid transmission of sound. In some buildings air is supplied and withdrawn from rooms by individual pipes that are small in diameter and extend without break from the air supply chamber to the rooms. This results in considerable friction between the walls of the pipes and the air, with a resultant weakening of the sound waves. Without some efficient control of the transference of sound through the ventilation system it is a waste of effort to construct sound-proof walls, double doors, and other contrivances for insulation.

When soundproofing a building all details should be considered with respect to the likelihood of transmission of sound. Each room, as far as possible, should be made an insulated unit by means of air spaces or air-filled materials that separate it from surrounding walls. Pipes and ventilators should be so installed as to minimize the chance of transfer of sound. Patent doors are now available that will close

the door space at top, sides, and bottom. In case a troublesome sound is generated in the room, it may be minimized by installing absorbing material on the walls.

The absorption of sound is an essential feature for soundproofing. Reflecting sound and scattering it still leaves it with energy. It must be absorbed; that is, converted into heat energy by friction, before it is eliminated as sound. This means that carpets, furniture, draperies, etc. should be present, or if greater absorption is desired, hairfelt or similar materials must be installed.

The insulation of sound is a complex problem and a successful solution is obtained only when all the possibilities of transfer of sound are anticipated and guarded against. While many things may be learned from further experience and much may be gained from additional theory, enough has been revealed to give encouragement to the belief that soundproofing may be prescribed in the future with some of the certainty that now attends the acoustic design of auditoriums.

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THE IGNITION TEMPERATURE OF COAL

BY

RAY W. ARMS



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ENGINEERING EXPERIMENT STATION

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THE IGNITION TEMPERATURE OF COAL

I. INTRODUCTION

1. *Outline of Work.*—There is no definite temperature at which coal bursts into flame. The phenomenon of flame from coal is due to the combustion of volatile matter driven off from the coal, the character of such matter varying under different conditions; hence, the temperature at which the coal flames varies widely and is dependent upon surrounding conditions, and therefore the flaming temperature of a coal does not serve as an indication of the ignition point. Strictly speaking, it is the true ignition point, but since it varies so widely its evaluation is meaningless. Moreover, any definition of ignition temperature based on the spontaneous emission of heat from coal is equally unsatisfactory, because experiments have proved that coal is giving off heat of combustion or some phenomenon akin to combustion at all temperatures. To be sure, the combustion at low temperatures is slow, but it is nevertheless evident. As the coal is raised in temperature by an outside source of heat it produces more and more heat from its own combustion, but there is no point in this process where the change in rate of heating is sufficiently abrupt to be regarded as indicating a definite ignition temperature.

The first purpose of this investigation, therefore, was to establish some definite point along the line of this process of heating which could be called the ignition temperature. In the effort to locate such a temperature heating curves were drawn to show the rate at which the coal increased in temperature when assisted by an outside source of heat. After a study of these curves the point at which the coal assumed a uniform glow was chosen as the most logical ignition temperature, not only because this point was found to be rather definite in the heating curve, but also because it could be checked in various types of apparatus by different means of temperature measurement and control, and by different operators.

With this glow point chosen as the ignition temperature the method for the determination became comparatively simple. The

sample of coal was heated gradually in a normal air supply until the glow appeared, the temperature of the coal itself being measured by means of a thermocouple. The furnace was shielded from air currents, but sufficient opening was left in the shields to provide an adequate air supply for the combustion of the coal. This apparently crude method is open to many objections, but it is surprising how closely the results can be made to check. Out of thirty runs by this method on the same coal, twenty-four agreed within ten degrees.

The accuracy of the method was next tested by experiment on several representative coals. Among the samples tried were bituminous coal, coke, anthracite, and lignite. The values for the glow points of these coals are shown in the following table:

TABLE 1
VALUES OF GLOW POINTS FOR DIFFERENT COALS

Kind of Coal	District	Glow Point	
		Deg. C.	Deg. F.
Bituminous	Georgetown, Ill.....	456	853
Bituminous	Bennett Station, Ill.....	496	925
Bituminous	Herrin, Ill.....	440	824
Lignite	Manitoba.....	526	979
Semi-bituminous	Castle Gate, Utah.....	528	982
Anthracite	Pennsylvania.....	600	1112
Coke.....		606	1123

2. *Influence of Various Factors on Ignition.*—Finally an attempt was made to determine the influence of various factors such as ash, moisture, sulphur, size of particles, air supply, etc., upon the temperature of ignition. The results of this work may be briefly summarized as follows:

Ash

The amount of ash present in the coal has no apparent influence on the temperature of the glow point. The ash influences the distribution of heat throughout a mass of coal and may change the time required to bring the coal up to a definite temperature, but, the proper temperature once reached, the coal will glow no matter what the ash content may be.

Moisture

Moisture has a great deal to do with the rate at which the coal is heated up to the point where it will burn, consequently with the shape of the heating curve; but, as the moisture is driven off before the glow point is approached, the actual temperature of the glow point can in nowise be affected by the quantity of moisture present.

Size of Particles

The glow point for all sizes of coal is the same, but in the process of determining this fact some difficulty was experienced in observing the glow points on the fine sizes. The air passages through finer mesh coal are so confined that the interior of such coal is not reached by the air. Furthermore, as its temperature rises, a pile of fine coal begins to cake on the surface, forming a crust that interferes still more with the passage of air to the interior. This made it rather difficult to determine the glow point of fine coal, but the results obtained indicate that there is no difference between its glow point and that of coarser sizes.

Quantity of Air

Glowing of coal is a form of combustion which requires oxygen, but the actual temperature of the glow point is independent of the quantity of oxygen supplied. Obviously, if the coal were heated in an atmosphere devoid of oxygen it would not glow at all, but would undergo destructive distillation. On the other hand, if an atmosphere of pure oxygen were supplied, the rapidity of the combustion would be increased but the temperature of the glow point itself would not be altered.

Sulphur

Separate runs were made on pure pyrite in the effort to determine its ignition temperature, which was found to depend much on its rate of heating. If heated very slowly practically all the sulphur can be driven off in the form of SO_2 without producing any visible flame. If the rate of heating is increased, a blue flame appears at a comparatively low temperature. Unquestionably, therefore, the combustion of sulphur adds to the mass of coal a heat which slightly increases the tendency of this coal to fire, but, as the heat of combustion of sulphur and the percentage of pyritic sulphur present

are comparatively low, the total effect of sulphur on the tendency of coal to fire is relatively small.* The experiments appeared to indicate that the presence of pyrite does not alter the glow point of the coal substance itself, but that its only effect is to add its heat of combustion to that of the coal.

Rate of Heating

Coal has a comparatively low heat conductivity, so that, in order to insure a uniform distribution of the heat throughout the sample, external heat must be applied slowly. If the sample is heated too rapidly it will burst into flame before the glow point is reached, due to the ignition at the edge of the pile of small pieces of coal which are at a higher temperature than the average of the pile itself. The best results in the present experiments were obtained when the sample was heated up to the glow point in about twenty minutes' time.

Volatile Products

The ignition points of the various gases volatilized from coal during distillation are fairly well known. In all cases the ignition points of these gases are higher than the glow point of the coal, so that no flame should be observed before the glow point is reached. However, if even a very small particle of coal is heated above the ignition temperature of any of the gases, a flame will result. Thus, in order to successfully determine the temperature of the glow point, it is first necessary to drive off most of the volatile products.

Residual Carbon

It is a well-known fact that various forms of carbon differ in the ease with which they may be ignited. Beyond a doubt the character of the carbon or coke resulting from the partial distillation of the coal has more to do with the temperature of the glow point than any other single factor. The exact reason for this was not determined, nor was any attempt made to differentiate the forms of carbon or coke resulting from various coals tested. Furthermore, when a coal starts to glow it is not yet completely coked. Samples of coal quenched at the glow point show that the individual particles have lost their angular shape and are only partly cemented together.

* "The Spontaneous Combustion of Coal." Univ. of Ill. Eng. Exp. Sta., Bul. 46, 1910.

3. *Acknowledgments.*—The author wishes to express his appreciation of the manner in which Mr. FRANK STAMBERG carried on the work of the investigation in the laboratory. Several of the samples of coal were furnished by PROFESSOR S. W. PARR of the Department of Chemistry of the University of Illinois, who also made valuable suggestions both regarding method of procedure and interpretation of results. PROFESSOR H. H. STOEK of the Department of Mining Engineering of the University of Illinois also furnished many helpful suggestions, and it was through his encouragement and assistance that the work progressed.

The work was done in the laboratory of the Department of Mining Engineering of the University of Illinois.

II. IGNITION TEMPERATURE

4. *General Significance.*—Under ordinary conditions the ignition temperature of a fuel is less important than the maintenance of combustion. No matter how large the mass of fuel may be, whether it is in a blast furnace, under a boiler, or in an ordinary household furnace, the origin of the heat necessary to raise this fuel to the ignition temperature is in the kindling which starts the fire. The flame of the match which ignites the kindling is at a much higher temperature than the ignition point of the coal; hence, the starting of an ordinary fire is a process depending upon the application of an outside source of heat, always at high temperature, to some portion of the mass of coal until a part of the coal itself burns and propagates heat to the remainder. At some time during this process the outside source of heat may be removed as, for example, when the kindling is completely burned. At all times during this process there is a zone of heat production at a temperature well above the ignition point. Whether this source of outside heat is in the form of kindling or in a form similar to the heat in the walls of a coke oven, it is only necessary to supply the heat until a certain part of the coal is burning and raising the temperature of the rest of the mass by its own combustion.

It is possible that the ignition temperature of fuel might assume a greater significance than this in connection with problems involving burning of pulverized coal. For example in the cement industry, where pulverized coal is used largely for heating the kilns, trouble is sometimes experienced when the supply of coal is temporarily cut off and the kiln allowed to cool. In resuming the supplying of coal to the cooled kiln, an explosive atmosphere, apt to cause damage, is sometimes produced. It is suggested that this may be prevented by supplying temporarily a fuel with a lower ignition temperature.

It is, however, in connection with the storing of coal in piles that the ignition temperature takes on a real significance. According to Professor S. W. Parr* and others, coal is actually burning at all times and

* "The Spontaneous Combustion of Coal." Univ. of Ill. Eng. Exp. Sta., Bul. 46, p. 50, 1910.

even at low temperatures. If the heat thus produced is sufficiently confined and air is supplied to the coal at the proper rate, the temperature of the interior of the mass will rise. As the temperature rises the rate of combustion also rises, and, after a certain safe temperature is passed, the process goes on quite rapidly until burning or flame results.

One of the original objects of this investigation was to determine the influence of the temperature of ignition of the various coals upon their liability to fire while in storage. At first thought it would seem that the ignition point would have a very important bearing on the tendency to fire. As a matter of fact, however, the glow points determined, with a few notable exceptions, bear no relation to the liability to fire. The same is true if some point on the heating curve other than the glow point is assumed to be the temperature of ignition. For example, the point at which the coal burns with a blue flame is a rather definite point which shows some variation with different coals; but even this variation bears no relation to the liability of the coal to fire.

However, these results are not in any sense conclusive, since several factors known to affect the firing qualities were not considered. For example, Wheeler has found* that the relative ignition temperature of coal depends upon the total oxygen content. Furthermore, experience shows that after coal has been stored for a certain time it becomes more or less immune from firing. Neither the oxygen content of the coal nor the time elapsed since it was mined was considered in the present experiments.

The temperature at which coal bursts into a yellow flame depends so much upon the rate of heating and other conditions, and varies so widely, that it is impossible to establish any quantitative relationship between it and the firing qualities of the coal.

The rate of heating or the shape of the heating curve of various coals differs widely, and this rate is far more important than any temperature which may be chosen as the ignition temperature. The shape of this heating curve is influenced by the volatile matter present, percentage of ash, moisture, sulphur, and oxygen, the air supply, the form of carbon in the partially coked residue, the size of the sample, and, perhaps, by other factors.

* See Table 7.

5. *Discussion of Ignition Temperature.*—In order to arrive at any satisfactory conclusion regarding the ignition point, some idea must first be obtained as to how “ignition temperature” may be defined.

At first thought it would seem that ignition point is the temperature at which self-heating begins and is maintained, but this, in the case of coal, seems to be a very indefinite point.

In the words of Wheeler:*

“If one adopts the generally accepted definition of Ignition Temperature as applied to gaseous mixtures, (that is, the temperature at which self-heating of the mixture begins to take place) the ignition temperature of most coals is the normal atmospheric temperature.

“A distinction between one coal and another could, no doubt, be made by determining the time taken for this self-heating from atmospheric temperature to result in flame under standard conditions.

“It may also be of value to know at what temperature a given coal must be heated in air, under specific conditions, in order that it shall burst into flame, and this temperature may, perhaps, be regarded as the ‘ignition temperature,’ the preliminary self-heating (corresponding with the ‘fire-flame period’ with gases) being ignored. Experimental difficulties, however, stand in the way, both of determining accurately the moment when flame appears in a mass of coal, and of ensuring that in a comparative test with different coals all the factors, on which the appearance of flame depends, are maintained constant.

“From a practical, as well as a theoretical point of view, it is sufficient to know at what temperature, under standard conditions, different coals begin to react with oxygen so rapidly that the ultimate appearance of flame is assured.”

Again J. F. Cosgrove† states that the ignition temperature of coal “is, of course, the temperature of ignition for the fixed carbon of the coal, because the gases are roasted out of the coal before the coke burns.”

J. S. S. Brame in his book on “Fuels” discusses the matter as follows:

“For active combustion to be initiated, a definite temperature, at least, must be maintained. The ignition point of all substances occurs no doubt at a

* “The Oxidation and Ignition of Coal.” Jour. Chem. Soc., Vol. 113, p. 949, 1918.

† “Coal, Its Economical and Smokeless Combustion.”

fixed temperature, but many conditions influence the ease with which the combustion may be started, mass and fineness of division being the most important. While a given coal, in a finely divided condition, may ignite at a low temperature, a lump of the same coal will require considerable heating before it will take fire, due to the smaller surface exposed to the air in proportion to the mass, which carries away the heat."

F. H. Sinnatt and Burrows Moore* give the following definition: "Temperature of ignition in oxygen is defined as that temperature to which fuel must be raised for its ignition to take place without the aid of external agents of inflammation." In this definition, as in some of the others, the temperature to which the coal is heated is used as the ignition point, but no account is taken of the heat due to the coal itself burning. In the experiments carried on by Sinnatt and Moore the time required to heat the samples of coal to glowing, flaming, and exploding temperatures was recorded, but in all cases these temperatures applied to the external source of heat and not to the temperature of the coal itself. As a matter of fact, if the coal is heated to a certain temperature it will carry on oxidation and combustion at an increased rate without any visible sign of such combustion, such as glowing or flaming. The temperature to which coal must be raised in order that it may carry on its own increase in temperature up to the ignition point is generally considered to be the ignition temperature.

6. *Various Suggested Ignition Temperatures.*—In the small amount of experimental work and printed discussion on ignition temperature of coal, several distinct points have been mentioned as possible criteria for comparing various coals.

- (1) The temperature at which self-heating begins.
- (2) The temperature to which coal must be raised in order that it may unite with oxygen and burn.
- (3) The temperature (zone) at which rapid self-heating begins.
- (4) The temperature to which coal must be raised in order that it may maintain its own combustion.

* Jour. Soc. Chem. Ind., Vol. 39, pp. 72-78, March 31, 1920.

- (5) Definitions on a time basis, such as the time required for a given external temperature to ignite the coal or cause it to glow.
- (6) The temperature of the glow point.
- (7) The blue flame temperature.
- (8) The crossing point of the outside heat, coal-heat curves. (Wheeler.)

The following objections may be raised to all of these methods of defining the ignition temperature.

(1) The temperature at which self-heating begins is actually atmospheric temperature, since coal is combining with oxygen even at low temperatures and producing small amounts of heat.

(2) A definition referring to the temperature at which coal will unite with oxygen and burn in reality defines the flaming temperature, as flaming is the usual visible evidence of combustion. The flaming of a coal, however, depends upon the character of the volatile matter driven off, which varies with the conditions of the heating. The flaming temperature, therefore, is very uncertain.

(3) The heating curves of all coals show an abrupt rise in the neighborhood of 200 deg. C; but there is no definite temperature during this change which may be assumed as the point where rapid self-heating begins. Hence this zone must be noted by its upper and lower limits or by its average temperature, either one of which would be rather indefinite.

(4) The temperature to which coal must be raised in order that it may sustain its own combustion without outside heat is one of the most logical points which could be chosen as an ignition temperature. However, as has already been pointed out, this temperature depends much on the size of the particles and on the air supply. In addition to the necessity of providing different values for different sizes of coal, there would probably be grave experimental difficulties in measuring the exact temperature at which combustion of the coal would be self supporting.

(5) Any definition based on time would not be a true definition but would simply serve as a criterion for comparing various coals.

(6) The glow points were found to be much easier to check with a fair degree of precision than was at first expected. As they are independent of variables such as heating rate they may be measured in various kinds of furnaces with the assurance that the results obtained will check any other operator's figures on the same coal. On the other hand the principal objection to the glow point as a definition lies in the fact that, before it is reached, a large portion of the volatile combustible matter has been driven off: and some of this matter driven off at the very lowest temperatures may have an ignition temperature lower than the glow point. In the experience of all investigators on the subject of ignition point, however, the semi-coked residue has been found to have a lower ignition temperature than any of the evolved gases. The temperatures of ignition of such gases as CH_4 , CO , etc., which are known to be given off by coal at low temperatures, are all higher than the glow points observed. True, it was found that the glow point bore practically no relation to the liability to fire while in storage, but this corresponds with the experience of Winnil and Graham,* who state that the most inflammable coal is not necessarily the most liable to spontaneous combustion.

(7) If after all the volatile matter has been driven off from a coal, the residue is brought up to a sufficiently high temperature it will flame with a non-luminous flame. Although this matter has not been investigated thoroughly it is probable that coals differ very slightly in the temperature at which this blue flame will appear. This is asserted in view of the fact that, in order to obtain the blue flame, practically all the volatile matter must be driven off previously.

(8) Wheeler has shown that the crossing point of the outside heat curve and the coal heating curve as a relative ignition temperature can be easily checked in his type of apparatus.† The objection to any methods of this kind, however, is that the results cannot be duplicated except in identical apparatus and perhaps by the same observer. Furthermore this "relative temperature" marks no definite event in the heating process. It occurs some-

* I. & C. Tr. Rev., Vol. 89, p. 336.

† See page 35 for a description of Wheeler's work.

time after the coal has begun to heat rapidly enough to cause the curve to cross the furnace curve but it does not mark the beginning of this rapid rise.

7. *Explanation.*—It is highly probable that the ultimate explanation of ease of firing, temperature of the glow point, or ignition temperature, however it is defined, must be sought in the chemical composition of the volatile products and the semi-coked residue. Brame in "Fuels" states: "Generally the proportion of hydrogen present is the determining factor in ignition point of fuel, this being well illustrated in the case of charcoal, which, if carbonized below a visible red heat, ignites at 427 deg. C., while if carbonized at a bright red heat, 900 deg. C., it ignites at approximately 537 deg. C."

In Wheeler's opinion adsorbed oxygen is the determining factor. Having made various tests on coal of different total oxygen percentages, he states that the relative ignition temperatures decrease as the oxygen content increases, but that no relationship exists between ignition temperature and other analytical data. Wheeler also makes the statement that the oxygen content given in his figures must be regarded as that of the saturated coals, because the treatment to which they had been subjected involved exposure. He also shows that a coal which takes up a large amount of oxygen shows self-heat at a lower temperature than low oxygen coals. He says, "The term self-heat may be seen to have its true significance so far as the highly oxygenated saturated coal is concerned if one considers that the loosely combined oxygen may, at a higher temperature, desire a more permanent attachment. If a reaction of the kind does take place, evolution of heat should be observed on a saturated sample if heated in an atmosphere of nitrogen."

Professor Parr suggests, in personal communication, that it is probable that oxygen adsorbed after the coal is mined has an important bearing on the glow point and the firing qualities of the coal. In his opinion, also, the nature of the gases given off just previous to the glow point will have an important bearing on the temperature of the glow, and these gases will probably show some alteration after the coal has been stored for some time. He therefore suggests that one series of experiments be undertaken to determine the chemical composition of such gases and the effect of storage

upon the glow point; and another to compare the glow points of freshly mined samples with those of samples saturated with oxygen.

Hollings and Cobb* obtained heating curves which show a marked exothermic reaction in nitrogen between 150 deg. and 250 deg. It is significant that the range 150 to 250, over which exothermic reactions in an inert atmosphere are manifest, should correspond so closely with the range over which rapid self-heating occurs in a stream of air.

* Jour. Chem. Soc., Vol. 107, p. 1109, 1915.

III. DESCRIPTION OF METHODS USED BY OTHERS

8. *Method of Sinnatt and Moore.*—Fig. 1 shows the apparatus used by F. S. Sinnatt and Burrows Moore for the determination of the relative temperatures of spontaneous ignition of solid fuels.

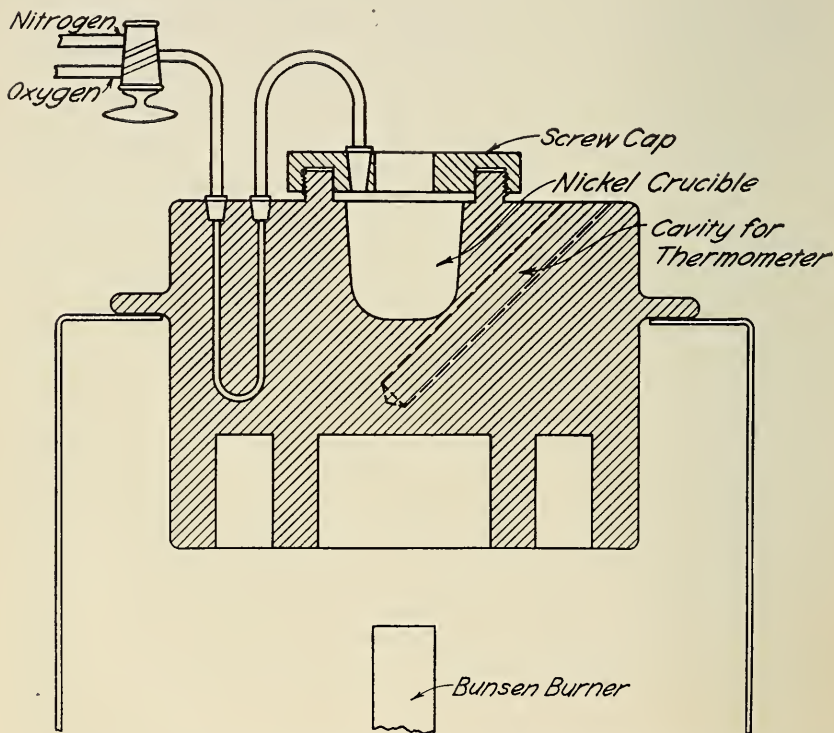


FIG. 1. APPARATUS USED BY SINNATT AND MOORE FOR DETERMINING IGNITION TEMPERATURE

The apparatus consists of a heavy cast iron vessel in which an enclosed nickel crucible may be raised to any desired temperature. The whole apparatus is enclosed in an asbestos-lined box to eliminate the influence of drafts. Before entering the combustion chamber the oxygen or other gas traverses the U-shaped tube bored into the casting.

The length of this pre-heating tube is sufficient to raise the temperature of the oxygen to that of the casting before it comes into the combustion chamber. A thermometer is placed in a cylindrical chamber arranged so that the mercury bulb is immediately below the combustion chamber. The casting is deeply grooved at the bottom in order to increase the surface exposed to the flame of a large bunsen burner which is used as a source of heat. The oxygen is dried by passing it through concentrated sulphuric acid. In certain experiments nitrogen was used to sweep out the crucible and a three-way tap was attached to the ignition meter so that nitrogen or oxygen can be passed into the combustion chamber as required.

Pulverized fuel was used in all these experiments and in each case a measured amount was dropped into the nickel crucible by means of a dropper made of glass tubing.

After the introduction of the sample a time record was kept of the following phenomena: first, commencement of visible active combustion, glowing of the mass; second, ignition as manifested by an explosion or production of flame; and third, extinction of the visible combustion. The special points noted were the time interval before visible combustion or glow occurred and the time interval before explosive or inflammatory ignition occurred. The progress of the combustion was watched through a central hole in the lid and the residue after visible combustion had ceased was examined preparatory to the introduction of a fresh charge. Tests were made on the same coal under various temperature conditions in the apparatus, and Tables 2 and 3 show some representative results obtained by these experiments.

The investigators, Sinnatt and Moore, make the following remarks regarding some of the coals:

“In the coals examined, with the exception of the two which are known to be liable to gob fires, there appear to be zones of temperature in which spontaneous ignition occurs after a much longer interval. It would appear that the coals yield volatile compounds at these ranges of temperature, which compounds have a higher temperature of ignition. It is suggested that such coals may possess the property at these temperatures of tending to extinguish any heating which may be taking place especially where the coal is in masses and the oxygen only in contact with the surface. Where, however, coal does not evolve such compounds, no such self-damping action is possible, and the result of the oxidation

TABLE 2

COMBUSTION PHENOMENA OBSERVED WITH COAL LIABLE TO PRODUCE
GOB FIRES

Time in Seconds for			Temp. Deg. C.	Remarks
Exinction	Glow	Ignition		
52	29	35	228	
55	20	35	230	
45	13	15	243	Very powerful explosion.
44	13	14	250	Powerful explosion, yellow flame.
40	11	14	267	Very powerful explosion, yellow flame.
40	10	14	278	
38	10	14	290	
39	10	14	300	Very powerful explosion, yellow-white flame.
38	9	13	310	
38	8	14	320	
25	6	9	330	
37	4	10	340	
35	3	3-14	350	Very powerful explosion at 14.
36	5	10	362	Very powerful explosion, yellow-blue-white flame.
30	2	2	378	
40	1	1	386	

TABLE 3

COMBUSTION PHENOMENA OBSERVED WITH ANTHRACITE

Time in Seconds for			Temp. Deg. C.	Remarks
Exinction	Glow	Ignition		
..	..	..	246	
..	..	..	248	
113	45	..	250	
125	32	32	256	
140	31	31	258	Yellow flame.
136	29	29	267	
120	23	23	278	
130	17	17	287	
90	19	..	296	
105	17	20	300	
110	15	45-50	310	
100	12	41	321	Yellow-white flame.
110	12	14	332	
108	9	12	342	
90	6	6	350	
100	6	28-35	362	
102	6	..	368	
120	7	50-58-70	384	Yellow-white flame at 50; blue flame at 70.
83	4	38	388	
106	5	52	390	Yellow-white-blue flame.
121	4	35	394	
111	6	17	400	Very powerful explosion.
90	5	5	412	

would be an additive one, consisting of the glowing of the solid substance and the ignition of the volatile constituents.

“In order to obtain an idea of the minimum temperature at which substances would either glow or ignite a time limit of four minutes was adopted, and the lowest temperature at which substances would either glow or ignite within this period was taken as the temperature of glowing or ignition. The time decided upon was sufficiently long for most practical purposes.”

The values obtained for a number of substances are given below.

TABLE 4
MINIMUM TEMPERATURES FOR IGNITION IN OXYGEN

Substance	Minimum Temperatures in Oxygen, Deg. C.			
	Glowing	Ignition	Holm's Figures	Brame's Figures
Anthracite coal.....	250	258	440	500
Hoo cannel coal.....	225	230		
Wigan Arley coal.....	184	188		370
Wigan yard coal.....	219	219		to
Indian coal.....	242	242		425
Coal liable to gob fires, No. 1.....	228	228		
Coal liable to gob fires, No. 2.....	228	228		
Gas coke.....	Above 398	Above 398		
Coalite.....	Above 396	Above 396		
Wood charcoal.....	248	248		
Irish peat.....	300	300		
Cellulose.....	324	324	360	
Fusain.....	367	367		
Paraffin wax.....		246	310	

The investigators continue:

“Early in the experiments it was found that the degree of fineness of the particles of the fuel had a considerable influence upon the minimum temperature at which spontaneous glowing would occur, and preliminary observations have been made to ascertain broadly the influence of the fineness of the fuels examined. The results are recognized as being quite preliminary but, as they show that the degree of fineness is a most potent factor, it is desirable to place them on record.”

An attempt was likewise made to ascertain the influence of the volatile matter present in the fuels on the temperature at which spontaneous ignition occurs. In order to do this the apparatus was

TABLE 5

MINIMUM TEMPERATURE AT WHICH SPONTANEOUS GLOWING OCCURS

Description	Degree of Fineness		
	1/200	1/100 to 1/200	Mixture of 2 parts 1/200 and 1 part 1/100-1/200
	Temp. Deg. C.	Temp. Deg. C.	Temp. Deg. C.
Indian coal.....	240	Above 380	242
Hoo canal.....	229	Above 390	277
Yard coal.....	219	Above 396	233
Coal liable to gob fires, No. 1.....	232	348	...
Coal liable to gob fires, No. 2.....	225	298	...

arranged so that nitrogen could be passed into the combustion crucible through the three-way tap shown in the diagram, until the volatile matter had been driven off from the coal. The procedure was as follows: The ignition meter was raised to any desired temperature, the nitrogen turned on, and a charge of coal introduced from the dropper. The coal was kept for 15 minutes in the atmosphere of nitrogen. At the end of this period the supply of nitrogen was interrupted and oxygen introduced into the crucible. The time interval before glowing or ignition occurred was then determined. The crucible was at once emptied of any residue and a fresh charge or blank was dropped in, having the oxygen continually flowing. The process was repeated for different temperatures and the results are given in Table 6.

Sinnatt and Moore say further:

“It may be observed that at temperatures only slightly above the minimum temperatures previously recorded for the occurrence of glowing, all the fuels, except the Hoo canal, which had been previously exposed to nitrogen failed subsequently to glow or ignite in oxygen within the time interval of four minutes whereas the control samples examined immediately afterwards behaved exactly as previously recorded. The conclusion to be deduced from these observations is that the volatile matter in the coal has considerable influence upon the liability of the fuel to glow. The non-appearance of the glowing in the nitrogen-treated charges appears especially important where the material has only been

heated to temperatures not appreciably above the minimum temperatures previously recorded for glowing to occur. It would therefore appear that the oxidation of the volatile constituents evolved contributes very largely to the heating effect, which leads to the mass of coal becoming heated, and consequently to the production of a glow.''

TABLE 6

TIME INTERVAL BEFORE GLOWING OCCURS IN NITROGEN AND OXYGEN

Exposed to Nitrogen for 15 Min. before Admission of Oxygen				No Previous Exposure to Nitrogen before Admission of Oxygen	
Air Dried Sample	No. of Seconds before Glow- ing	Remarks F—flame E—explosion	Temp. Deg. C.	No. of Seconds before Glow- ing	Remarks
Arley coal 200 mesh	20	Feeble E at 20 sec. Yellow F.	398	1	More powerful E at 43 sec. Blue F.
	10	Feeble E at 10 sec.	378	1	
	35	Residue partly ash.	285	3	More powerful E at 25 sec.
		No glowing.	240	13	Explosion at 25 sec.
		No glowing.	226	25	
		No glowing; unburnt residue.	260	8	
Anthracite coal 200 mesh		No glowing.	388	7	Yellow F.
		No glowing.	362	12	Feeble E.
		No glowing.	325	22	
		No glowing.	300	30	
Hoo cannel 200 mesh	8	Feeble E at 12 sec.	382	1	Fairly powerful E.
	10	Very feeble E at 12 sec. ..	363	3	Fairly powerful E.
	11	Very feeble E.	340	3	No ignition
		No glowing.	270	16	No ignition
Irish peat 200 mesh	20	No glowing; residue charred	390	1	
		No glowing; residue charred	385	1	
		No glowing; residue charred	350	2	
Cellulose 200 mesh	15	No glowing; residue charred	396	1	
		No glowing; residue charred	369	1	
		Feeble E at 15 sec.	340	30	Powerful E at 30.
Fusain 200 mesh		No glowing.	389	6	Yellow F.
		No glowing.	370	10	

9. *Remarks on Method of Sinnatt and Moore.*—In the method just described for obtaining the ignition temperature it is obvious that the temperatures obtained are those of the external source of heat and not of the coal itself. The very small sample used, about 0.0052 g., did not in any way affect the temperature of the apparatus

to an extent which could be measured. It is impossible with this type of apparatus to measure the temperature of the coal itself at any stage in the process. The chief value of the method lies in the possibility of determining the relative ease with which coals fire while in storage, but the reasons for the ease of firing are not made known. It is to be noted in Tables 2 and 3 that the glow points are in all cases reached before ignition takes place.

The results obtained by these men, although taken in a manner very different from that of the present investigation correspond in every way with those recorded later in this paper.

Present experiment, however, determined that the glowing temperatures were far above those recorded by Sinnatt and Moore in their table. This explains why these investigators obtained no glow after the volatile matter had been driven off, as shown in Table 6. It has been found by several investigators that the giving off of the volatile products in an atmosphere of oxygen or air is accompanied by exothermic reactions which add their heat to the mass of the coal. Therefore the explanation of the failure to obtain the glow point is that, after the volatile matter had been driven off in an atmosphere of nitrogen, very little heating effect could be obtained from the exothermic reactions, while the semi-coked residue which was later submitted to the action of oxygen was at too low a temperature to glow, and its combustion in oxygen was going on too slowly to raise the temperature of the coal mass to the glow point. Had the oxygen been admitted at a trifle more rapid rate, the glow would undoubtedly have appeared. In Tables 2 to 6 the temperatures indicated are those of the apparatus. Undoubtedly if the temperature of the coal sample had been measured it would have been found to be considerably above that of the apparatus when the coal glowed.

10. *Method of Wheeler.**—The apparatus used by Wheeler is shown in Fig. 2. It consists of a tube holding the sample of coal, and an electrically heated sand bath surrounding the sample tube. Air is drawn through the sample at a constant rate; the temperature of the coal mass itself and the external sand bath being taken simultaneously. Experiments were made on 40-g. samples of 150-mesh and 240-mesh coal. The temperature of the sand bath was slowly raised

* Jour. Chem. Soc., Vol. 113, p. 949, 1918.

at a uniform rate and the temperatures inside and outside the coal taken at intervals.

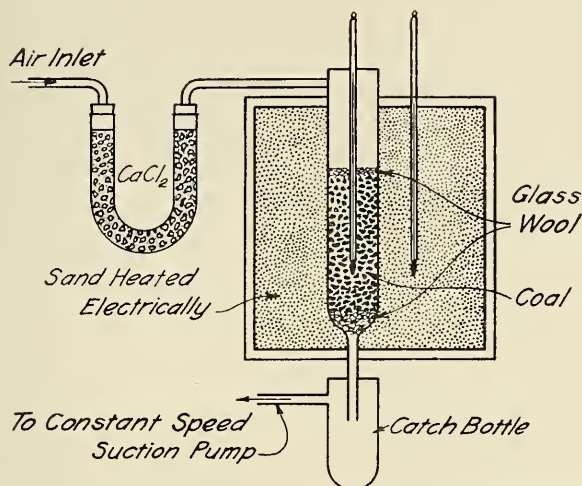


FIG. 2. APPARATUS USED BY WHEELER FOR DETERMINING IGNITION TEMPERATURE

The results of these experiments were plotted as two time-temperature curves, one showing the heating rate of the coal sample and the other showing the heating rate of the external sand bath. In all cases the temperature within the coal eventually began to rise rapidly above that of the sand bath, but, due to the kind of thermometers used, it was impossible to carry the experiment to very high temperatures. It may, however, be assumed that after the heating curve began to rise rapidly it would only be a matter of time before the coal would flame. A curve, characteristic of these results, is shown in Fig. 3.

Wheeler says:

“It will be seen that the temperature at which the reaction velocity becomes so rapid that inflammation of the coal is imminent is clearly indicated to be at about 205 deg. C. It is evident also from the gradual approach of the two curves that the heat due to the oxidation of the coal was sufficient, despite the loss to outgoing air, to raise the temperature of the coal above that normally due to the heat received from the sand bath when the reaction was proceeding at a temperature of 125 deg. C. The latter temperature might be regarded as

the ignition temperature of the coal, (that is, the temperature at which self-heating begins) under the conditions of the experiment.

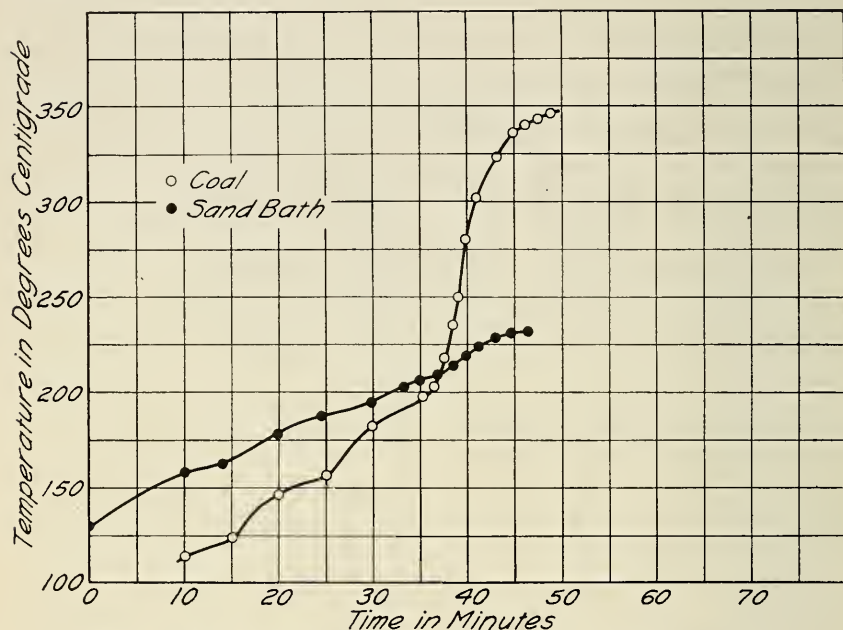


FIG. 3. HEATING CURVES OF COAL AND SAND BATH (WHEELER)

“For comparative purposes, however, it was found preferable to record as the relative ignition temperature the point at which the coal temperature curve and the sand bath curve cross, as this point is clearly defined. It is the temperature at which rapid self-heating begins and is therefore in conformity with Nernst’s definition of the ignition temperature for gaseous mixtures.”

Wheeler treated several samples of coal having various oxygen contents and determined that the most highly oxygenated fuels are those most liable to self-heat. His results are shown in the following table, in which the oxygen contents were taken on the basis of ash-free, dry coal and must be regarded as those of the “saturated” coals as the treatment to which the coals had been subjected involved exposure.

11. *Remarks on Method of Wheeler.*—The heating curves obtained by Wheeler are similar to those obtained here except for the fact that much higher temperature readings could be obtained by the

use of the thermocouple than with glass thermometer. Wheeler proved unquestionably that the crossing point of the coal-heat, outside-heat curves is a definite point and that he is justified in using this point

TABLE 7
RELATIVE IGNITION TEMPERATURES FOR VARIOUS OXYGEN CONTENTS

Oxygen Content Per Cent	Rel. Ig. Temp. Deg. C.	Oxygen Content Per Cent	Rel. Ig. Temp. Deg. C.
11.1	165	8.3	182
11.1	165	8.1	180
11.1	167	8.0	183
10.6	177	7.6	192
10.5	176	7.4	188
10.3	176	7.3	185
9.9	177	7.0	192
9.9	179	6.7	195
9.9	179	6.6	206
9.5	178	6.4	200
9.2	178	5.6	210
8.8	183	5.4	200
8.8	187	5.1	217
8.7	186	4.9	195
8.6	183	4.7	220
8.5	185	3.9	200

as a means of comparison for different coals. It is to be noted, however, that these results can be duplicated only in the identical apparatus used by Wheeler, as the shape of the heating curve is affected by the characteristics of the furnace and the conditions under which the experiment is run. Similar to Wheeler's experience, present experiments obtained heating curves showing abnormal tendencies between the range 150-250 deg. C. Fig. 13, page 54, shows this bulge in the heating curve very markedly.

IV. GLOW POINT

12. *Glow Point as Ignition Temperature.*—If a sample of coal is heated carefully so that the rate of heating is uniform throughout the sample it will be found that at a certain very definite temperature the sample will assume a uniform glow. This glow takes place rather suddenly, if one neglects the occasional glowing particles on the edge of the pile, and it is quite certain that it is not entirely due to incandescence of carbon alone, since the color of the glow is much brighter than black bodies assume at this temperature. Furthermore the change from the black coal to the brightly glowing mass is too rapid to be explained by incandescence. After coal has been heated for about twenty minutes under the conditions of the experiment, the change from black to glowing coal takes place sometimes within a half minute of time. With much sulphur present a rather confusing glow occurs earlier than this, but with low sulphur coals the appearance of the glow is quite abrupt. With a few unexplained exceptions the temperatures observed for the glow points of samples of the same coal check fairly closely.

Strange to say there is no very abrupt rise in the heating curve beyond the glow point. To be sure the sample of coal is still rising in temperature, but the rather abrupt rise that might be expected after rapid combustion begins is not apparent. Professor Parr suggests that this might be explained by assuming that at this point adsorbed oxygen, which is probably already partly combined with the coal substance, completely combines with the carbon to form carbon dioxide. The additional heat thus produced is probably not sufficient to alter the temperature at a rate which can be measured by this method. Whatever the explanation of the glow point may be, it occurs at a very definite place in the heating curve and may therefore be accepted definitely as the ignition temperature.

13. *Determination of Nature of Glow Point.*—An attempt to determine the intensity of the action at the glow point was made by submitting a sample of coal above the glow point and one below the glow point to a blast of oxygen. No definite statements can be made regarding the results of this experiment because of the difficulty of

handling a single small particle of coal, determining its temperature, and keeping it from being blown away by the blast. The procedure was to place a small sample of coal in a wire cylinder, raise it above the glow point, and then allow it to cool until the glow just disappeared. As nearly as could be determined the sample was always cooled by the blast of oxygen if its temperature was below that of the glow point, but if it was glowing in but a small part its combustion would rapidly accelerate until it was burning fiercely.

Another experiment made to determine the exact nature of the glow point was to heat a sample of coal in the furnace to about 300 deg., at which time the furnace current was shut off and temperature readings continued. The results of several trials indicated that the coal heated to a temperature just short of the glow point, then the rise stopped. It must not on that account be supposed that further heating is necessary to carry on combustion; indeed, if the concentration of oxygen be increased but slightly the heating will continue even beyond the glow point. The experiment does indicate, however, that the glow point is reached only with a certain amount of difficulty, and that, even if the heating is started, the glow point will not be reached unless a certain concentration of oxygen is exceeded.

These experiments to ascertain the nature and characteristics of glow point furnish in nearly all cases added argument for establishing the glow point as the ignition temperature. The glow point is, moreover, the first visible evidence of combustion, as it occurs in all cases before flame or explosion takes place.

Professor Parr believes that the temperature of the evolved gas has something to do with the glow point and advises a further series of tests to determine the nature of this gas and the influence of weathering on the ignition temperature.

V. DESCRIPTION OF APPARATUS

14. *General Discussion.*—The apparatus used in these experiments is shown in Figs. 4, 5, and 6. It consists of an electric furnace of the resistance type, a motor generator set furnishing direct current, a method of regulating the voltage of this generator by means of shunt field resistance, a Hoskins thermocouple, and a millivoltmeter.

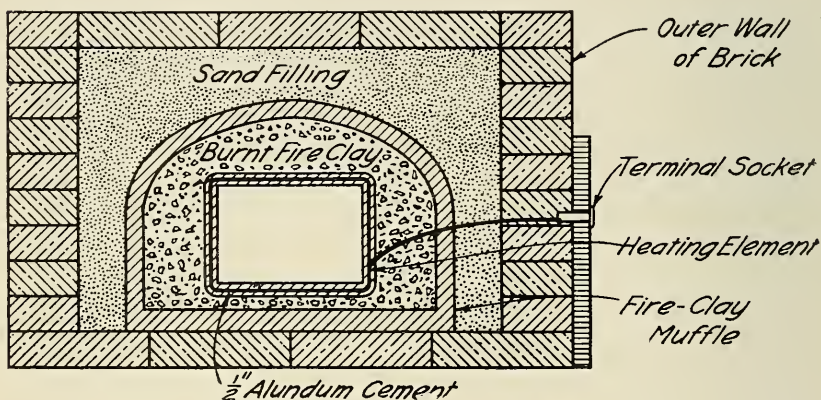


FIG. 4. SECTION OF FURNACE

15. *The Furnace.*—The furnace that was used was made purposely for these experiments and is shown in Figs. 4 and 6. The heating element was made by wrapping 70 feet of special resistance wire, No. 14 B. & S. gauge, having a resistance of 625 ohms per mil-foot and a total resistance of 10.78 ohms, on a reinforced paper form. After the element was wound it was cemented with alundum cement which was allowed to dry before the form was removed. The element was then placed in an ordinary muffle and the space between the wires and the muffle filled in with burned fire-clay. The ends of the wire were brought through holes bored in the muffle to terminals placed on the outside of the furnace. The muffle was then set on a stand and walled in with brick.

A thin layer, $\frac{1}{4}$ in., of alundum cement was spread on the wires inside the furnace as a lining, and this construction brought the

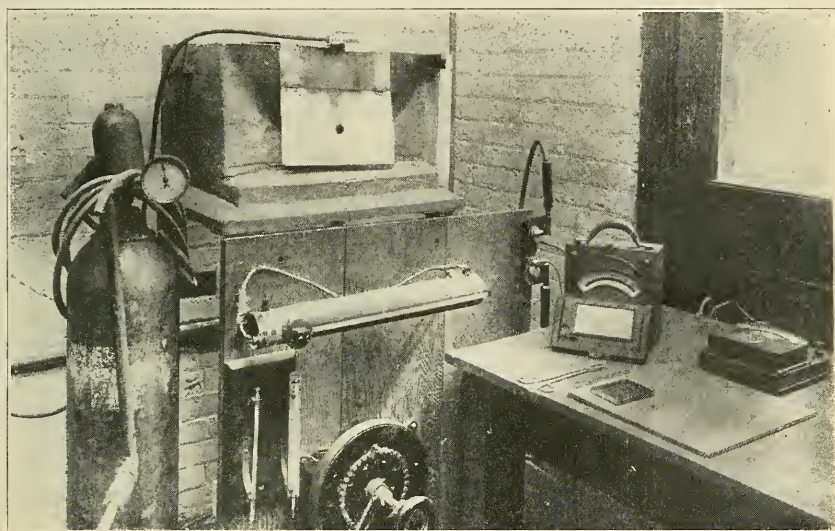


FIG. 6. APPARATUS USED IN PRESENT INVESTIGATION

heating element very close to the interior surface. Because of this the furnace heated rapidly. As the inside was worn off it had to be plastered up occasionally to avoid danger of short circuiting the wires inside the furnace.

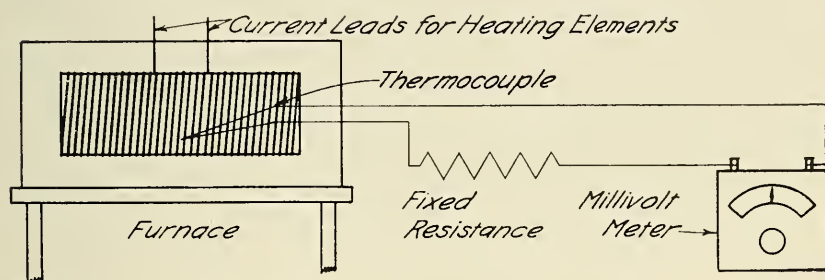


FIG. 5. WIRING DIAGRAM OF FURNACE

16. *Power Required.*—Because of its ease of measurement and control direct current was chosen as the power for the heating element. The laboratory was already equipped with a 250 volt D. C. compound generator driven by a 440 volt A. C. motor. The voltage of this generator could be regulated by the shunt field control. The switchboard of this set was provided with a field rheostat, but it was found that the successive points on this rheostat gave jumps in the voltage which were larger than desired. A smaller field rheostat, shown in Fig. 6, was therefore wired in series with the switchboard rheostat for the purpose of giving finer adjustments of the voltage. With this auxiliary rheostat it was possible to regulate the voltage so that the current flowing through the heating element was constant at all times.

It was found that 15 amperes gave the best results in the experiments, heating the coal sample as rapidly as possible without showing any great amount of lag due to the low conductivity of the coal. In order to get 15 amperes through the furnace a voltage of about 160 was required, depending somewhat upon the temperature of the heating element. An ammeter was placed in the circuit as shown in the wiring diagram.

VI. TEMPERATURE MEASUREMENT

17. *Method Employed.*—The thermocouple used was a Hoskins base metal couple, the negative side of which is an alloy called Chromel, composed of chromium and nickel, and the positive side Alumel, made of aluminum and nickel. Within the range of temperature desired, this couple has a guaranteed millivoltage of approximately 40 mv. As the millivoltmeter used has a maximum scale reading of 20 mv., it was necessary to use the couple in series with a resistance in order to reduce the scale reading. The couple, resistance, and millivoltmeter were standardized against the melting points of three pure metals.

The placing of a resistance in series with the couple gave the added advantages of high resistance temperature measurements. When the resistance of the couple circuit is extremely low the millivoltmeter reading is affected by the variation of the resistance of the hot wire, imperfect connections, a change in the length of the connections, and many other things. If a high resistance in the circuit is kept at a constant temperature the above mentioned factors will not affect the result appreciably.

The gases produced from the burning of the coal were very corrosive and the couple frequently broke. However, due to the high resistance mentioned above, frequent re-calibrations proved that after the wire was re-welded there was very little difference in the reading for any given temperature.

18. *Calibration of Thermocouple.*—The metals used in the calibration of the couple were aluminum, melting point 659 deg. C., lead, melting point 327 deg. C., and tin, melting point 232 deg. C. These three metals were used because their melting points fell within the limits of temperature most used in this work, and because they gave the most satisfactory calibration points. It was found that zinc gave an uncertain calibration point.

19. *Fixed Junction Correction.*—No attempt was made to keep the fixed junction of the thermocouple at zero degrees, but room temperatures were observed on all the tests and the correction for

room temperature made by the method described by L. H. Adams before the A. I. M. E., September, 1919. In this method the millivolts corresponding to room temperature are added to the observed millivoltmeter reading, giving a value of the voltage which will correspond with the true temperature of the hot end of the couple. In this same paper Adams includes a table showing temperature difference per millivolt with this type of couple. This difference varies slightly for different temperatures, but there is less variation than in noble metal couples or "Constantan" combinations. In other words the calibration curve is nearer a straight line for the Chromel-Alumel couple than for the other types.

TABLE 8
TEMPERATURES FOR VARIOUS MILLIVOLTMETER READINGS
DEGREES CENTIGRADE

M. V.	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
00.	0	46	90	134	179	223	268	312	358	404	451	498	543	587	630
05.	2	48	92	136	181	225	270	314	360	406	453	500	545	589	632
10.	5	50	94	138	183	227	272	317	363	409	456	503	548	591	634
15.	7	53	97	141	186	230	275	319	365	411	458	505	550	594	636
20.	9	55	99	143	188	232	277	321	367	414	460	507	552	596	638
25.	12	57	101	145	190	234	279	324	369	416	463	510	555	598	641
30.	14	59	103	147	192	237	281	326	372	418	465	512	557	600	643
35.	16	61	105	149	194	239	283	328	374	421	467	514	559	602	645
40.	18	64	108	152	197	241	286	330	376	423	469	516	561	605	647
45.	21	66	110	154	199	243	288	333	379	426	472	519	564	607	649
50.	23	68	112	156	201	246	290	335	381	428	474	521	566	609	651
55.	25	70	114	158	203	248	292	337	383	430	476	523	568	611	653
60.	28	72	116	161	205	250	294	340	386	433	479	525	570	613	655
65.	30	75	119	163	208	253	297	342	388	435	481	528	572	615	657
70.	32	77	121	165	210	255	299	344	390	437	484	530	574	617	659
75.	35	79	123	168	212	257	301	347	392	440	486	532	577	620	
80.	37	81	125	170	214	259	303	349	395	442	488	534	579	622	
85.	39	83	127	172	216	261	305	351	397	444	491	536	581	624	
90.	41	86	130	174	219	264	308	353	399	446	493	539	583	626	
95.	44	88	132	177	221	266	310	356	402	449	496	541	585	628	

The table of temperature differences just mentioned could not be employed directly because of the high resistance used, but a similar table was prepared based on Adams's temperature differences and our own calibration, and is given above as Table 8. This table was used in connection with all temperature determinations of the glow points of the various coals. In making the experiments on the heating curve of the furnace and the heating curves of the standard coal a slightly different resistance was used and a separate calibration was

made. The table calculated from this is not included. Several recalibrations of the thermocouple during the tests on coals showed practically no deviation from the values given in the above mentioned tables.

Taking all factors into consideration, such as the accuracy of the calibration, loss of heat, lag of the couple, approximations in the calculations, and similar sources of error, the readings are thought to be correct to about 5-8 degrees.

VII. METHOD OF MAKING TESTS

20. *General Procedure.*—The method used in these experiments was as follows: a sample was placed in a flat pan and the thermocouple inserted in the top of the coal so that the junction was entirely submerged, see Fig. 7; the shields were then placed over the furnace and the current turned on; observations of the temperature were made at half-minute intervals and temperatures of glowing, flaming, or other phenomena, were noted.

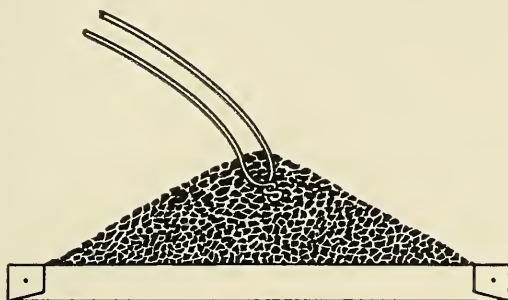


FIG. 7. DEPTH OF IMMERSION OF THERMOCOUPLE

21. *Weight of Sample.*—For the determination of glow point alone it was unnecessary to standardize the weight of the sample as the glow point is independent of the weight. In order to get the heating curves of various coals for purposes of comparison, all the conditions, including the weight of the sample, were standardized. It was found that about 35 g. of coal could be heated at a fairly uniform rate without any great amount of lag in any part of the pile; furthermore this weight was large enough to make possible a good immersion for the couple.

The pan in which the sample was placed was rather flat, with low sides, so that the air had free access to the sample and all parts of the pile could be observed from the front of the furnace.

22. *Setting of Thermocouple.*—It was found rather important to have the proper immersion of the thermocouple in the sample in order to check results. The object of course was to get the

average temperature of the sample itself, or more properly to observe what happened to that coal which immediately surrounded the couple. If the point of the couple was too deep it was in no way certain that what was observed on the surface was also taking place within the pile. On the other hand if merely the tip of the couple was placed in the coal, temperatures observed applied to the gases coming off the coal rather than to the interior of the pile itself.

23. *Distribution of Heat and Temperature Lag.*—In starting these experiments it was uncertain just how much dependence could be placed on temperature measurements made in this manner. The number of degrees by which the thermocouple lagged, the temperature of the coal itself, the distribution of temperature throughout the sample, the lag of the top of the coal pile behind those individual particles next to the hot furnace floor and the difference in temperature between the surface of any particle and its interior were all uncertain factors. In order to eliminate some of the uncertainty regarding the lag of the couple and the distribution of the heat throughout the pile, a cooling curve was taken on a given coal sample whose heating curve had previously been observed, and the glow point on the cooling curve noted. In observing this the temperature at which the coal lost its glow and returned to its black aspect was noted. In every case these cooling curve glow points were the same as those observed on the heating curves. These results indicate that the lag of the couple was very slight and that under the conditions of the experiment the distribution of the heat throughout the sample was fairly even.

The lag of various parts of the coal pile depends on the rate of heating. The first work done was to determine the proper rate of heating and the proper size of particles. It was found that with 35 g. of sample, 15 amperes supplied to the furnace heated the sample at a fairly uniform rate and the glow observed on low sulphur coals was fairly uniform throughout the pile.

No attempt was made to determine the lag of the interior of the individual particles behind the surface because this is relatively unimportant: that is, if we assume that glow point and flame, and as a matter of fact all oxidation, are superficial phenomena, it matters very little what the temperature of the interior of the particle may be. Experiments on large and on very small particles showed that

the total amount of surface exposed to the action of oxygen bore no relation to the temperature of the glow point. This fact also gives some weight to the statement that the temperature of the interior of the particle is of no importance.

Some error is probably introduced by radiation of heat from the walls of the furnace to the thermocouple, especially during the early stages of the heating when the furnace is at a higher temperature than the coal. About 8 in. of the couple was exposed in this way, protected only by small, two-holed, porcelain sleeves $\frac{5}{8}$ in. long, extending to within an inch of the coal sample. At the glow point, however, the error introduced by radiation is probably small, inasmuch as the walls of the furnace and the sample of coal are both at a temperature higher than the air temperature represented by the heating curve of the furnace.

VIII. DESCRIPTION OF EXPERIMENTS

24. *Heating Curve of Furnace.*—The first work that was done was to compare the heating rate of various currents on the empty furnace. After several quantities of current were tried, both on samples of coal and on the empty furnace, it was concluded that 15 amperes gave the most satisfactory heating curve. To determine the heating curve of the furnace, six runs were made under exactly similar conditions. The furnace was allowed in each case to cool to room temperature and the temperature of the room itself was kept fairly constant by steam heat. The shield on the front of the furnace was in the form of a door hinged at the top, so that, when

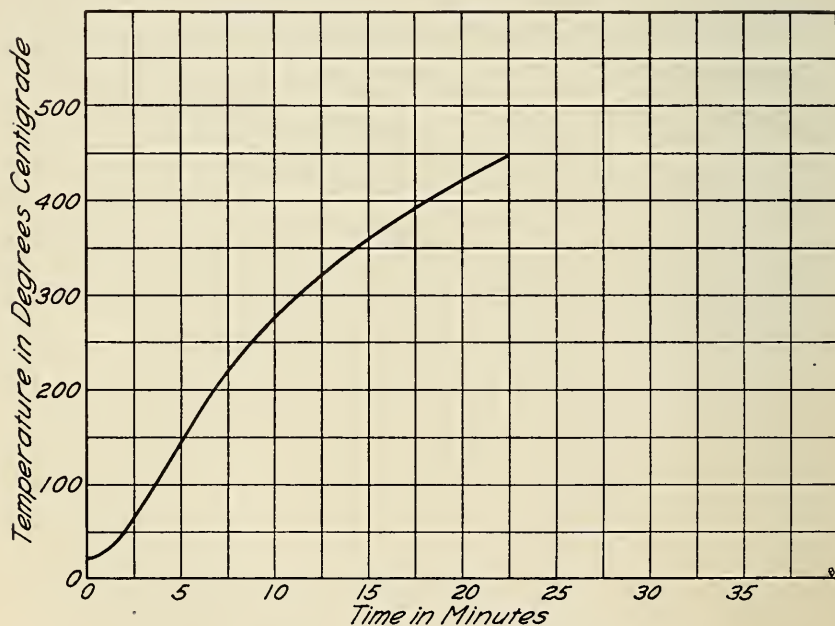


FIG. 8. AVERAGE HEATING CURVE OF EMPTY FURNACE

lowered, it always assumed the same position. This door was made of sheet metal having in the center a $\frac{3}{4}$ -in. hole through which the action within the furnace could be observed. The shield was not tight

fitting; air could enter the furnace around the edges and through the hole, and also by an opening provided in the back of the furnace for the admission of the couple. The furnace was placed in a corner of the laboratory away from air currents, so that even with openings through which air might pass to cool the furnace, individual observations of heating curves differed but slightly from the average. The maximum deviation of any single heating curve from the average of all was less than 15 deg. The heating curve of the furnace calculated from these six runs is shown in Fig. 8.

25. *Work on Standard Coal.*—A low sulphur and ash coal was hand sorted from available samples in order to provide a relatively pure coal which could be used in standardizing a method for the determination of the glow point. On analysis this coal was found to contain 8.60 per cent ash and 1.25 per cent sulphur. This coal was prepared by crushing to a maximum size $\frac{1}{4}$ in. and then screening through the following screens: $\frac{1}{4}$ in. and $\frac{1}{8}$ in., and 14, 28, 48, 100, 150, and 200 mesh. Thirty-five gram samples were then heated at a constant rate based on a 15 ampere supply of current to the furnace. Care was exercised to give the same immersion to the couple in every case. Three or four check runs were made on each size to determine the effect of size of particles on the glow point. The results of these runs are shown in the following tables:

TABLE 9
GLOW POINT TEMPERATURES, STANDARD COAL
DEGREES CENTIGRADE

Size of Coal	Run No.				
	1	2	3	4	5
$\frac{1}{4}$ in.— $\frac{1}{8}$ in.	469	469	463	469	454
$\frac{1}{8}$ in.—14 mesh.	469	473	445	...	...
14—28 mesh.	466	467	471	435	...
28—48 mesh.	517	510	456	501	...
48—100 mesh.	472	470	472	480	...
100—150 mesh.	466	468	...	...	...
150—200 mesh.	470	466	468	477	...
Through 200	466	472	469	478	...

Average Temperature—472 deg. C.

TABLE 9A

GLOW POINT TEMPERATURES, STANDARD COAL
DEGREES FAHRENHEIT

Size of Coal	Run No.				
	1	2	3	4	5
¼ in.—⅜ in.....	876	876	865	876	849
⅜ in.—14 mesh.....	876	883	833	...	...
14—28 mesh.....	871	873	880	815	...
28—48 mesh.....	963	950	853	934	...
48—100 mesh.....	882	878	882	896	...
100—150 mesh.....	871	874	...	...	...
150—200 mesh.....	878	871	874	891	...
Through 200 mesh.....	871	882	876	892	...

In at least one run on each of these sizes the furnace was started from the room temperature and observations of the temperature made at half-minute intervals to determine the heating curves. As these heating curves vary but little they were averaged to represent the heating curve of the standard coal. This heating curve was plotted against the average heating curve of the furnace and is shown in Fig. 9.

The results obtained for the glow point on the sizes finer than 100 mesh were uncertain because there soon formed over the pile a crust of glowing coal which made it impossible to determine just what was going on around the thermocouple. This is explained by the fact that in the smaller sizes very much more surface is exposed to the action of oxygen, and the surface of the pile is actually at a higher temperature than the interior. The temperature of this crust, however, is not transmitted to the thermocouple; hence results on the finer sizes are apt to be false. No attention was paid to this initial glow of the crust but as soon as it seemed that the pile was glowing to a depth equal at least to the immersion of the thermocouple the temperature was noted. In every case the results on the fine coal were very uncertain and it would require a great deal of practice before an operator could distinguish the proper point and check himself.

The most consistent results were obtained on the larger sizes from ¼ in. to ⅛ in., and ⅛ in. to 14 mesh. With this method any

one of these sizes could be adopted for use as a standard, and in the subsequent work on individual coals these sizes were used.

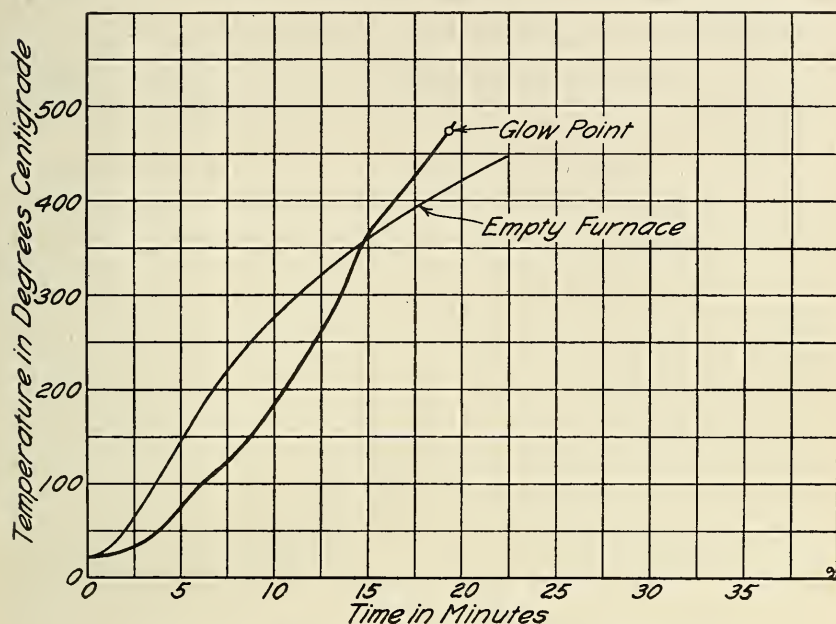


FIG. 9. AVERAGE HEATING CURVE OF STANDARD COAL

26. *Effect of Air Supply.*—To test the influence of various oxygen percentages on the temperature of the glow point the coal was heated under various conditions.

Case I.

The furnace was heated in the ordinary manner but a small stream of oxygen was admitted through the shield. This was done to give the same effect as a larger air supply, without the objectionable cooling effects of a blast of air.

Case II.

The furnace was heated in the usual manner to a temperature of about 300 deg., at which point the current was shut off. Oxygen was then admitted slowly to determine if the glow point could be reached without the action of outside heat.

Case III.

Conditions of Case III were similar to those of Case II except that a blast of oxygen was played on the sample.

In all three cases the action of oxygen was to increase the heating rate. This, however, was done at the expense of accuracy, inasmuch as the pile heated locally and a uniform glow was difficult to obtain.

In Case I a satisfactory glow point was observed by keeping the quantity of oxygen admitted very low. This glow point was found to be the same as when the coal was heated under ordinary conditions.

In Cases II and III when the current was shut off at about 300 deg., it was found that if sufficient oxygen was admitted to the sample the glow could be produced without the assistance of external heat, but if insufficient oxygen was admitted, or no oxygen at all, the coal would heat to some extent but the glow would not appear.

In Case III if the blast of oxygen was strong enough and the coal sample had not yet reached the glow point the latter would actually be cooled off, but if the glow point was first obtained and the coal then submitted to the blast of oxygen it would burn fiercely. This would indicate that the rate at which coal is raised to the temperature of the glow point is largely dependent upon the air supply, but that the actual temperature of the glow point, no matter how it is approached, is not altered.

27. *Effect of Ash.*—To determine the effect of ash on the glow point a sample of coal was mixed with pure shale washed from coal, and run under standard conditions. The following table of results

TABLE 10
EFFECT OF ASH ON GLOW POINT

Glow Points of Coal 35 g. Sample		Glow Points, 25 g. Coal Mixed with 10 g. Shale	
Deg. C.	Deg. F.	Deg. C.	Deg. F.
525	977	526	979
521	970	524	975
512	954	520	968

shows that the temperatures for the glow points under the two conditions were practically the same.

One peculiar feature of this experiment is illustrated in the heating curves shown in Fig. 10. It will be seen that high refuse coal heated more rapidly in the early stages than clean coal, but at a retarded rate towards the end of the process. This retardation was due to the fact that the quantity of coal in this experiment was less

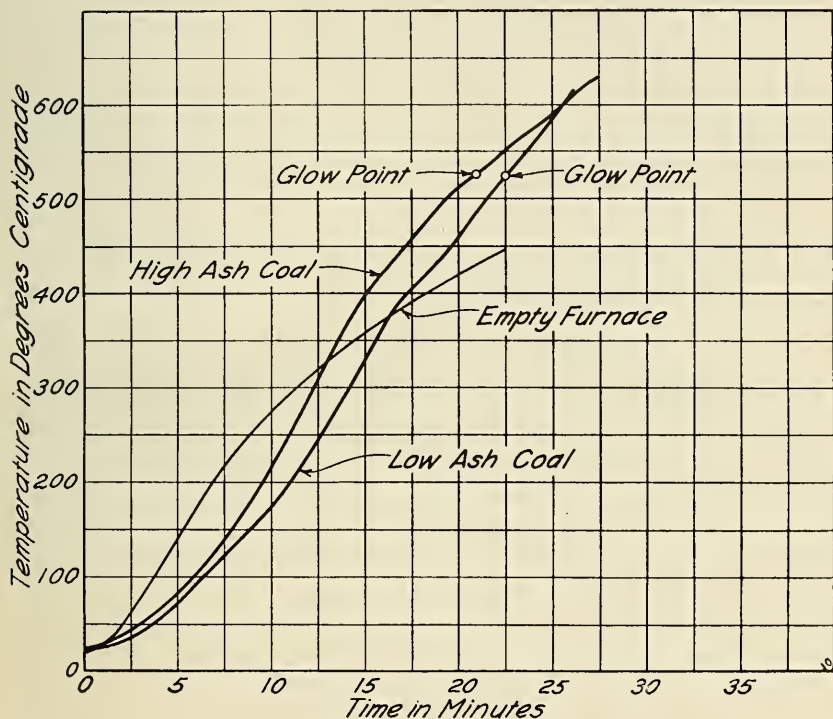


FIG. 10. EFFECT OF HIGH ASH ON HEATING CURVE

by 10 g. than in the other test; hence the quantity of heat derived from the coal after self-heating began was less. The rapid heating of the high refuse material early in the experiment may be explained by all or any of the following considerations:

- (1) Shale has a lower specific heat than coal. No figures on the specific heat of shale are known, but similar substances, such as

brick, kaolin, and ashes, have specific heats ranging from 0.120 to 0.224, whereas that of coal is given by the same authorities as 0.314. To be sure the relative thermal conductivities of these substances enter into the discussion to some extent since it is a rate of heating that is being observed. From the best figures obtainable the conductivity of coal is approximately double that of shale, which fact would tend to offset the opposite relation of the specific heats to some extent.

(2) Pure coal requires more heat in the early stages to drive off moisture and other volatile matter.

(3) In the high refuse samples, more air is supplied to the individual particles or the air which reaches any particular piece is higher in oxygen than in the case when the coal particles are more concentrated.

If the true explanation for this rapid heating of high ash coal could be found it might explain the prevalence of gob fires, refuse pile fires, and fires in piles of dirty coal.

28. *Effect of Sulphur.*—The effect of sulphur was tested in like manner by adding to the coal an amount of pyrite equivalent to about 6 per cent sulphur in the sample, the particles of pyrite being the same size as the particles of coal. As this sample was heated, the individual particles of pyrite and the coal just adjoining them glowed at a comparatively low temperature. Undoubtedly these individual particles were at or above the temperature of the glow point but their accumulated heat had a negligible effect on the temperature of the entire mass. From this it was assumed that the glow point of the coal substance proper is independent of the percentage of sulphur. It must be admitted, however, that this constitutes an objection to the use of the glow point as a criterion for ignition temperatures since it is well known that sulphur in pyrite burns at a lower temperature than coal and consequently assists in its combustion and ignition.

29. *Effect of Moisture.*—The effect of moisture was studied by saturating a 35 g. sample of coal and running it under standard conditions. As in the case of the experiments on the high ash coals the tabulated results show that the glow points for the high moisture samples are practically the same as for the original coal.

TABLE 11

EFFECT OF MOISTURE ON GLOW POINT

Glow Points of Coal		Glow Points of Coal Saturated with Moisture	
Deg. C.	Deg. F.	Deg. C.	Deg. F.
526	979	515	959
525	977	529	984
512	954	526	979

Fig. 11 shows the heating curve under these conditions. In this curve the evaporation of the moisture is clearly indicated at 100 deg., but in spite of this retardation in heating the coal reaches the glow point sooner than that which was not dampened. In this

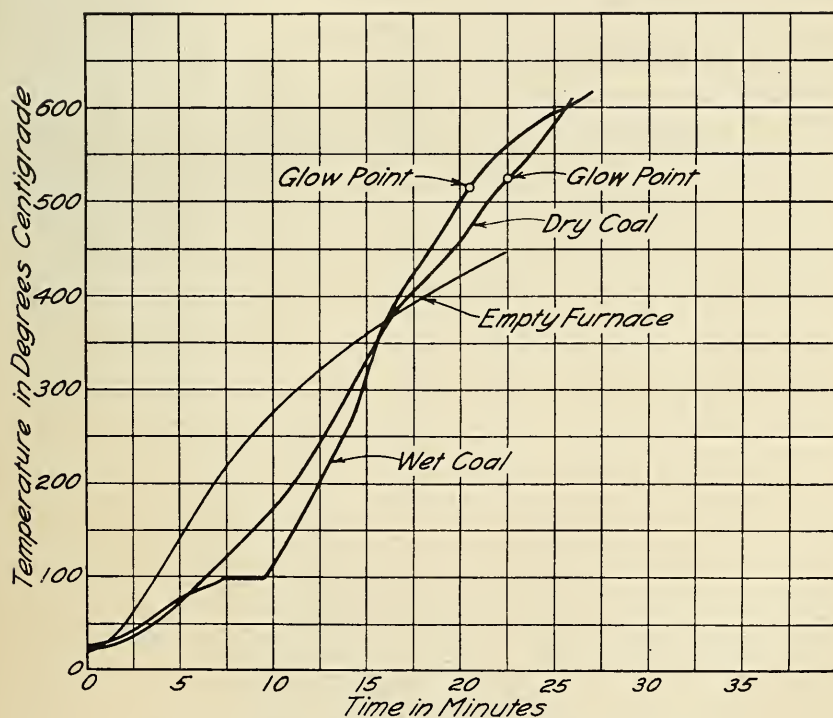


FIG. 11. EFFECT OF HIGH MOISTURE ON HEATING CURVE

experiment the coal showed an unusual tendency to flame before the glow point was reached. Several trials had to be made before the glow point could be obtained prior to the flame. It would seem, therefore, that there is some weight to the contention that better combustion is obtained from coal which has been previously moistened. Certainly in these experiments the moistened coal was more inflammable.

IX. RESULTS AND DISCUSSION

30. *Tests on Ten Different Coals.*—In addition to the standard coal ten different coals were tested and their glow points determined as shown in Table 12.

TABLE 12
GLOW POINTS OF COALS TESTED

Coal from	Glow Points		Average		Difference in Deg. from Average
	Deg. C.	Deg. F.	Deg. C.	Deg. F.	Deg. C.
Herrin, Williamson County.....	433	811	440	824	—7
	451	844	...	...	11
	437	819	...	...	—3
Georgetown, Vermilion County.....	453	847	456	852	—3
	453	847	...	...	—3
	466	871	...	...	10
Bennett Station, Vermilion County.....	451	844	...	...	—5
	498	928	496	925	2
	494	921	...	...	—2
Old Ben No. 8.....	526	979	521	970	5
	525	977	...	...	4
	512	954	...	...	—9
Lignite, Manitoba, Canada.....	535	995	526	978	9
	522	972	...	...	—4
	520	968	...	...	—6
Castle Gate, Utah.....	535	995	528	982	7
	524	975	...	...	—4
	525	977	...	...	—3
Lovington, Moultrie County.....	526	979	530	986	—4
	534	993	...	...	4
Lincoln, Logan County.....	559	1038	558	1036	1
	557	1035	...	...	—1
	598	1108	600	1112	—2
Anthracite, Pennsylvania.....	598	1108	...	...	—2
	605	1121	...	...	5
Coke.....	607	1125	606	1123	1
	607	1125	...	...	1
	605	1121	...	...	—1

The average glow points shown in Table 12, with the exception of those for anthracite and coke, show no apparent relation between the glow point and the liability to fire while in storage. For example the lignites and the coal from Castle Gate, Utah, which acts very much like a lignite in coking experiments, both have a comparatively high glow point, whereas the coal from Herrin, Williamson

County, Illinois, which ordinarily shows no unusual tendency to fire while in storage, has the lowest glow point of all the coals.

31. *Determination of Various Glow Points.*—So far in these experiments no attempt had been made to determine the oxygen of the coals nor the amount of oxygen absorbed after the coal was mined. It had been attempted simply to develop a method for the determination of ignition temperatures and to show that this method would give consistent results. Wheeler had shown that relative ignition temperature varied with the total oxygen content. Before making any positive statements, therefore, particularly concerning the relation between the glow point and the tendency to fire in storage, it was necessary to make a series of tests determining the glow point of the same coal both freshly mined and after it had been exposed to the action of oxygen for various periods.

Coal No. 1.

(Herrin-Watson Coal Co., Herrin, Williamson County, Illinois.)

This coal had the lowest glow point temperature of any of the coals tested. Three runs were made on the $\frac{1}{4}$ - $\frac{1}{8}$ -in. size, giving results as shown in the table. The heating curve showed no unusual characteristics and is not included.

Coal No. 2.

(Sharon Coal Company, Georgetown, Vermilion County, Illinois.)

In taking the heating curve of this coal it was observed that apparently strong exothermic reactions were taking place before the glow appeared. In order to be sure of this fact and to see how nearly the heating curves could be duplicated, a second curve was taken on this coal. The two heating curves plotted against the furnace heating curve are shown in Fig. 12.

Coal No. 3.

(Bennett Station, Vermilion County, Illinois.)

The peculiarity of this coal may be seen in the heating curve, Fig. 13, which displays rather unusual evidence of exothermic reactions before the glow point is reached. In this particular coal the glow appears on a rather steep portion of the curve, whereas in most of the other samples the glow point marks a flattening of the curve rather than a steeper inclination.

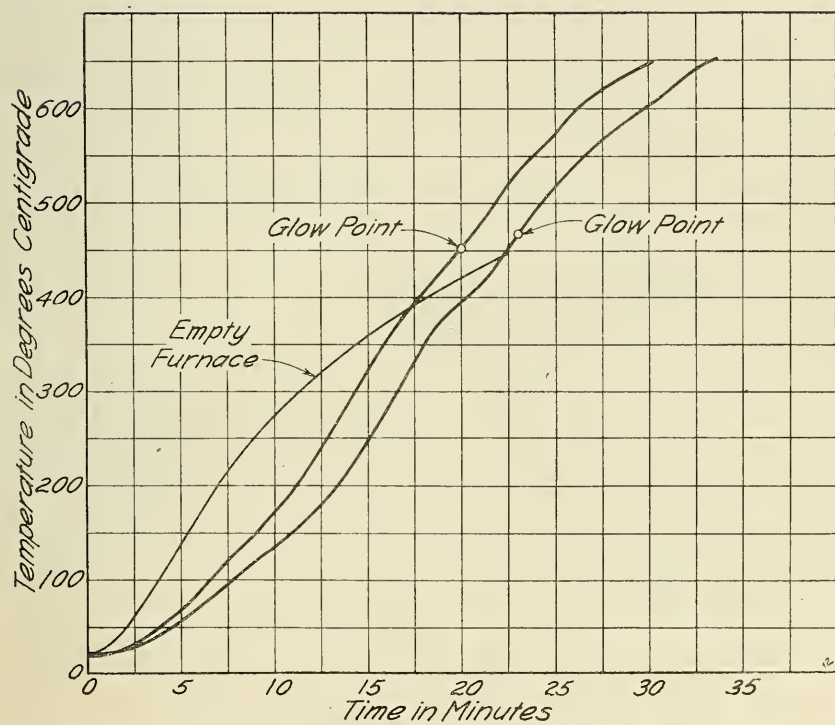


FIG. 12. HEATING CURVES OF COAL No. 2, SHOWING SIMILARITY OF CURVES FOR CONSECUTIVE RUNS

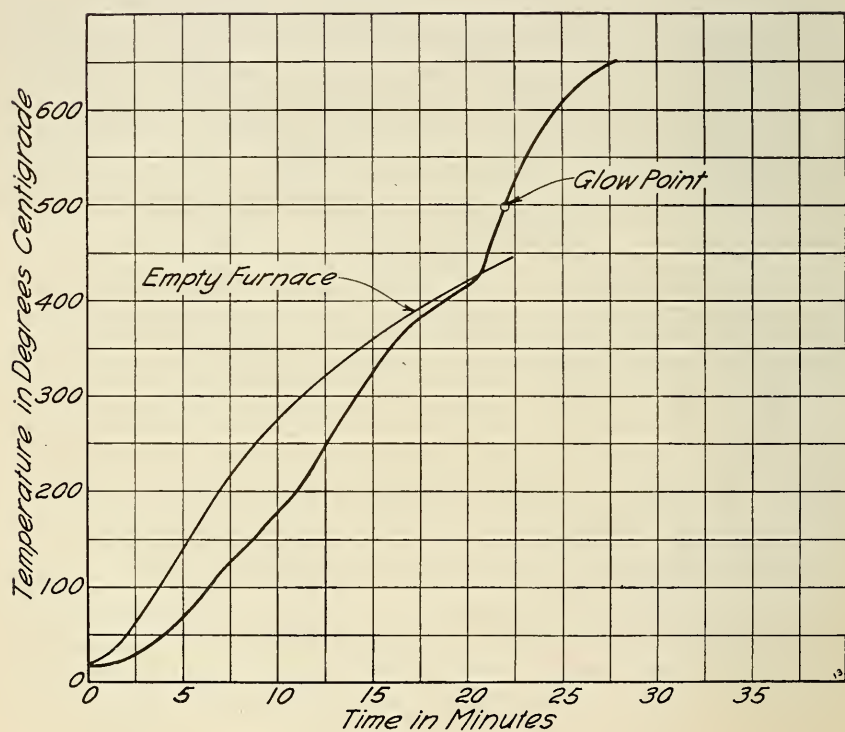


FIG. 13. HEATING CURVE, COAL NO. 3

Coal No. 4.

(Old Ben Corporation, Mine No. 18, Johnson City, Illinois.)

This coal was used in the experiments to show the effect of high ash and high moisture on the heating curve. The heating curve may be seen in Figs. 10 and 11.

Coal No. 5.

(Lignite from Manitoba, Canada. Heating Curve, Fig. 14.)

Up to a temperature of about 150 deg. this curve is comparatively flat, as could be expected from the nature of the coal. During this range moisture and other volatile matter requiring heat for vaporization were being driven off. After this process was complete, however, there was a large discrepancy in temperature between the coal and the furnace; consequently the coal heated rapidly to the glow point.

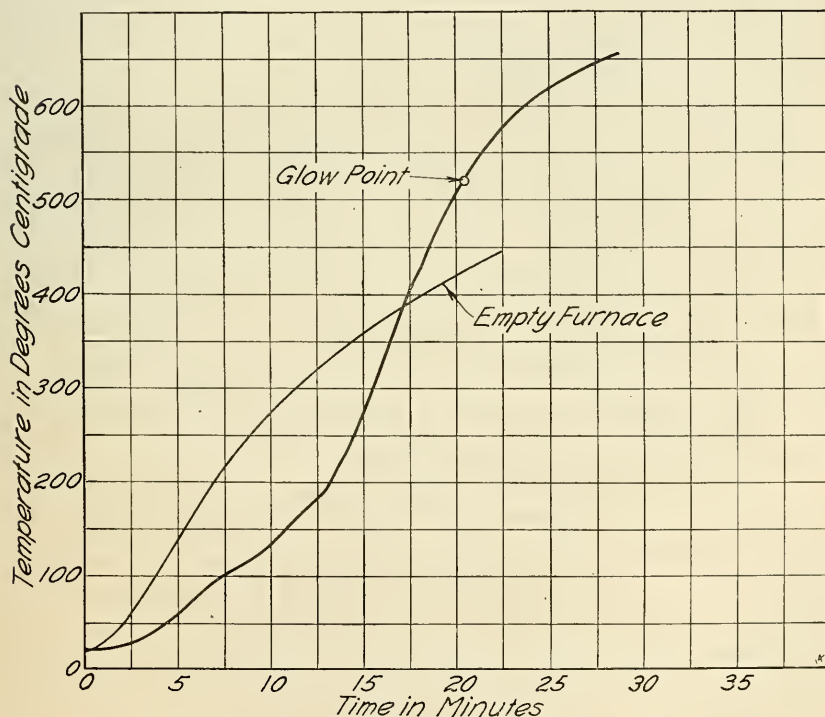


FIG. 14. HEATING CURVE, COAL No. 5

Coal No. 6.

(Bituminous Coal. Castle Gate, Utah. Heating Curve, Fig. 15.)

In coking experiments made under the direction of Professor Parr this coal acted very much as a lignite. The moisture content, however, is only 3.5 per cent, which is much less than that of lignite; consequently the lower range of the curve does not show the flattening tendency that is apparent in the lignite curve. As may be seen in Table 12, the glow point of this coal was practically the same as that of the lignite.

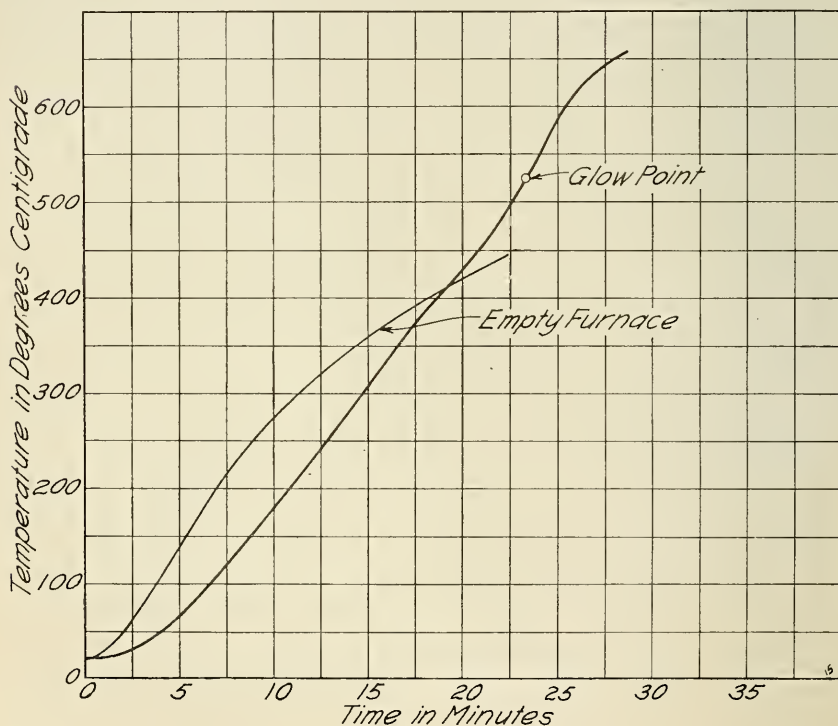


FIG. 15. HEATING CURVE, COAL NO. 6

Coal No. 7.

(Lovington, Moultrie County, Illinois. Heating Curve, Fig. 16.)

Coal No. 8.

(Lincoln, Logan County, Illinois. Heating Curve, Fig. 17.)

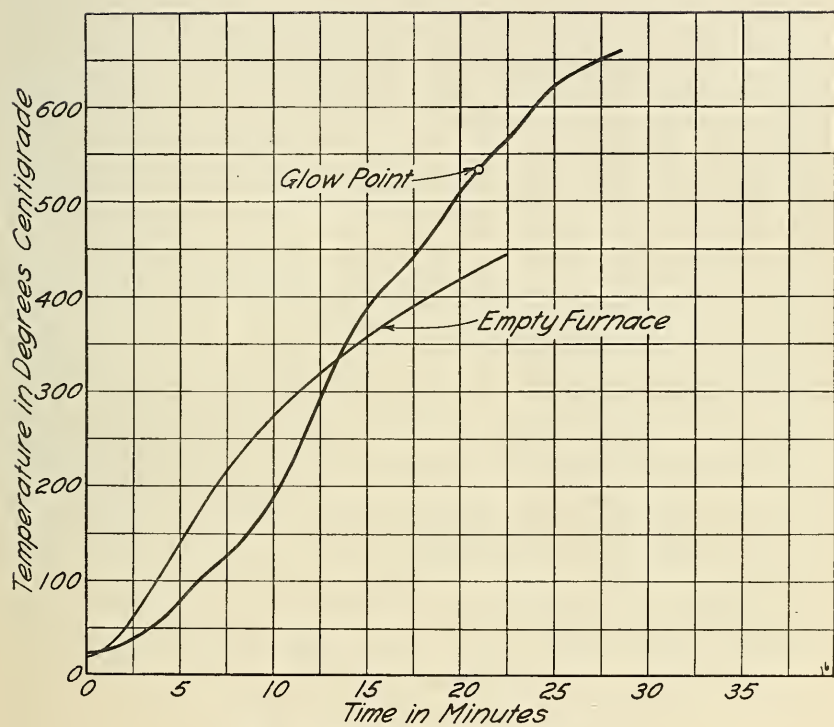


FIG. 16. HEATING CURVE, COAL NO. 7

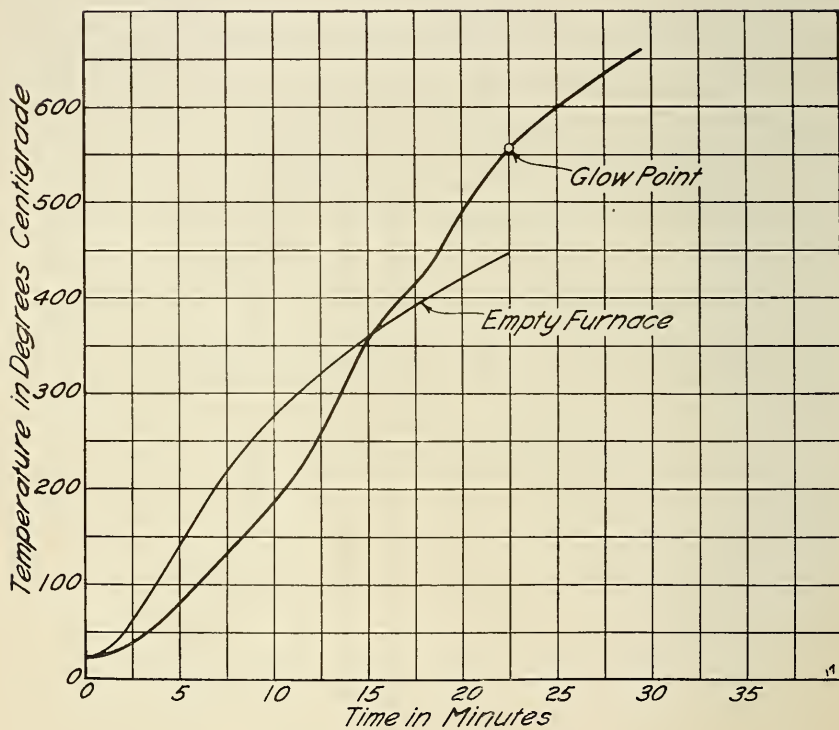


FIG. 17. HEATING CURVE, COAL NO. 8

Coal No. 9.

(Anthracite. Pennsylvania.)

The glow point of this coal was much higher than any of the bituminous coal or the lignite, as might be expected if the glow point is any criterion of ignition temperature. The heating curve as shown in Fig. 18 is an extremely smooth curve from the lower temperatures to the glow point, showing no exothermic reactions because of the relatively small amount of volatile matter present. As the heating of anthracite coal to the glow point was more a process of furnishing heat to a substance with a definite specific heat than heating something which contains certain volatile constituents requiring heat for their vaporization, its heating curve more nearly parallels that of the empty furnace. To be sure at a temperature of about 200 deg., anthracite began to furnish heat and its heating curve to gain on the furnace

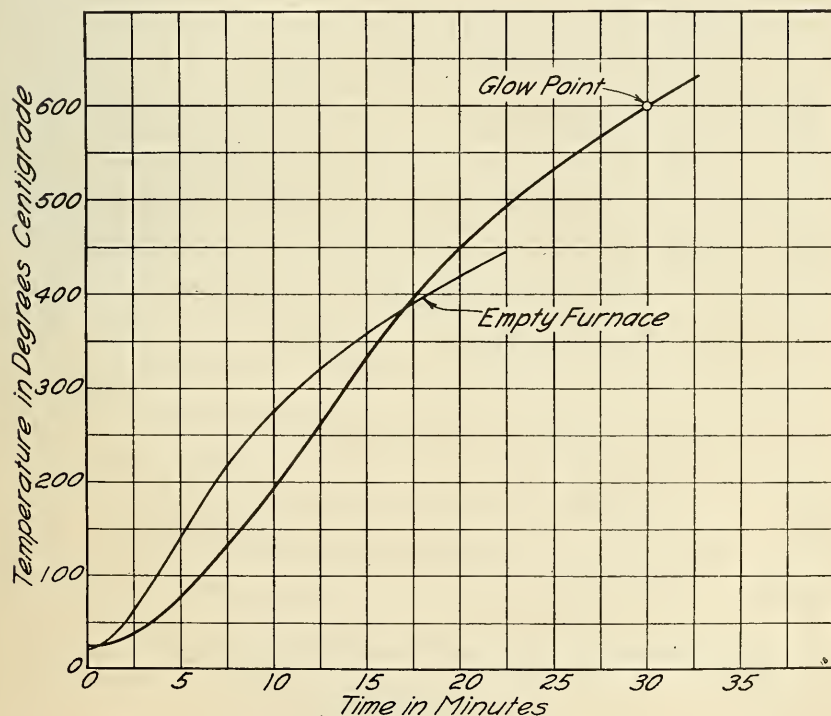


FIG. 18. HEATING CURVE, COAL NO. 9 (ANTHRACITE)

curve, but this gain is not as abrupt nor as well marked as in the case of the higher volatile substances illustrated in the previous diagrams. It is more apparent than ever in this heating curve and in that of coke, Fig. 19, that the glow point was merely an incident in this heating and did not mark a noticeable change in the heating rate.

Coke.

(Heating Curve, Fig. 19.)

This heating curve is even smoother and closer to the curve of the furnace than that for anthracite. However, at approximately the same temperature, 200 deg., self-heating began, causing a gain in the coke heating curve over the furnace curve.

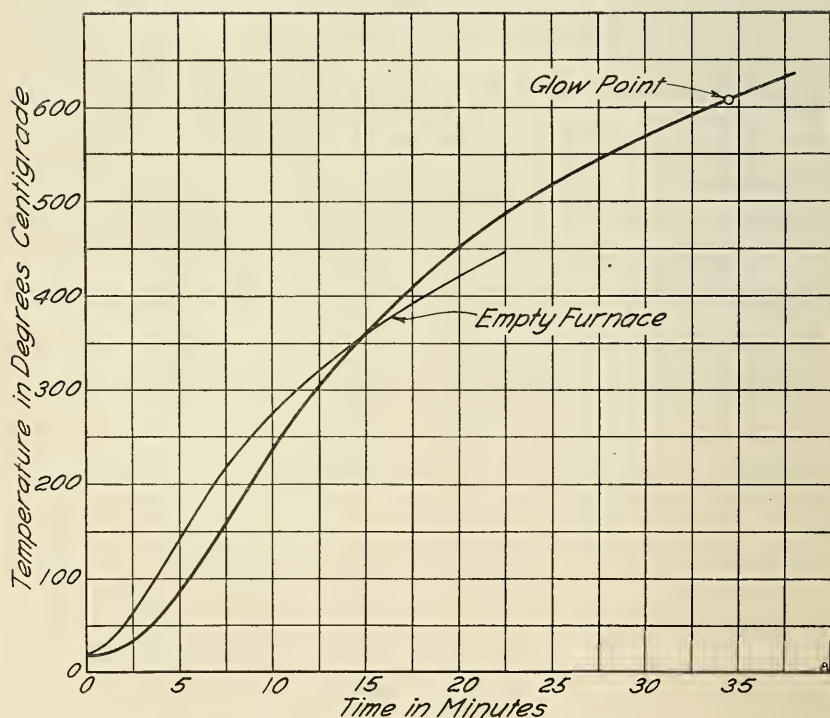


FIG. 19. HEATING CURVE, COKE

X. CONCLUSIONS.

32. *Summary.*—

(1) The “Ignition Temperature” of coal means nothing unless it is applied to some definite point in the process of heating the coal, which can easily be determined and duplicated..

(2) The temperature at which the coal glows seems to be the most logical point to choose as the ignition temperature. It is easily observed, can be duplicated with a fair degree of accuracy, and marks the beginning of visible combustion.

(3) The glow point is probably affected by the oxygen content of the coal, and perhaps by other agencies made active by weathering.

(4) The glow point is not affected by ash, moisture, size of particles, slight variations from the normal air supply, or rate of heating.

(5) There is no indication at present that the glow point bears any direct relation to the liability to fire while in storage. Perhaps if a series of tests were made on the glow points of freshly mined coal new information would be brought to light which would lead to the discovery of some more definite relation between glow point and the firing qualities of the coal.

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AN INVESTIGATION OF THE PROPERTIES OF CHILLED IRON CAR WHEELS

PART I WHEEL FIT AND STATIC LOAD STRAINS

BY
J. M. SNODGRASS
AND
F. H. GULDNER



BULLETIN No. 129

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AN INVESTIGATION OF THE PROPERTIES
OF CHILLED IRON CAR WHEELS

PART I
WHEEL FIT AND STATIC LOAD
STRAINS AND STRESSES

CONDUCTED BY
THE ENGINEERING EXPERIMENT STATION
UNIVERSITY OF ILLINOIS

IN COÖPERATION WITH
THE ASSOCIATION OF MANUFACTURERS OF
CHILLED IRON CAR WHEELS

BY
J. M. SNODGRASS
AND
F. H. GULDNER

ENGINEERING EXPERIMENT STATION

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AN INVESTIGATION OF THE PROPERTIES OF CHILLED IRON CAR WHEELS

PART I. WHEEL FIT AND STATIC LOAD STRAINS

I. INTRODUCTION

1. *Preliminary.*—The car wheel is one of the important factors which have made possible the great development of transportation facilities that has occurred within the past seventy years. Although during this time many types of car wheels have been utilized, only five have come into common use, thus demonstrating their general fitness to meet the severe service conditions now existing. These five types are the rolled steel wheel, the forged steel wheel, the cast-steel wheel, the built-up wheel, and the chilled iron wheel. The first three types are manufactured by processes and of materials indicated by their names. The built-up wheel, as the name suggests, consists of either a spoked wheel of cast-iron, or a disc, upon the periphery of which is attached a tire of rolled steel.

The chilled iron wheel is essentially a cast-iron wheel whose tread, or that part of the wheel which comes in contact with the track, consists of an extremely hard metal capable of resisting both excessive deformation and wear, while the remainder of the wheel consists of relatively soft and easily machined cast-iron. The hardened tread is produced by chilling the outer circumference or tread of the wheel when the metal is poured. A more detailed description of the chilled iron wheel, with such additional information regarding its manufacture and properties as may be of service in the interpretation of the results of this investigation, is presented in Appendix A, page 75. In service, the use of the built-up wheel is confined almost wholly to passenger coaches, locomotives, and tenders, whereas the rolled, forged, and cast-steel wheels, and the chilled iron wheel are in use under locomotive tenders, passenger coaches, and freight cars. For various reasons the use of steel wheels predominates in passenger service, while the relative cheapness of chilled iron wheels, together with certain other advantages they possess, makes their use almost universal for freight service.

Seventy years of transportation history show a very great increase in car capacity, and, as a consequence, greatly increased wheel loads both in freight and in passenger service; yet the same period of time does not show a proportionate increase in the size and presumably in the strength of the car wheels used, a fact which is especially true with relation to the chilled iron car wheel used in freight service. As an example, the wheel load in freight car service has increased from 5 000 lbs. on the 10-ton capacity car of 1870, to 25 000 lbs. on the 70-ton capacity car of the present time, an increase of 400 per cent. During the same time the weight of the wheel has been increased from 525 lbs. for the 5 000-lb wheel load to 850 lbs. for the 25 000-lb. wheel load, or an increase of 60 per cent. From comparisons of this kind it is evident that increase in wheel load has not been accompanied by a corresponding increase in the weight of the wheel. As the chilled wheel of today is carrying the imposed loads with a low percentage of failures, it seems evident that unduly large factors of safety may have been present in the earlier period. Furthermore, the conclusion may be drawn that, if the policy of increasing wheel loads continues without providing proportionate increase in wheel weight, a time will eventually arrive when the wheel no longer can withstand the imposed loadings.

Considering the relatively small amount of trouble that car wheels in general have given heretofore, and referring in particular to the chilled iron wheel, it is not surprising to find that but little experimental work has been done with regard to car wheel problems.* As a consequence, the stresses and strains in car wheels due to various service conditions have been largely a matter of speculation and conjecture, based mainly on the relatively small amount of information available from failures of wheels in service. By a careful analysis of these failures, railroads and wheel manufacturers, independently at first, then in small groups, and later in conjunction with the Master Car Builders Association, have, with comparatively slight changes in design and through moderate additions of metal, been able to produce wheels which have at all times satisfied service requirements.

* Report of Tests to Determine the Stress in the Plate of Cast Iron Wheels Due to the Heat Produced by the Brake Shoe. L. E. Endsley. Proc. West. Ry. Club, p. 194, Mar. 27, 1914.

Standardization of Chilled Iron Crane Wheels. F. K. Vial. Proc. A. S. M. E., Dec. 1914.

Owing to the desirability of obtaining definite information concerning the magnitude and distribution of stresses in chilled car wheels and of determining the limitations of these wheels as used today, and with a further view of improving the chilled iron wheel in order to meet future requirements, a coöperative agreement was entered into by the ASSOCIATION OF MANUFACTURERS OF CHILLED CAR WHEELS and the UNIVERSITY OF ILLINOIS. Essentially the agreement provided that the Association furnish funds for all special equipment required and defray all expenses incident to carrying on the investigation, and that through its Engineering Experiment Station, the University conduct the research, provide the use of available laboratories, shops, and office facilities, and publish a report of the results. The principal items of the agreement are presented in Appendix C, page 94. After the approval of the agreement, committees under whose direction the work should be carried on were formed, and a conference was held in March, 1916, at which a tentative program was laid out.

2. *Scope of Investigation.*—The original program was revised and extended at various times until it has embraced tests to determine:

- (1) the strains caused by mounting the wheel on its axle;
- (2) the strains caused by the static or wheel loads;
- (3) the ultimate breaking strength of flanges, and strains caused by flange pressure;
- (4) the strains, due to temperature gradients in the wheel, caused by brake application;
- (5) incidental problems related to the above.

The present bulletin gives the results of a series of strain-gage tests in connection with the items (1) and (2) just mentioned; that is, tests made to determine the strains produced within the wheel by mounting it on its axle, and by the application of wheel loads. The strain within the wheel caused by forcing it on an axle was first determined for two 33-in. 725-lb. M. C. B. wheels. The same pair of mounted wheels was then subjected to static loads ranging from 20 000 to 200 000 lbs. per wheel, and the resulting strains noted. The loading effect was produced by applying the load to the axle by means of a testing machine, and allowing the wheels to transmit it to a pair

of rails, the conditions being similar to those found in service. A similar set of tests was carried out with a pair of 33-in. 740-lb. Arch Plate type of wheels. Additional tests for the purpose of obtaining more complete information concerning the mounting strains than was given by the above mentioned tests were then made upon a pair of 33-in. 725-lb. M. C. B. wheels. The reporting of the results of the mounting and static tests upon the wheels just mentioned is the purpose of the present bulletin. It is intended to publish additional reports dealing with tests to determine the ultimate strength of flanges, the effect of flange pressure, and the effect of brake applications, together with the related problems of brake friction and the thermal expansion of cast-iron.

3. *Acknowledgments.*—The Association of Manufacturers of Chilled Car Wheels appointed GEORGE W. LYNDON and F. K. VIAL, President and Consulting Engineer, respectively, of the Association, as a committee to assist in promoting the investigation. In addition to the financial help given by the Association, the investigation owes much to the assistance rendered by this committee in the way of technical advice, helpful interest, and effective coöperation throughout the progress of the work. The Engineering Experiment Station of the University of Illinois appointed EDWARD C. SCHMIDT, Professor of Railway Engineering, and ARTHUR N. TALBOT, Professor of Municipal and Sanitary Engineering, as a committee in charge of the investigation. Professor Schmidt withdrew from the University to enter the United States Military Service during the late war, and was replaced on this committee by JOHN M. SNODGRASS, Professor of Railway Mechanical Engineering. In the direction of the investigation this committee has had frequent conferences with the committee of the Association of Manufacturers of Chilled Car Wheels.

The greater part of the experimental work was carried on by F. H. GULDNER, who became connected with the work a few months after its inception. R. E. TURLEY and O. S. BEYER, JR., both of whom were connected with the experimental work during the earlier part of the investigation, did much in the way of developing methods, making some early tests, and getting the work under way. The committees are further indebted to H. F. MOORE, Professor of Engineering Materials at the University of Illinois, for his assistance and advice upon various matters relating to the investigation.

II. PROBLEM OF CAR WHEEL UNDER LOAD

4. *General Statement Concerning Wheel Loading.*—A car wheel in service is subjected to conditions which produce stress within the wheel. These stress-producing conditions may exist in a great variety of combinations, and give rise to stresses of a more or less complex nature. The principal causes of stress in a car wheel are:

- (1) manufacturing processes, which may cause initial stresses;
- (2) forcing the wheel on the axle, or mounting;
- (3) the proportion of the car loading supported by one wheel;
- (4) the lateral pressure on the wheel flanges produced by rounding curves, by wind, or by the unevenness of the track;
- (5) non-uniform temperatures in the wheel caused by brake application;
- (6) centrifugal force, when the speed is high.

Rotation, moreover, complicates the problem still further by introducing impact and repeated stresses.

Important initial stresses, if existent, can be traced to improper manufacture. The process of annealing* has for its object the elimination of this type of stress, and, if it is properly performed, either removes the initial stresses entirely or reduces them to small magnitude. Concerning the magnitude and distribution of this type of stress, relatively few definite data exist.†

After being cast, the wheel is bored slightly smaller than its axle and forced upon it. In general, this mounting of the wheel on its axle produces compression in radial directions and tension in tangential directions throughout the entire wheel. The intensity of these stresses is greatest at the bore and decreases toward the tread until the intersection of the inner and outer plates is reached, at which point there may be a slight increase in magnitude, beyond which a decrease again occurs. At the rim or tread of the wheel these stresses become negligible.

* See Appendix A, p. 45.

† Interstate Commerce Commission. Report of the Chief of the Bureau of Safety, Covering the Investigation of an Accident which Occurred on the New York Central Railroad near Waterloo, Indiana, March 21, 1917, pp. 8-10.

On being placed in service, the wheel doubtless encounters conditions that produce an exceedingly complex stress distribution within it. The static load, or proportional part of the total weight of the loaded car, is transmitted by the axle to the wheel and through it to the rail. As a consequence, tensile and compressive stresses of varying intensity are set up in all parts of the wheel. The compressive stress produced by the static load is at a maximum on the radial line connecting the center of the wheel and the point where the rail is in contact with the wheel. No general statement can be made as to where the actual maximum stress occurs on this radial line.

The order in which additional stresses occur in the wheel is necessarily determined by the conditions under which the wheel is operating. Lateral pressure on the wheel flange when rounding a curve adds to or modifies stresses already existing in the wheel. Both the static load and flange thrust stresses may be considerably augmented by impact, as in the case when heavily loaded cars operate on imperfectly aligned track, or strike guard rails and crossings at high speed. The application of the brakes for purposes of car retardation generates heat that must be dissipated by the brake shoe and the wheel. Within the wheel this dissipation of heat produces unequal temperatures or gradients between different points, with resultant stresses. If not properly handled by operating men, long continued brake application, such as occurs in mountainous regions, may occasion abnormal stresses and be a source of trouble, with attendant possibilities of serious disaster. Centrifugal force or a hot journal may play a part in causing undue stress within a wheel. Due to rotation, moreover, all the stresses present, excepting initial stresses and those due to mounting, occur as repeated stresses, thereby still further complicating the general problem.

Of the more important means of producing internal stress in the wheel it may be generally stated, with the exceptions subsequently noted, that the mounting of the wheel and the static load tend to produce compressive stresses in a radial direction, which stresses may be wholly or partially relieved or even reversed by tensile stresses due to centrifugal force and brake application. On the outer face of the wheel the stresses due to flange thrust are cumulative with, while on the inner face they are opposed to, those caused by centrifugal force and brake application.

Throughout the investigation the attempt has been made to simulate service conditions in the methods used in loading the wheels in so far as the laboratory facilities would permit. In addition to this, most of the wheels tested were subjected to conditions exceeding in severity those which would result from normal wheel service. The wheels tested were for the most part taken at random from the stock of the Griffin Wheel Company, and are assumed to be representative of their respective types.

5. *Evaluation of Stresses Existing in a Loaded Car Wheel.*—

An examination of the section of a chilled car wheel indicates that, for purposes of analysis, it might be considered as a series of curved plates in which the outer edge of one becomes the inner edge of a succeeding one, or vice versa; furthermore, that the stresses existent in any individual section would be a function of those occurring in adjacent sections. Attempts have been made to derive the theory underlying curved plates, but as yet no satisfactory analysis has been made. Until such an analysis is available it will probably be impossible to predict the intensity or distribution of stresses in a car wheel by analytical methods. Hence, when it becomes either necessary or desirable to know the conditions of stress in a shape as complex as that of a car wheel, an experimental method aids, although it does not completely succeed, in solving the problem.

From the foregoing it can be seen that the general problem is a complex one and that, due to the irregular shape of the wheel, it cannot at present be satisfactorily analyzed by theoretical methods. Accordingly the attempt has been made to determine, as far as is possible by experimental methods, the individual effects caused by the principal loads producing stress in a car wheel, and in several cases to determine their cumulative or combined effect. Having found the individual effects, combinations can then be estimated for any assumed operating condition with greater precision than has heretofore been possible.

It is evident that the problem of the car wheel under load is one of compound stress; that is, stress in more than one direction. Throughout this report the term stress has been used to indicate the stress which would exist in the material under observation if subjected to either simple tension or simple compression, and, under these conditions, deformed to an extent equal to the measured strain. The

strains measured in these investigations constitute the fundamental data taken, and these measured strains are for the most part due to the effect of more than one stress acting within the material; that is, they are due to compound stress. Concerning compound stress there are three theories more or less generally accepted, none of which, however, can be considered wholly acceptable in the determination of the actual stresses existing within cast-iron subjected to such loads as car wheels necessarily carry. These three theories are known respectively as the maximum shear, the maximum stress, and the maximum strain theory.* It seems safe to assume that the maximum shear theory does not apply to cast-iron, and should not be used in connection with the problem in hand. The application of the maximum stress theory would in general lead to a lower evaluation of the existing stresses than would an application of the maximum strain theory. The computed stresses which are presented and discussed throughout the report are determined from the measured strains and a stress-strain relation determined from specimens tested under simple tension and simple compression. As reported, therefore, they are not in general the stresses which exist in the wheel, and may not be used as a measure of the resistance developed in any section in the way that a simple stress is used. The values of the stress are given to allow the reader to use these approximate values of a more familiar concept than strain. The word stress has throughout the discussion frequently been qualified by the use of the expression "corresponding simple" when referring to the stresses computed from the measured strain and the stress-strain relation used; and this expression has been used to call attention to, and to emphasize the fact that the stresses under consideration are for the most part computed from measured strains resulting from compound stress and from a stress-strain relation based upon simple tension and compression.

As the available information concerning the theories does not appear to indicate definitely that any of them, either separately or in combination, should be applied in the case of cast-iron, it is felt desirable that the matter of the actual severity of these stresses should be considered as held in abeyance at this time. The report, moreover, is not intended to present figures which can be taken as exact

* For a brief discussion of these theories see, "The Strength and Stiffness of Steel Under Biaxial Loading." Univ. of Ill. Eng. Exp. Sta., Bul. 85, 1915.

values of the actual stresses existing where the strains from which the values given were calculated are produced by compounded stress. No attempt has been made to report exact values for the stresses existing in the wheels under conditions of actual service, as this report is concerned only with the stresses produced through two kinds of wheel loading, namely, mounting and static loading. The stresses resulting from these two forms of loading may be materially modified by additional stress-producing factors to which a wheel may be subjected in service. The usual stress condition produced in the parts of the car wheel by mounting, and through the application of static loads, is that of material subjected to tension in one direction and compression in a direction at right angles thereto. This condition produces a deformation or strain in the direction of either stress, which is greater than would be produced by the stress in that direction acting alone. Any estimates regarding factors of safety, based upon the stresses reported for the mounting and static load tests, and considered as applying to the chilled iron wheel or its parts, could therefore be properly considered as conservatively estimated; that is, if the actual stresses could be determined, it is probable that this would not involve a reduction in the estimated values of the factors of safety, but would rather justify increased values for them. In this report no attempt has been made to indicate values for factors of safety.

III. PHYSICAL PROPERTIES OF CHILLED CAR WHEEL IRONS

6. *Selection and Treatment of Specimens.*—In order to assist in interpreting the data obtained in the tests made on the car wheels, a number of specimens of wheel irons were taken, both from wheels tested, and as coupons from the foundry ladle. Some of the properties of these specimens which were of most importance to the investigation were determined.

In general, in discussions pertaining to the strength of materials it is customary to consider the matter in terms of stress, where stress is that internal force which, when a body is subjected to external forces, tends to hold the molecules in their original relation and to preserve the integrity of the body. The fundamental data obtained in these tests were, however, strains or deformation measurements. Hence, to facilitate interpretation of the strain data in terms of stress, it became necessary to have a knowledge of the stress-strain relation of the metal used in car wheels. This relation was determined by applying a known load to the metal under test, and measuring the corresponding elongation or contraction; then, by calculation from the load and the deformation, were obtained the stress per unit area and the strain per unit length, respectively. Measurement of the strain, along with a knowledge of the relation between stress and strain, permitted an evaluation of the stress produced. Accordingly the stress-strain data, and in addition the ultimate strength and modulus of elasticity, were determined for the car wheel and coupon specimens obtained. Since the stress-strain relation is dependent largely upon the composition of the metal, chemical analyses were made to determine total carbon, combined carbon, silicon, manganese, phosphorous, and sulphur. Hardness tests, both by the Brinell method and with the scleroscope, were also made in an attempt to associate the quality of hardness with other properties of the metal. The origin, shape, and treatment of the specimens, together with the results of the chemical and physical tests, are shown in Table 1 and in Figs. 1 to 5 inclusive.

On account of the irregular shape of the wheel (see Fig. 1, *a* and *b*) in the region between the hub and the inner side of the tread, it was difficult to remove specimens of any considerable length. For this reason the specimens removed from the wheel and used for the

TABLE 1

PHYSICAL AND CHEMICAL PROPERTIES OF CAR WHEEL IRONS AND COUPONS

SPECIMEN NUMBER	WHEEL NUMBER	FROM	CHEMICAL ANALYSIS						HARDNESS		STRESS-STRAIN			REMARKS
			TOTAL C	SI	MN	P	S	BRINELL	SHORE	MOD OF ELAS AT 5000 LB PER SQ IN IN MILLIONS	ULTIMATE STRENGTH LB PER SQ IN	KIND OF TEST		
B2		33-725	0.41	3.71	0.55	0.63	0.364	0.213	137	37.9	19	28000	Tensile	See Figs. 1a, 2a, & 3
D1		Wheel	0.67	3.52	0.57	0.63	0.350	0.214	143	35.8	18	26300	"	
D2		"	0.30	3.66	0.55	0.63	0.362	0.215	130	36.1	22	27700	"	
D3		"	0.48	3.73	0.56	0.62	0.337	0.214	128	38.0	20	27700	"	"
D4		"	0.57	3.66	0.56	0.65	0.354	0.222	140	38.2	28	26900	"	"
D5		"	0.50	3.63	0.59	0.64	0.350	0.216	133	34.8	19	26300	"	"
D6		"	0.82	3.68	0.58	0.63	0.350	0.215	144	37.4	17	24200	"	"
F1		"	0.61	3.70	0.56	0.65	0.351	0.222	141	35.2	16	28000	"	"
F2		"	0.47	3.75	0.59	0.63	0.363	0.219	134	35.6	22	26600	"	"
F3		"	0.47	3.71	0.60	0.63	0.359	0.221	110	34.4	17	24900	"	"
F4		"	0.58	3.66	0.56	0.65	0.340	0.224	138	35.4	15	26400	"	"
F5		"	0.51	3.79	0.60	0.63	0.351	0.216	122	35.4	18	26400	"	"
F6		"									17	23300	"	"
B4		"										29500	"	"
B5		"										28600	"	"
B6		"										25200	"	"
E1		"	0.60	3.59	0.57	0.64	0.350	0.214	120	30.0	17	62500	Com-	See Figs. 1a, 2b, & 5
E2		"	0.40	3.68	0.59	0.61	0.363	0.213	131	37.0	21	75000	pressive	
E3		"	0.41	3.75	0.60	0.64	0.373	0.216	136	37.0	17	74200	"	
E4		"	0.61	3.74	0.61	0.62	0.338	0.220	144	39.8	25	77700	"	"
E5		"	0.52	3.60	0.59	0.64	0.360	0.215	138	35.5	20	75200	"	"
E6		"	0.59	3.65	0.59	0.64	0.350	0.215	151	40.1	17	75000	"	"
C1		"	0.75	3.63	0.60	0.67	0.347	0.216	142	39.8	15	77400	"	"
C2		"	0.41	3.70	0.58	0.63	0.369	0.212	128	34.8	16	73900	"	"
C3		"	0.50	3.69	0.55	0.59	0.345	0.204	130	35.0	19	73200	"	"
C4		"	0.70	3.71	0.56	0.67	0.354	0.227	137	36.4	19	82900	"	"
C5		"	0.46	3.75	0.57	0.64	0.350	0.218	130	35.1	19	72900	"	"
C6		"	0.74	3.77	0.58	0.63	0.368	0.206	128	35.8	17	74200	"	"
A ₁	16129	Wheel	1.20	3.64	0.61	0.47	0.340	0.224			18	26000	Tensile	See Figs. 1b, 2c, & 3
A ₂	"	"	"	"	"	"	"	"			15	27100	"	
A ₃	"	"	"	"	"	"	"	"			19		"	
B ₁	22121	"	1.12	3.53	0.72	0.56	0.340	0.252			16	32600	"	"
B ₂	"	"	"	"	"	"	"	"			18	32800	"	"
B ₃	"	"	"	"	"	"	"	"			19	32700	"	"
1PP		Coupon	1.08	3.75	0.57	0.60	0.330	0.197			16	26200	"	See Figs. 2d & 3
3PP		"	0.86	3.64	1.44	0.63	0.330	0.176			14	26500	"	
1	51092	"	1.53	3.82	0.54	0.38	0.256	0.174	164	24.9	24	24500	"	Sand Cooled, See Figs. 2c & 4
1A	"	"	1.54	3.82	0.52	0.39	0.259	0.175	150	25.3	21	22700	"	
1A	"	"	1.28	3.86	0.53	0.38	0.250	0.176	190	32.6	18	23300	"	
1A		"	1.30	3.79	0.51	0.39	0.253	0.174	184	31.3	21	24000	"	Wheel & Coupons Pitted Std. Lgth. Time
2	51091	"	1.13	3.88	0.53	0.38	0.257	0.176	242	40.2	25	32200	"	"
2A		"	1.13	3.82	0.51	0.39	0.266	0.176	207	37.2	19	24900	"	Sand Cooled
2A		"	1.30	3.84	0.51	0.39	0.257	0.175	242	41.0	18	33200	"	
3	51090	"	Trace	3.85	0.52	0.39	0.252	0.187	91	18.8	16	15200	"	
3	"	"	"	3.88	0.53	0.38	0.258	0.190	89	18.4	20	15400	"	Wheel & Coupons Taken from Molds & Cooled with Water. Wheel Broke.
3A		"	1.36	3.77	0.51	0.40	0.254	0.178	204	33.2	18	23800	"	Sand Cooled
1A1		"	1.23	3.82	0.50	0.58	0.180	0.130			16	25500	"	Wheel & Coupons Pitted Triple Std. Time
2A1		"	0.84	3.79	0.57	0.54	0.190	0.132			17	24400	"	
3A3		"	0.76	3.79	0.66	0.55	0.220	0.128			17	23400	"	
4A1		"	0.82	3.79	0.58	0.63	0.210	0.186			19	24500	"	"
5A1		"	0.72	3.82	0.63	0.65	0.200	0.170			15	22600	"	"
6A1		"	0.79	3.81	0.55	0.56	0.190	0.143			14	20800	"	"
182		"	0.80	3.69	0.55	0.59	0.230	0.124			16	27400	"	"
281		"	0.90	3.68	0.61	0.67	0.240	0.163			18	26000	"	"
382		"	0.89	3.68	0.74	0.60	0.260	0.138			15	24200	"	"
3C1		"	0.87	3.77	0.79	0.48	0.360	0.164			17	27100	"	"
L3		"	0.74	3.52	0.64	0.61	0.200	0.151			22	31800	"	"
L4		"	1.04	3.32	0.63	0.59	0.320	0.249			28	31600	"	"

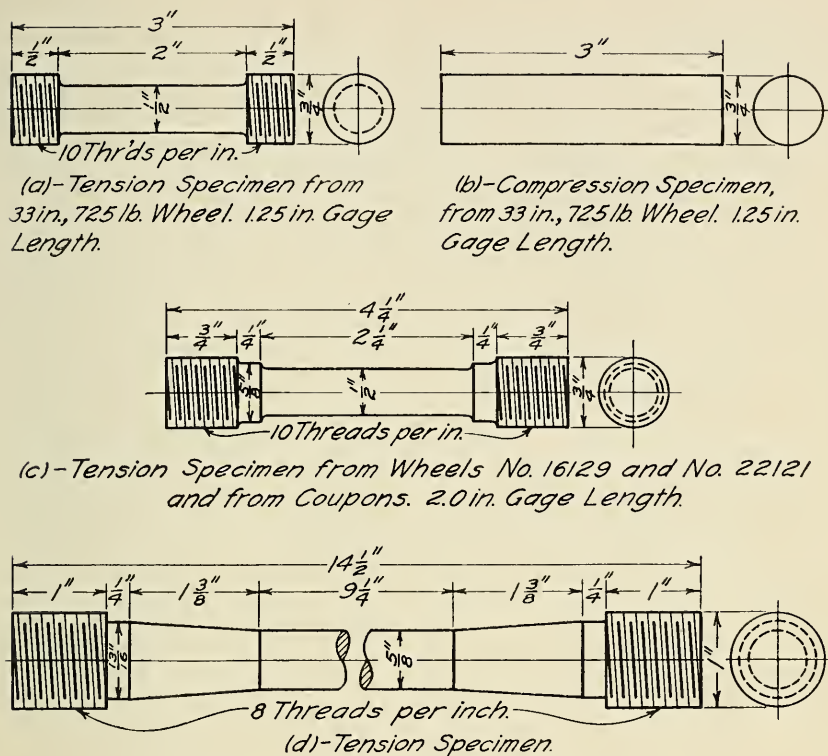


FIG. 2. DETAILS OF TEST SPECIMENS

tension tests did not conform to the dimensions recommended by the American Society for Testing Materials. The treatment to which the specimens were subjected varied. In the case of specimens taken from the wheel, the removal was made after the wheel had been cooled or annealed, as shown in the table. The coupons were cast in sand molds, and were either allowed to cool in the sand, or, after solidification, were placed in the pit with the wheel they represented. The number of specimens already tested is small, but it is hoped that in time a sufficient number of like specimens may be subjected to tests so that a more complete study of the physical properties of car wheel irons may be made.

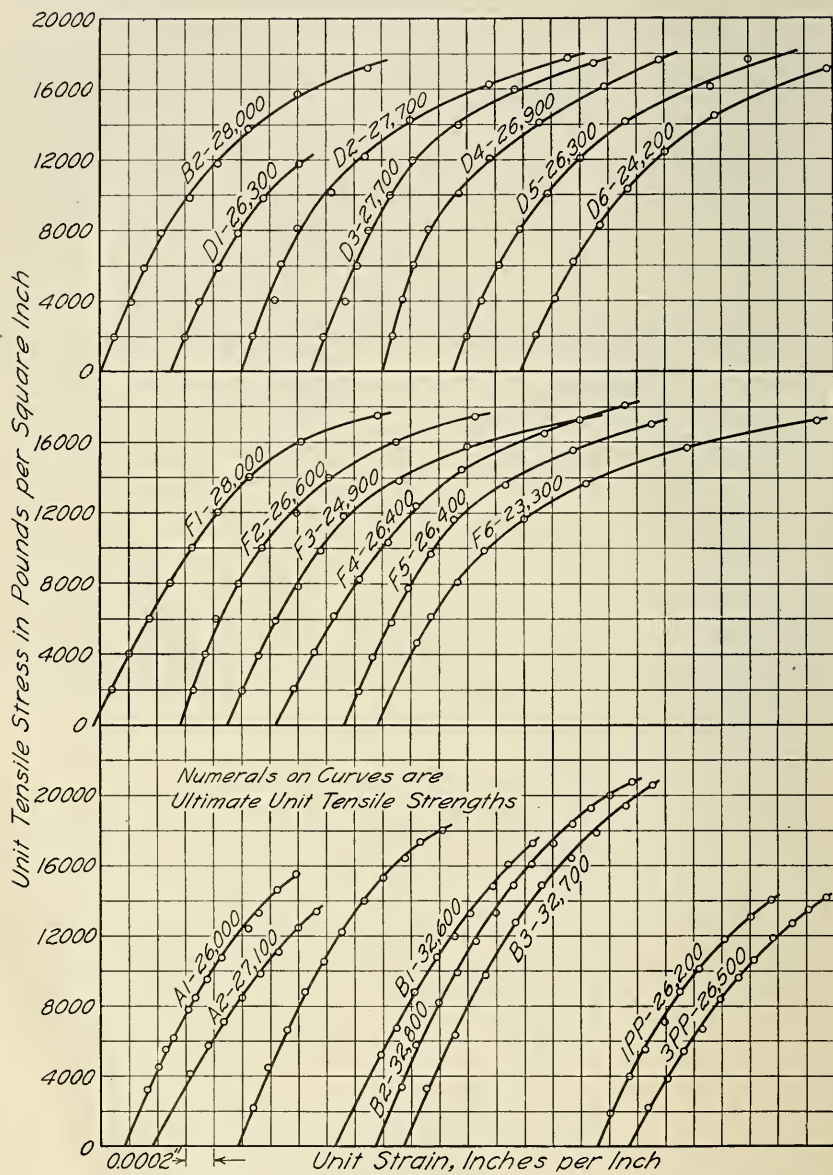


FIG. 3. STRESS-STRAIN RELATION FOR WHEEL IRONS AND COUPONS (TENSION)

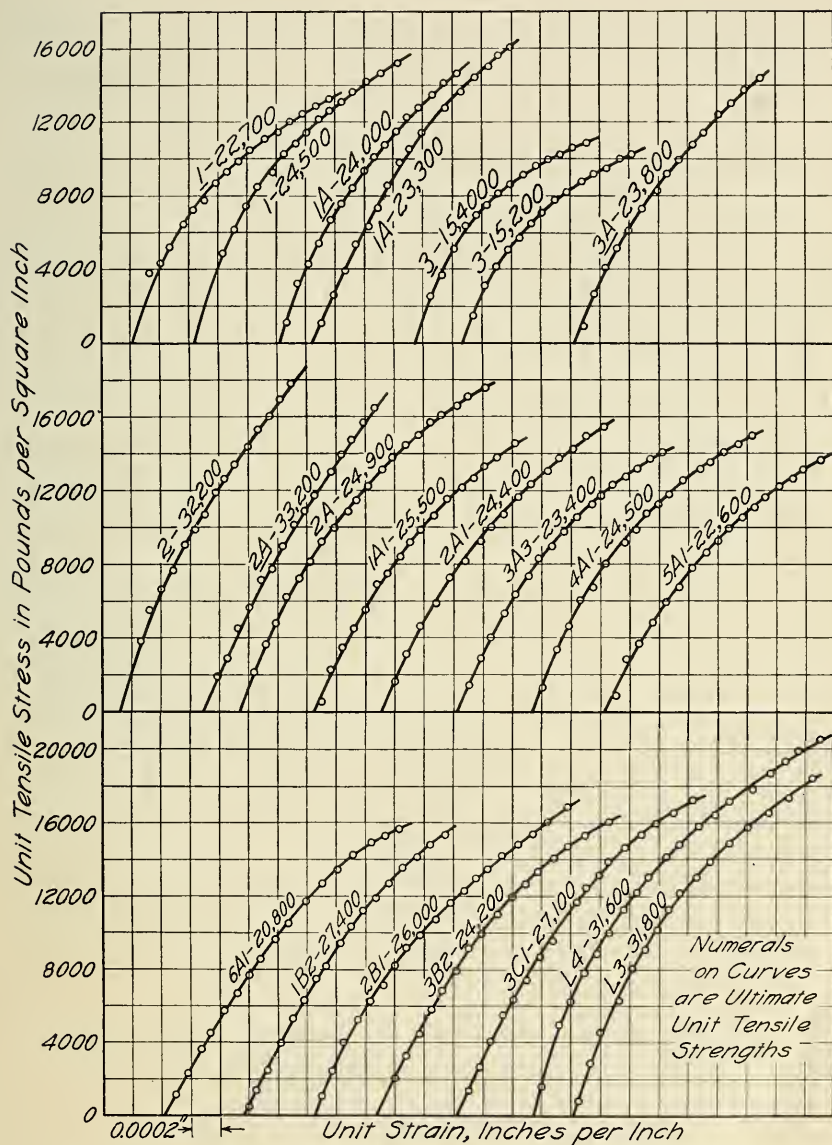


FIG. 4. STRESS-STRAIN RELATION FOR WHEEL IRON COUPONS (TENSION)

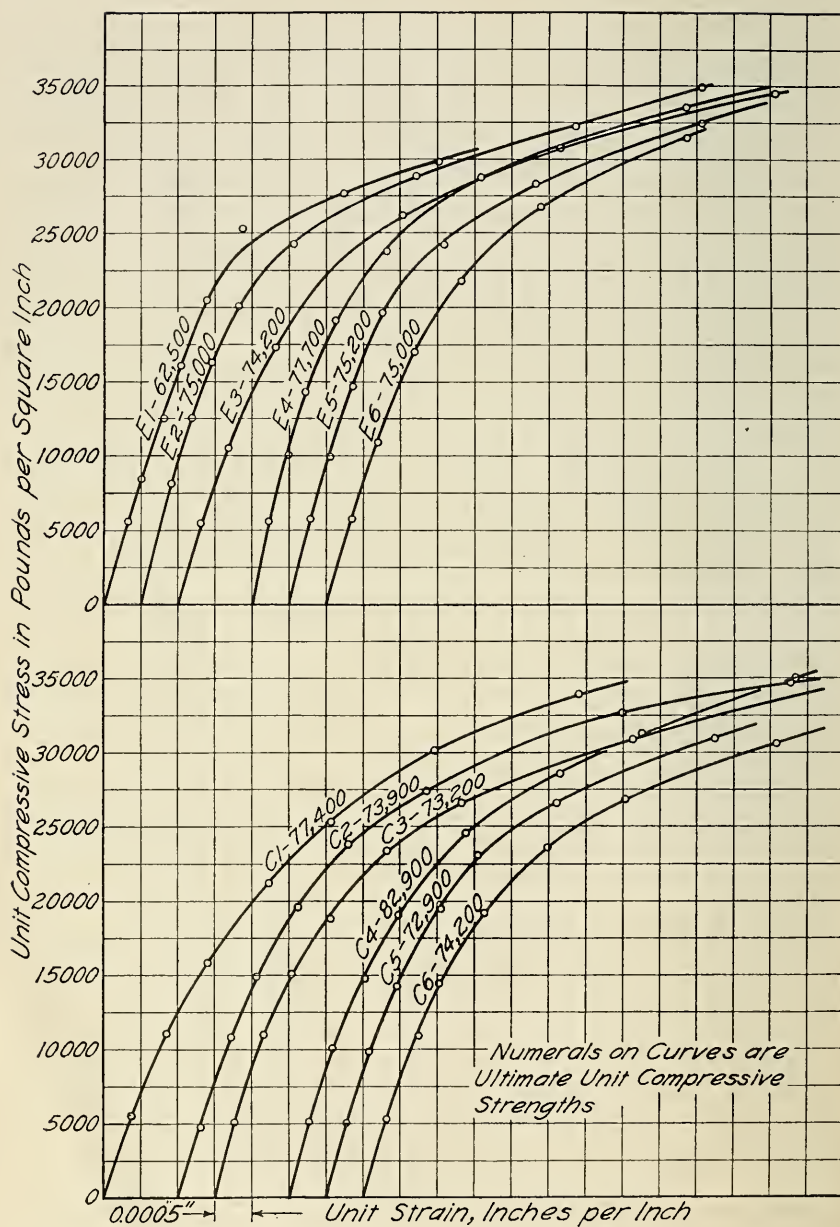


FIG. 5. STRESS-STRAIN RELATION FOR WHEEL IRONS (COMPRESSION)

7. *Chemical Analyses.*—The chemical analyses were made in the laboratory of the Griffin Wheel Company, Chicago. The amounts of the various elements present in the specimens are shown in Table 1 and are expressed in per cent.

8. *Stress-Strain Relation.*—For these determinations a 100 000-lb. Riehle Universal Testing machine, using spherical seated holders or bearing blocks, was used. The machine was stopped for readings.

A Ewing extensometer reading directly to 0.000071 in. per in., made by the Cambridge Instrument Company, was used in measuring the strains on the specimens shown in Fig. 2, *a* and *b*. For the 2-in. specimens an extensometer reading directly to 0.000025 in. per in. was used, and for the 8-in. specimens an instrument reading directly to 0.000062 in. per in.* The modulus of cast-iron being a variable, it was thought that the secant modulus† corresponding to a stress of 5 000 lbs. per sq. in. would permit suitable comparison, and the moduli are thus reported.

As no two of the specimens, even when cut from the same wheel, gave identical stress-strain relations, and as it would obviously be impractical to cut a test specimen from each of the positions in the wheel at which the strains were measured, it was necessary, in order to evaluate the stress from the strains as measured on the car wheel, to deduce two stress-strain curves which might be assumed to be representative of wheel iron in tension and in compression, respectively. For this purpose, the results for the 13 tension specimens, B_1 to F_6 inclusive, (see Table 1) were averaged, and from the mean a curve was drawn. In a similar way the mean compression stress-strain curve was obtained from the 12 compression specimens, E_1 to C_6 inclusive. As stated above, these two curves, Fig. 6, were assumed to show, respectively, representative tensile and compressive stress-strain relations for chilled wheel irons, and are the curves upon which the stresses hereafter given are based.

9. *Hardness.*—The hardness by the scleroscope method was determined with a Shore scleroscope made by the Shore Instrument

* An illustration of the latter two instruments is shown in "The Relation between the Elastic Strength of Steel in Tension Compression and Shear." Univ. of Ill. Eng. Exp. Sta., Bul. 115, Fig. 4, p. 12, 1919.

† *Secant modulus* is defined as the slope of a straight line connecting the origin of the stress-strain curve and some arbitrarily chosen point on the curve, the curve itself being plotted with unit stress as ordinates and unit strains as abscisse.

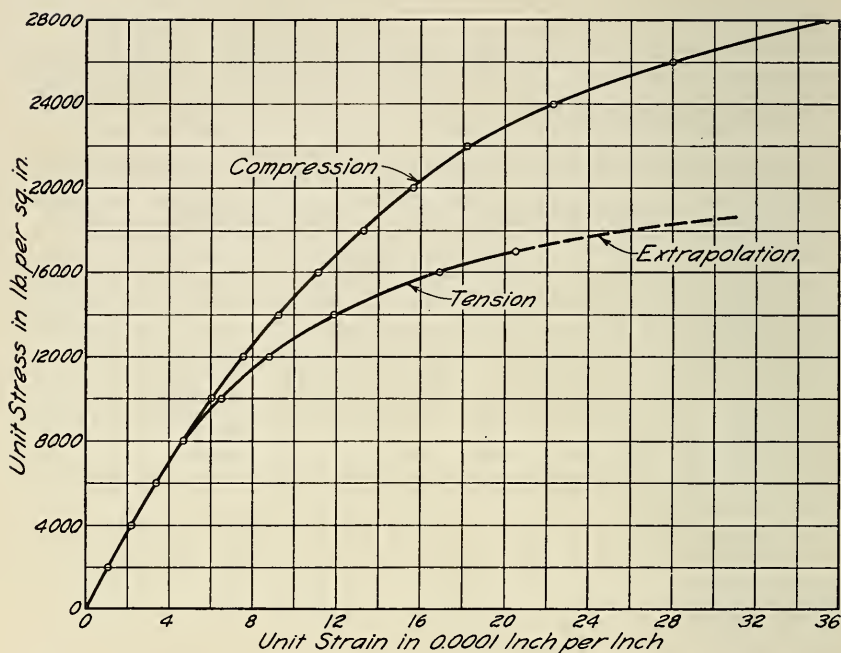


FIG. 6. ASSUMED MEAN STRESS-STRAIN RELATION FOR CHILLED WHEEL IRONS USED IN EVALUATION OF STRESS

and Manufacturing Company. This instrument measures the hardness by the measured rebound of a small weight carrying a blunt diamond which is dropped from a fixed height (vertically) on the material. The scleroscope data given in Table 1 represent the mean value of 10 readings taken on each specimen. The apparatus used for the Brinell tests was manufactured by Aktiebolaget Alpha, of Stockholm, Sweden. This test consists of forcing, under a definite pressure, a hardened steel ball of definite size into a flat plate of the material to be tested, and measuring the diameter of the indentation. The Brinell hardness is a function of the diameter of the indentation thus made. In the case of the tabulated Brinell data, they are the average of 3 or 4 observations using a 12-mm. ball with a pressure of 3000 kg. for 30 sec. The average hardness shown by the 36 samples cut from the wheel, Fig. 1a, at various distances from the center of the wheel is given in Fig. 7.

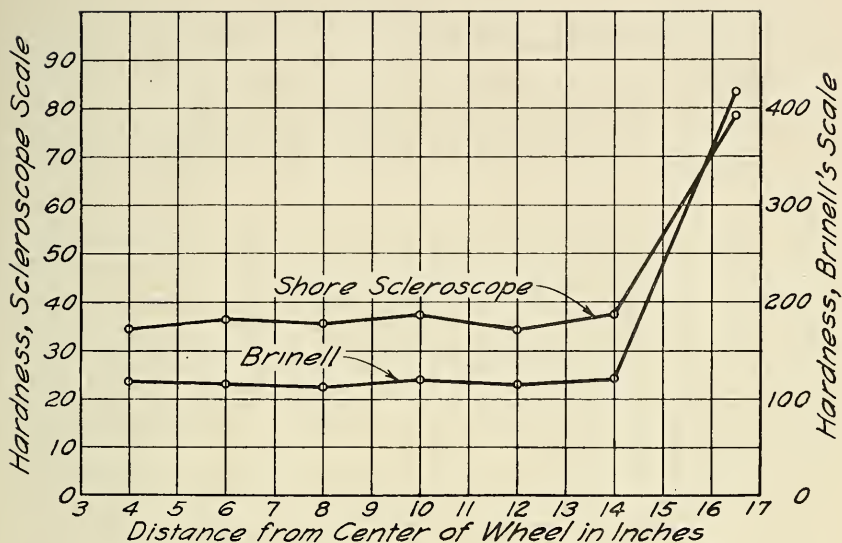


FIG. 7. MEAN HARDNESS OF 33-IN. 725-LB. CHILLED WHEEL
AT VARIOUS RADII

10. *General Remarks Concerning Properties of Chilled Wheel Irons.*—It is recognized that proper application of chemical knowledge in the work of the wheel foundry has been an important factor in enabling the chilled iron wheel to meet the demands of modern railway operation. As a result car wheel manufacturers maintain chemical laboratories and control the chemical composition of wheels within fairly well defined limits. The railroads, however, with a few exceptions, do not embody chemical control in their specifications, but rely almost wholly on the results of physical tests in the acceptance of wheels.* Inspection of Table 1 shows that the chemical composition varies at different positions within the wheel. For instance in Table 1 the specimens, B_2 to C_6 inclusive, as removed from the plate of the wheel shown in Fig. 1a, show the following range in the percentage of the several elements:

* Specifications for Cast-iron Car Wheels. Proc. M. C. E. Assn., Vol. 52, p. 490. 1918.

	Per Cent	
Total Carbon	3.52	to 3.79
Combined Carbon	0.30	to 0.82
Silicon	0.55	to 0.61
Manganese	0.59	to 0.67
Phosphorous	0.337	to 0.373
Sulphur	0.204	to 0.227

With respect to the importance or effect of these variations no conclusions have been reached. For the same specimens the Brinell hardness varied from 110 to 151, while the scleroscope hardness ranged from 30.0 to 40.1. The secant modulus of elasticity of the several specimens had a minimum value of 15 000 000, whereas the maximum value was 28 000 000 lb. per sq. in. An appreciable variation from 23 300 to 29 500 lb. per sq. in. likewise existed in the ultimate strength of the specimens when they were tested in tension. In compression, the ultimate strength ranged from 62 500 to 82 900 lb. per sq. in. In the three specimens cut from wheel No. 22 121, the average ultimate tensile strength, 32 700 lb. per sq. in., is approximately 23 per cent greater than the average, 26 620 lb. per sq. in., found in the wheel represented by specimens B_2 to B_6 inclusive. Considering the important effect that tension plays in the failure of cast-iron, it would seem desirable to produce wheels having a high ultimate strength in tension, that is, having the property of toughness, provided, of course, this could be done without materially affecting the hardness, or the wearing qualities of the metal in contact with the rail.

If all the specimens given in Table 1 are considered, a variation of 100 per cent between the minimum and maximum moduli of elasticity in tension is seen to exist. A similar condition also exists with respect to the values of ultimate strength which are reported. The compression data likewise show considerable variation in the moduli and in the ultimate strengths recorded. This suggests that the metallurgy of wheel irons still offers a fertile field for improvement in quality, in order that wheels may meet the probable demands of future railway service.

No distinct relation was found between the ultimate tensile strength and the Brinell or scleroscope hardness, nor could a constant relation be determined between the Brinell and scleroscope hardness. This latter fact is readily apparent from inspection of Fig. 7. Over

the distance 4 to 14 in. from the center of the wheel, the ratio of Brinell to scleroscope hardness is approximately 115 to 36 or 3.2, whereas at a radius of 16.5 in., that is, on the tread of the wheel, this ratio is 5.3. This figure also strikingly indicates that the extreme hardness produced by chilling is confined to the tread, and that the metal in the plate is relatively soft and of nearly uniform hardness.

IV. APPARATUS AND METHODS USED IN MOUNTING AND STATIC LOAD TESTS

11. *The Strain-Gage, and Its Use in Measuring Strains in Car Wheels.*—In the determination of strains developed by the various methods of stressing a car wheel, measurements were taken thereon in both radial and tangential directions with a Berry Strain-Gage illustrated in Fig. 8.

This gage consists of a frame carrying a fixed cone-shaped point, a multiplying lever, and an indicating dial. The short arm of the multiplying lever also carries a cone-shaped point, and the long arm of the lever actuates the plunger of the dial. This instrument measures the change of the distance between two small holes. These holes are the extremities of a gage-line along which the strain is desired. In making observations with the strain-gage, the cone points are inserted in the gage holes and the dial readings noted for each of the gage-lines on the wheel. These readings are first taken with the wheel in an unstrained condition. Another series of observations is then taken on the same gage-lines with known loads on the wheel. The difference between the initial and final readings divided by the product of the gage length and the multiplication ratio (as obtained by calibration) of the strain-gage lever, gives the actual unit strain for the load in question.

As a formula this expression becomes:

$$e = \frac{R_i - R_f}{l \times m}$$

where e = unit strain;

R_i = initial reading of the dial;

R_f = final reading of the dial;

l = length of gage-line;

m = multiplication ratio of strain gage.

The relative values of R_i and R_f , together with a consideration of the characteristics of the strain-gage used, indicate whether the strain is tensile or compressive. From the unit strain thus found and by reference to the stress-strain diagrams, Fig. 6, an approximate value of the corresponding simple stress, variously called the "true

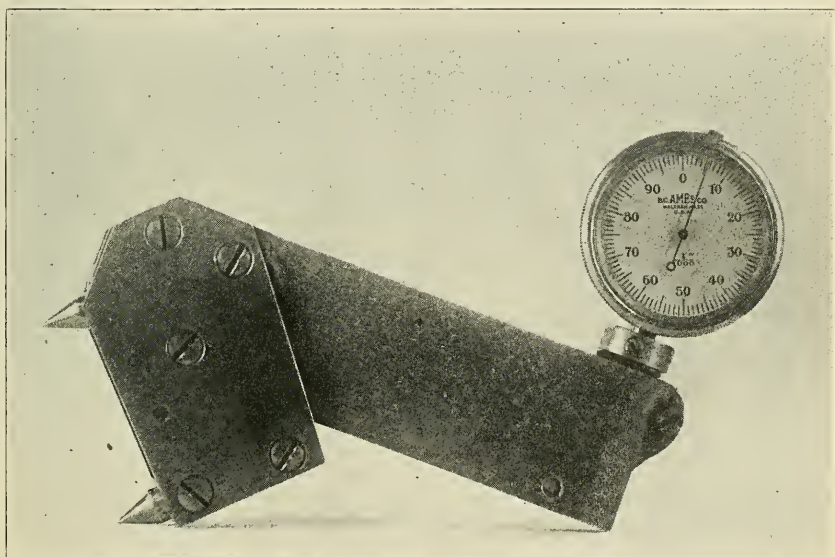


FIG. 8. BERRY STRAIN-GAGE

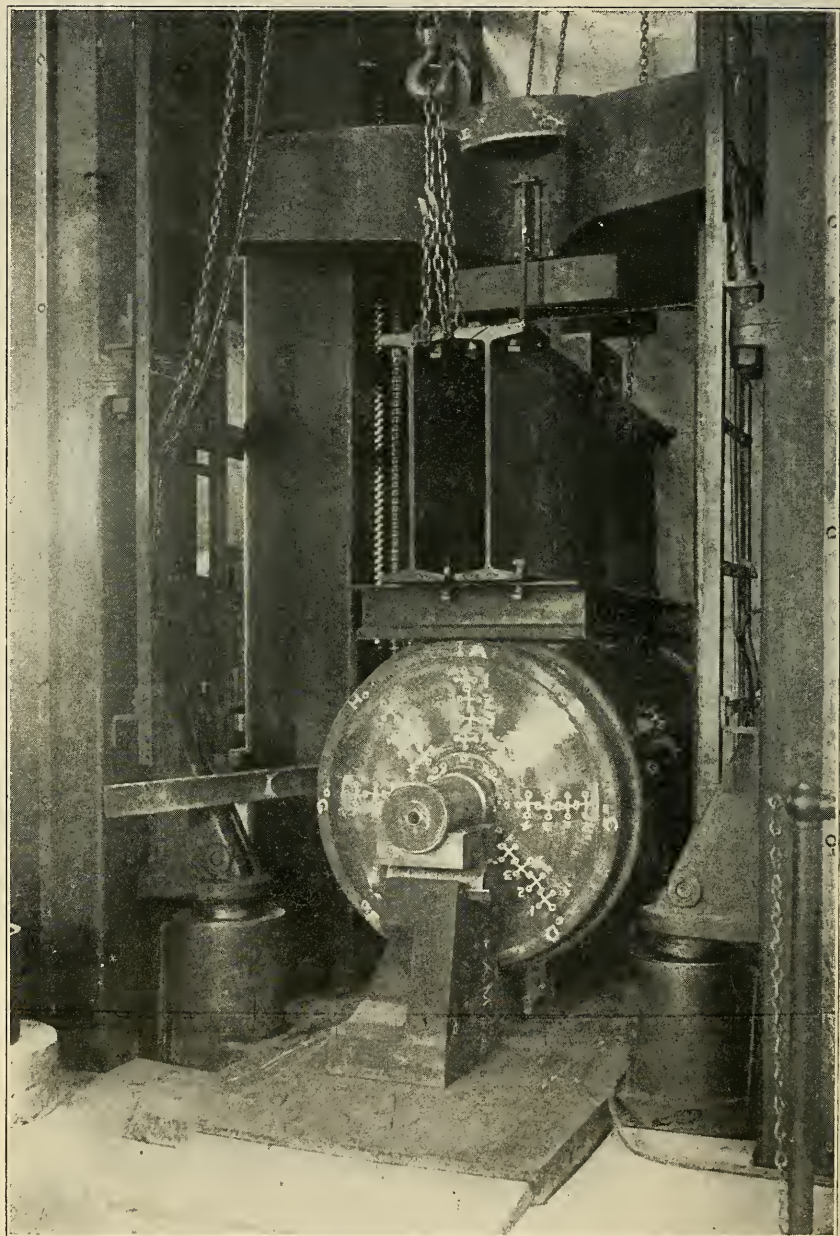


FIG. 9. 600 000-LB. TESTING MACHINE WITH PAIR OF WHEELS
IN POSITION TO RECEIVE STATIC LOAD

stress," "reduced stress," or "ideal stress," is determined. Throughout this report any stress mentioned is in terms of the corresponding simple stress, or the stress that would exist if the metal were subjected to simple tension or compression and deformed to a degree equal to the measured strains. Both the initial and final strain-gage observations were taken in terms of a standard gage-line called a standard bar. This bar was made of the same material, in this case cast-iron, as that in which the strain was desired, and its use provided a means of compensating for changes in temperature of the material being tested and progressive changes due to other causes. It also furnished a check for noting any changes that might occur in the relative position of the dial and frame of the strain-gage. In these tests the gage-lines were established in both radial and tangential directions on the wheel by drilling holes 0.055 in. in diameter, approximately 1/16 in. in depth, and 2 in. between centers. The instrument used had a dial graduated to 0.001 in. and a multiplication ratio of five. Hence the unit strain could be read directly to 0.0001 in. per inch, and by estimation to 1/10 of this figure.

Throughout the report certain gage-lines are shown for which no strains are given. For these the readings were not taken. The omissions are due to the fact that the initial strain-gage readings, i. e., those taken with the wheel in an unstrained condition, could not be duplicated when several trials were made. This was due to slight imperfections in the gage holes, to scale on the metal, etc. Since all the measured strains are dependent on these initial readings, the unreliable readings were discarded and no further observations were made on these gage-lines.

12. *Preparation and Methods of Testing Wheels for Determination of Strains Due to Mounting and Static Loads.*—A pair of wheels was prepared with gage holes determining lines in both radial and tangential directions and on several radii. With the strain-gage, readings were taken on each of the gage-lines before mounting the wheels on the axle. The two wheels were then pressed upon the seats of a standard 5½-in. by 10-in. M. C. B. axle in a 600 000-lb. Riehle testing machine. Autographic diagrams of the pressure during mounting were taken. After mounting, observations on each of the gage-lines were again made, and, as stated above, the differences, after proper

corrections, between the latter readings and the initial readings are measures of the unit strains.

After the strains incident to mounting had been determined, the wheels were placed in the testing machine, as shown in Fig. 9, and subjected to various known static loads.

For each of the static loads thus applied a complete series of strain readings was taken. In this case the difference between the initial readings, the observations taken before mounting, and the observations at the load in question, is the total strain in the wheel caused by the combined effects of mounting and the static load. It will be noted that, except for the inversion of the wheel and rail, the apparatus arranged for the static tests loads the wheel in the same manner as in actual service.

V. RESULTS OF MOUNTING AND STATIC TESTS

13. *Corresponding Simple Stresses in Two 33-in. 725-lb. M. C. B. Wheels Due to Mounting Wheels on Axle, together with Combined Effects of Mounting and Static Loads.*—The location of the gage-lines on these two wheels is shown in Figs. 10 and 11 and the gage-lines are designated as follows: the radial lines on which the gage-lines are located are lettered *A* to *H*, and these letters become the first figure in the designation; the relative position of the gage-line on the radius is next indicated by a numeral, which in turn is followed by either the letter *R* or *T*, signifying respectively a radial or a tangential gage-line; thus *B4R* denotes the gage-line or radial *B*, fourth from the tread and in a radial direction, and *B4T* indicates the gage-line similarly located but in the tangential direction.

The pressures required to mount the wheels on the axle together with the fit allowance are shown in Fig. 12, while the equivalent simple stresses resulting therefrom are given in Figs. 13 and 14.

Diagrams similar to Fig. 12 are regularly taken by both the manufacturers and railroads when mounting wheels. They serve as a check against placing improperly mounted wheels in service. With wheels properly fitted, this curve will show a nearly uniform increase in the pressure required for forcing the wheel on the axle from the instant the axle enters the wheel up to its final position. In addition, the final pressure as recorded indicates whether the recommendation of the M. C. B. Association in this respect has been fulfilled. If this pressure is much below that recommended, there is a possibility of the wheel becoming loose on the axle; whereas, if it is in excess of that recommended, cracked wheels may result. In view of these facts considerable attention is generally given to the final pressure required. For this type of wheel the M. C. B. Association specifies a final pressure ranging from 45 to 65 tons. The pressures recorded in the tests were 45.8 and 61.5 tons, or within the allowable range. Attention, however, is called to the fact that for a wheel seat diameter of 7 in. and a hub length equal to 7 in., the manufacturers' practice is to make the fit allowance 0.011 to 0.017 in., which results in a range of mounting pressures of about 45 to 65 tons. It will be noticed that the allowance on wheel No. 671237 was 0.021 in., and the mounting pressure at a

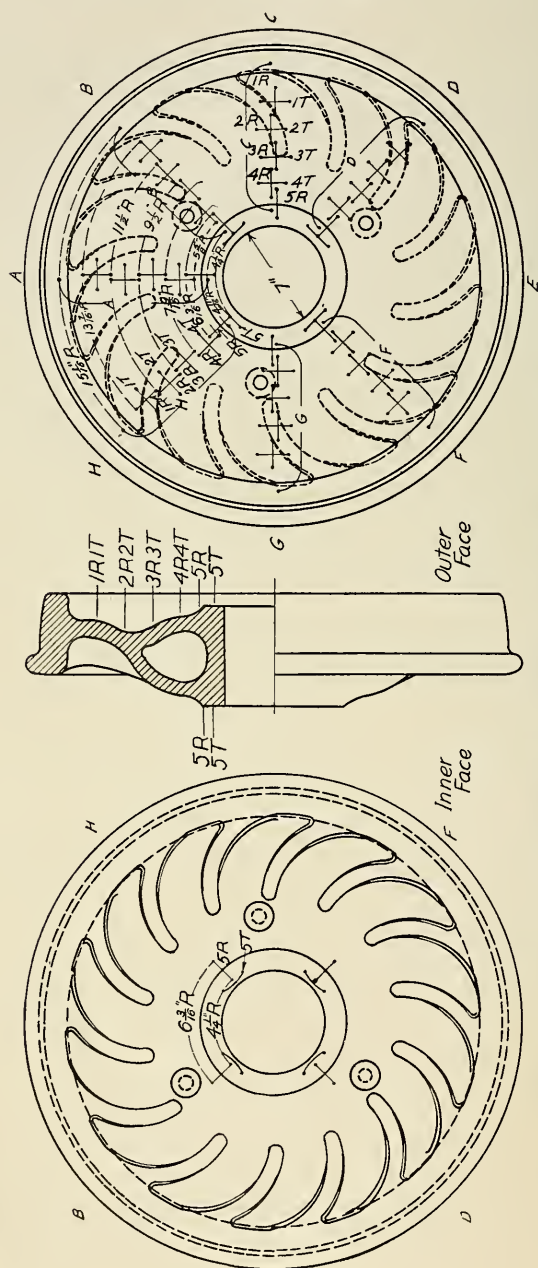


Fig. 10. LOCATION OF GAGE-LINES ON 33-IN. 725-LB. M. C. B. WHEEL No. 671 237

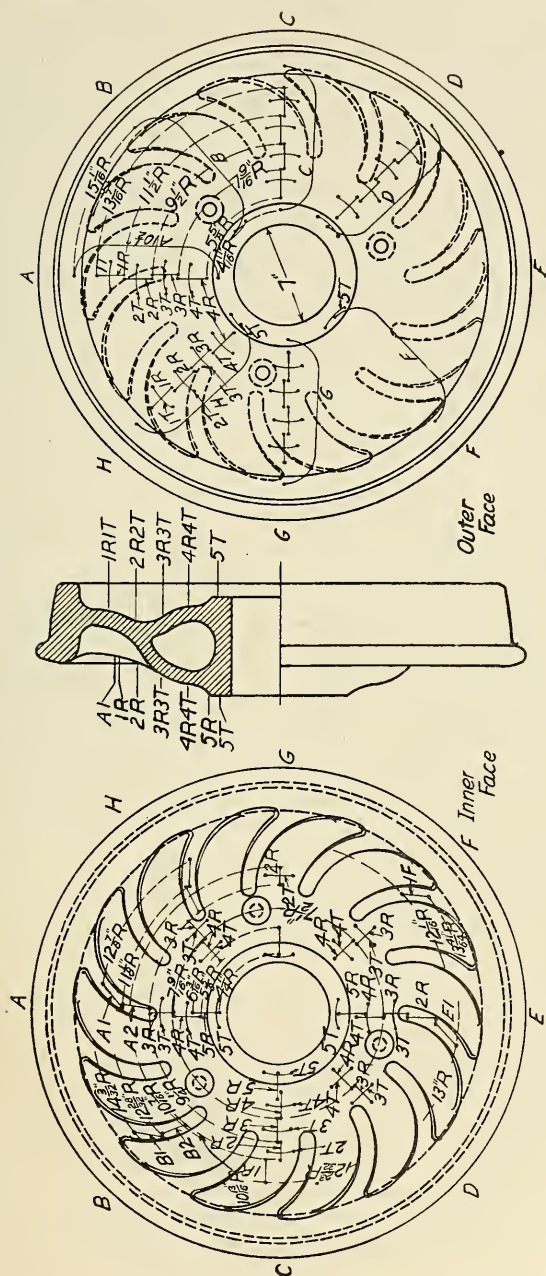


FIG. 11. LOCATION OF GAGE-LINES ON 33-IN. 725-LB. M. C. B. WHEEL No. 671 449

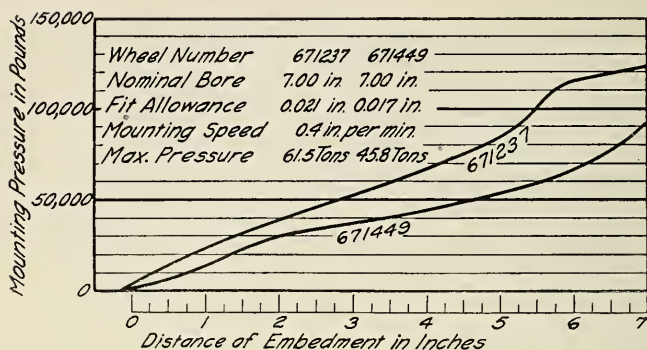


FIG. 12. AUTOGRAPHIC DIAGRAMS OF PRESSURES REQUIRED TO MOUNT TWO 33-IN. 725-LB. WHEELS ON AXLE

speed of 0.4 in. per minute was 61.5 tons; i.e., the allowance was 25 per cent greater than the manufacturers' maximum allowance, while the pressure required was 5 per cent less than might be recorded by commercial types of wheel presses, which, however, operate in some cases at much higher speeds. Likewise, in wheel No. 671 449 the fit allowance equalled the manufacturers' maximum, and in commercial work the expected pressure would be about 65 tons. The maximum recorded pressure for this wheel, however, was about 45.8 tons, or roughly equal to that which would occur with the minimum fit allowance of 0.011 in. A similar condition existed in later mounting tests. This apparently indicates that the pressure required to mount the wheel is a function of the mounting speed. It is also probable that the magnitude of the mounting pressure is dependent on the alignment of the wheel bore and axle during mounting. As no spherical blocks were used in these tests when mounting the wheel—nor are they used on wheel presses—it is possible that the maximum recorded mounting pressures are higher than would be the case if perfect alignment between bore and axle existed. Hence it would appear that fit allowance might be a better criterion for mounting wheels than pressure where the mounting speed falls considerably outside the range of speeds customarily used in wheel shops, and in cases where poor alignment may occur. Under no circumstances, however, would it be advisable to discontinue the use of the final mounting pressure as a check against the placing of improperly fitted wheels in service.

In the case of wheel No. 671 237 the maximum tensile unit strain, due to mounting (Fig. 13), was on gage-line *B4T* on the *B* radial of the outer face. The magnitude of this strain was 0.00222 which corresponds to a simple tensile stress of 17 400 lb. per sq. in. Inspec-

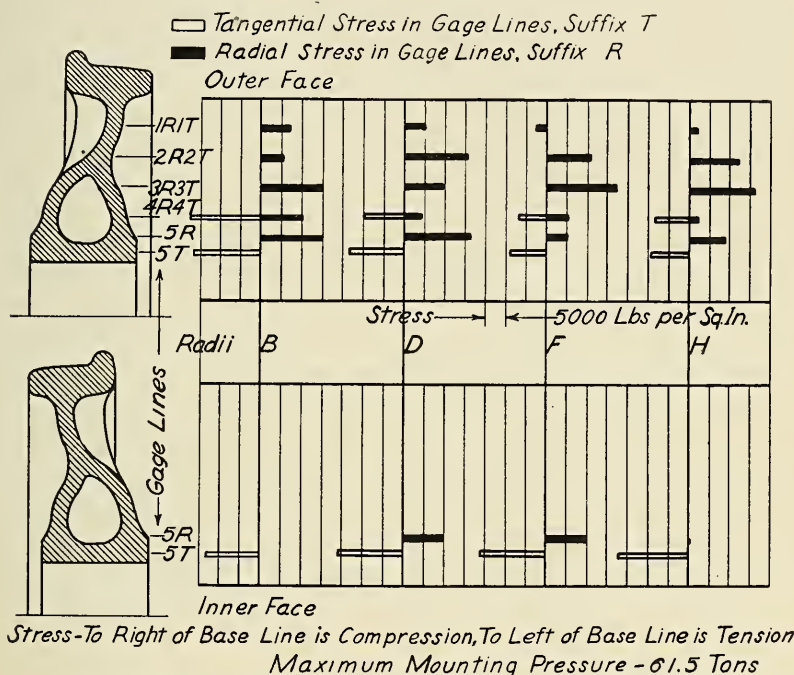


FIG. 13. CORRESPONDING SIMPLE UNIT STRESSES IN 33-IN. 725-LB.
 M. C. B. WHEEL NO. 671 237 CAUSED BY MOUNTING ON AXLE

tion of Fig. 10 shows that the measurement was taken in close proximity to a chaplet, and this may partially account for the large value. On the inner face the same value is nearly reached on gage-line *H5T* where the strain was 0.00218, and the simple tensile stress corresponding thereto is 17 300 lb. per sq. in. With respect to the compressive strains the maximum was on the outer face on gage-line *F3R* and equalled 0.00127, which corresponds to a simple compressive stress of 17 600 lb. per sq. in. This is in the region near the intersection of the inner and outer plates. A relatively short radius

of curvature, 3 in., exists on a portion of this gage-line, so that curved plate action may partially account for the values found in this region.

Fig. 14 presents the corresponding simple unit stresses produced in wheel No. 671 449 by mounting. A greater number of strain measurements were made on wheel No. 671 449, and a better idea of the magnitude of the stresses produced by forcing the wheel on the axle is possible than in the case of wheel No. 671 237. On the outer face the larger tangential or "hoop" stresses are in general near the bore, and they decrease toward the rim. This is readily apparent on radii *G* and *H*, and indicates a distribution of stresses which would be expected. There are, however, variations from this arrangement or distribution along radial lines *A*, *C*, and *D*. On these lines the tangential stress does not at all points show a decrease from bore to rim; at certain points the stresses are slightly greater than at other points nearer the bore. Bearing in mind, however, the fact that cast iron from the ordinary cupola cannot be cast so as to be of uniform strength, as was evidenced by the results of the previously mentioned tests made on specimens cut from various parts of the wheel, such variations could have been anticipated. Similar conditions occur on the inner face with respect to the tangential stresses. On this wheel the maximum recorded tensile strain occurred on the inner face in gage-line *A5T*. Its value was 0.00161 and the corresponding stress is 15 700 lb. per sq. in. A study of the radial strains, Fig. 14, on the outer face of wheel No. 671 449 reveals a distribution similar to that found in wheel No. 671 237. On all of the radial lines a tendency towards a concentration of stress appears on the outer face on gage-line *3R*, or near the intersection of the inner and outer plates. The maximum recorded strain occurred on gage-line *H3R*, and was equal to 0.00116, corresponding to 16 500 lb. per sq. in. On the inner face the stresses on both the radial and bracket gage-lines are in general compressive and of relatively small magnitude.

After being mounted, the wheels were subjected to static loads up to 200 000 lb. per wheel, or about 10 times the load they would be subjected to in practice. The maximum stresses in wheel No. 671 449, caused by combined mounting and static loads, are shown in Figs. 15 and 16. In Fig. 15 are shown the corresponding simple stresses caused by superimposing the effects of static loads, ranging from

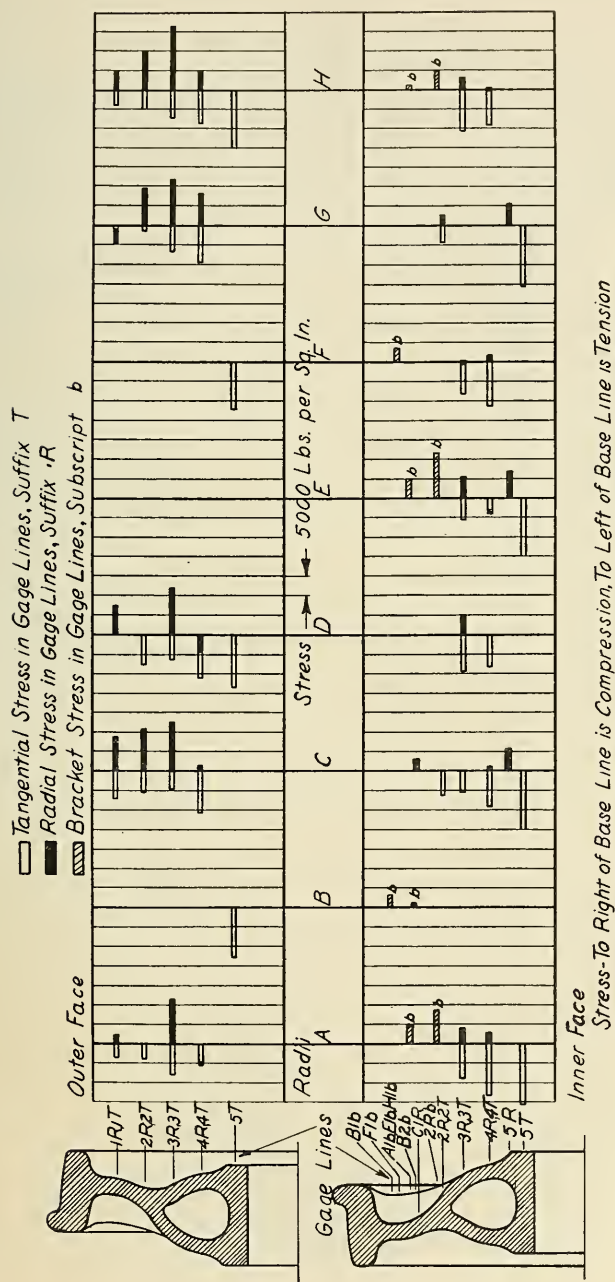


FIG. 14. CORRESPONDING SIMPLE UNIT STRESSES IN 33-IN. 725-LB. M. C. B. WHEEL No. 671 449 CAUSED BY MOUNTING ON AXLE

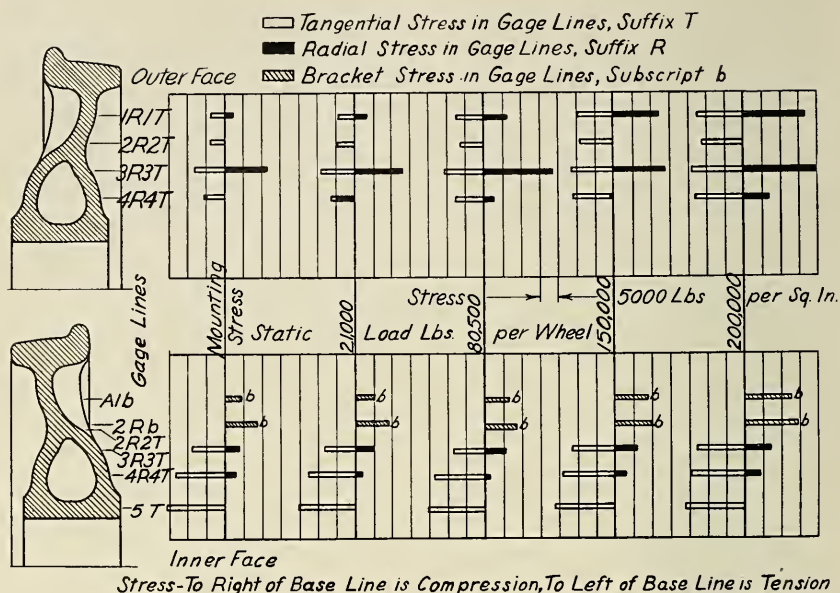


FIG. 15. CORRESPONDING SIMPLE UNIT STRESSES IN 33-IN. 725-LB.

M. C. B. WHEEL No. 671 449 DUE TO COMBINED EFFECTS OF
MOUNTING AND STATIC LOADS. LOADS APPLIED ON *A* RADIAL

21 000 to 200 000 lb., upon those due to forcing the wheel on the axle. To prevent the axle being bent, it was given additional support between the wheels for wheel loads above 40 000 lb. In the tests represented by Fig. 15 the load was supplied on the *A* radial gage-line, i. e., the several loads were transmitted from the axle along this radial gage-line to the rail. The strains along this radial gage-line were larger than the strains along the other radial gage-lines; consequently, presentation of figures relating to the strains along the other radial gage-lines has been omitted from the discussion. In Appendix B, pages 81 to 85, Tables 2, 2a, 3, and 3a, there are presented in tabular form the numerical values of the strains measured on the various radial gage-lines of wheels No. 671 237 and 671 449. Fig. 16 presents for wheel No. 671 449 information similar to that in Fig. 15, except that for Fig. 16 the static loads were applied along the *G* radial gage-line.

With respect to the radial gage-line along which the load is applied, the stresses due to a static load, when imposed upon those

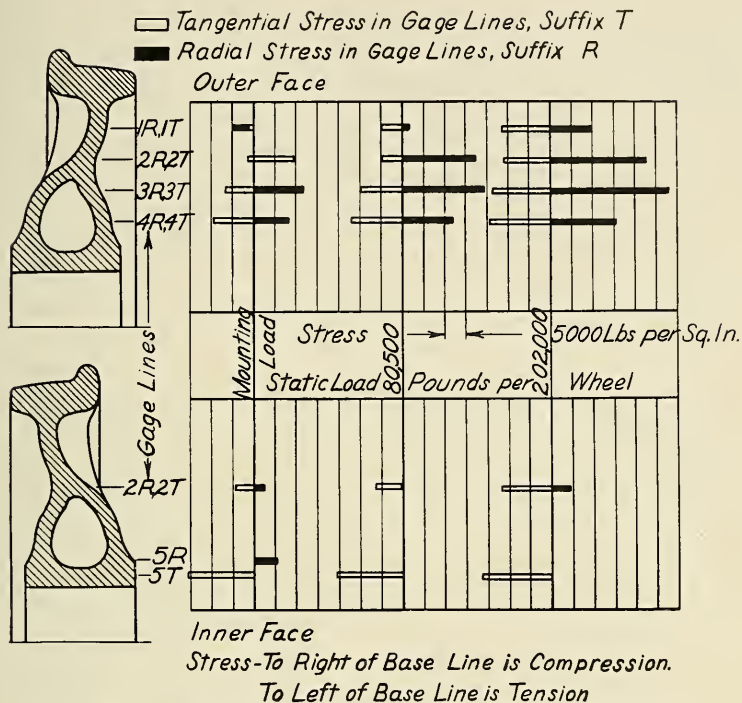


FIG. 16. CORRESPONDING SIMPLE UNIT STRESSES IN 33-IN. 725-LB. M. C. B. WHEEL NO. 671 449 DUE TO COMBINED EFFECTS OF MOUNTING AND STATIC LOADS. LOADS APPLIED ON *G* RADIAL

caused by mounting, are similar in kind to those of mounting, i. e., compressive on the radial and tensile on the tangential gage-lines. In ordinary practice the 33-in. 725-lb. wheel of this type may be subjected to a load not exceeding 20 125 lb. With a static load of 21 000 lb. the stresses are but slightly different from, and may be either greater or less than, those of mounting. This indicates that mounting is a much more important factor in producing stress than the maximum car load—neglecting impact and indirect effects—to which wheels are subjected in normal railroad service. With the static load equal to 200 000 lb. applied on the radial gage-line *A*, the maximum unit tensile strain due to the combined effects of mounting and load, was found to be on the tangential gage-line *A5T* on the inner face of the wheel, and to be equal to 0.00185. The corresponding unit stress is

16 500 lb. per sq. in., or an increase of only 800 lb. per sq. in. above that caused by mounting alone. The maximum increase in tensile strain occurred on the tangential gage-line *G1T* on the outer face when the 200 000 lb. load was applied along the *G* radial. For this loading, the increase in unit tensile stress on the several gage-lines over that due to mounting was as follows:

Gage-Line, Outer Face	Increase in Tensile Stress per sq. in.
<i>G1T</i>	10 500 lb.
<i>G2T</i>	9 100 lb.
<i>G3T</i>	6 900 lb.
<i>G4T</i>	4 700 lb.

As gage-line *G1T* was nearest the tread and *G4T* nearest the hub, these figures indicate that the influence of the static load decreases as the distance from the rail becomes greater. As a whole the observations relating to stresses produced by the static loads indicate that the influence of these loads (as expressed by increase of the tensile stresses) decreases as the distance from the rail toward the axle increases, although occasional exceptions to this rule occur.

Slightly different conditions exist on the radial gage-lines with respect to magnitude and distribution of the stresses. On the radial gage-lines the stresses are compressive, and the variations over the *G* radial gage-line for the corresponding 200 000-lb. load were:

Gage-Line, Outer Face	Increase in Compressive Stress per sq. in.
<i>G1R</i>	15 000 lb.
<i>G2R</i>	13 300 lb.
<i>G3R</i>	13 800 lb.
<i>G4R</i>	7 400 lb.

Here again the maximum increase in stress occurs nearest the tread and the minimum increase nearest the bore. It will be recalled that, in mounting, the opposite condition was found; namely, that the greatest stress occurred at the bore and the least at the tread. It is further evident from the above figures that the variation in stress in the radial direction differs from that in the tangential direction in that the latter shows an almost uniform decrease in the intensity when traversing the section from *G1T* to *G4T*, while in the former no such

uniform decrease occurs. For the combined effect of mounting and the 200 000-lb. static load the maximum recorded compressive strain was on radial gage-line *G3R*, and amounted to 0.00267. The corresponding simple compressive stress is 25 600 lb. per sq. in., or 13 800 lb. greater than the stress produced by mounting alone. The results on which Figs. 13 to 16 inclusive are based, together with the measured strains, are given in Appendix B, Tables 2, 2a, 3, and 3a. Figures similar to Figs. 15 and 16, which relate to wheel No. 671 449, are not submitted for wheel No. 671 237, due to the fact that the measurements of strain taken during the mounting test of wheel No. 671 237 were so meagre; the numerical values of these, however, are given in Appendix B, Table 2a.

In summarizing, these tests would indicate that, directly, the wheel load of ordinary service does not materially alter the existent strains caused by mounting, although, indirectly, the wheel load is a factor in producing stress, due to the bearing it has on flange pressure, impact, etc. Load application results in increasing the compressive strains already existent in the wheel to a greater extent than it increases the tensile strains. It is further evident that in the absence of speed and track curvature, very heavy wheel loads may be sustained without greatly increasing the magnitude of the tensile stresses which were produced by mounting.

14. *Corresponding Simple Stresses in Two 33-in. 740-lb. Arch Plate Wheels Due to Mounting Wheels on Axle, together with Combined Effects of Mounting and Static Loads.*—From the results of the tests on the wheels mentioned in the preceding section it was impossible to determine the exact position of either the maximum tensile or the maximum compressive stress. Furthermore, the relatively small number of strain lines prevented a clear conception of the stress distribution across the wheel section. Accordingly, in subjecting a pair of 33-in. 740-lb. Arch Plate wheels to the mounting and static load tests, they were prepared with approximately twice as many gage-lines as were used on the previously tested wheels. This was done by placing the 2-in. gage-lines 1 in. apart, instead of 2 in. apart as in the former tests. The locations of the gage-lines on wheels Nos. 04 474 and 04 476 are given in Figs. 17 and 18 respectively. The system used for identification of the gage-lines is shown in the figures,

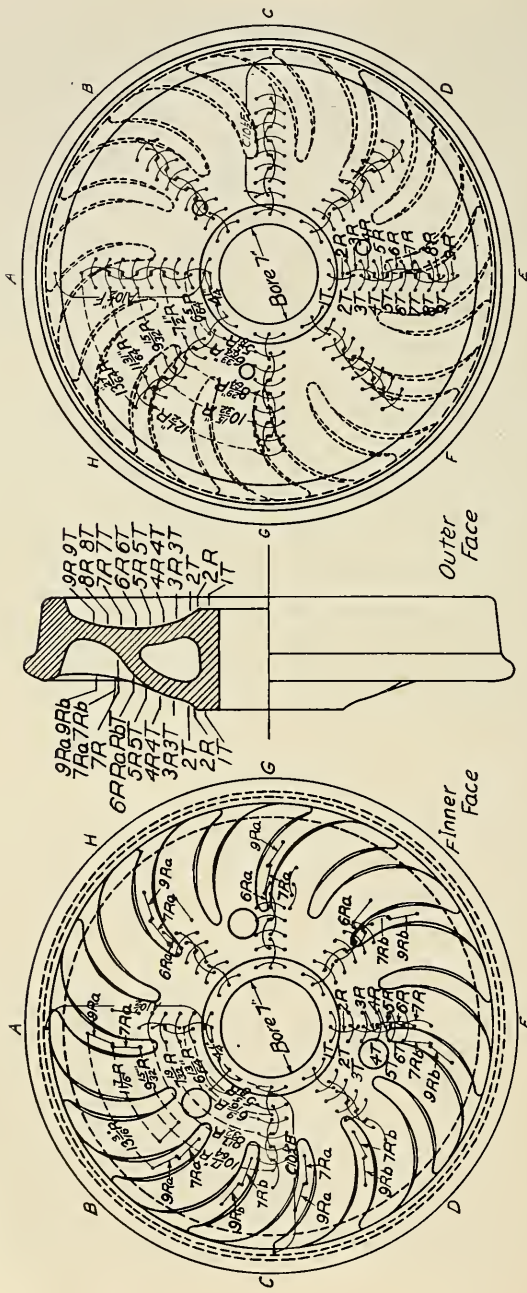


FIG. 17. LOCATION OF GAGE-LINES ON 33-IN. 740-LB ARCH PLATE WHEEL No. 04 474

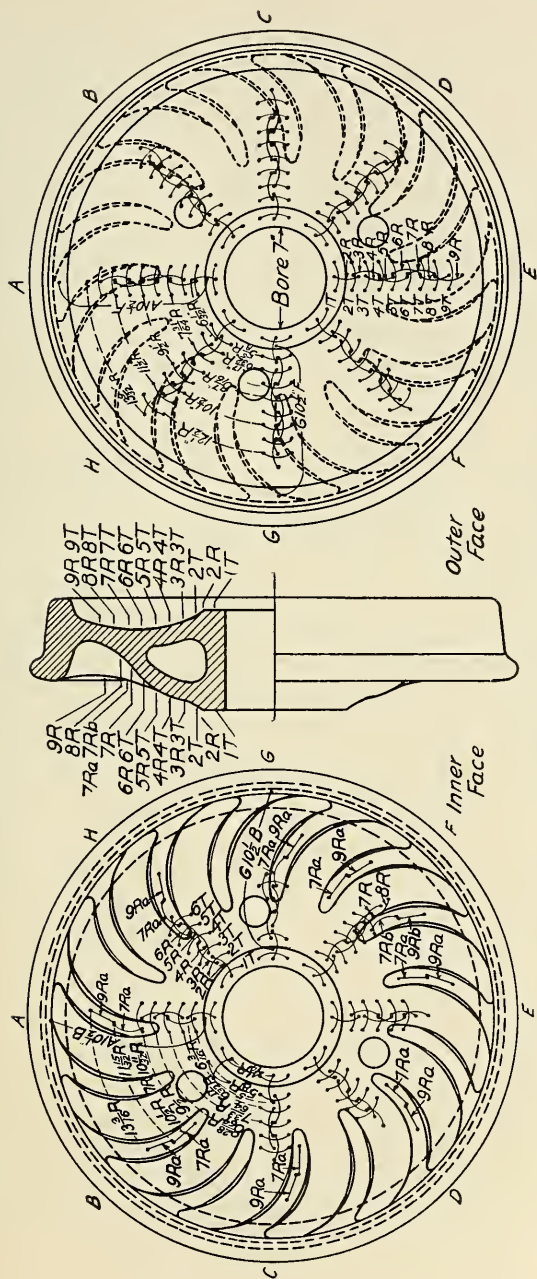


FIG. 18. LOCATION OF GAGE-LINES ON 33-IN. 740-LB ARCH PLATE WHEEL No. 04 476

and is similar to that used on the 33-in. 725-lb. wheels* except that the gage-lines are numbered from the hub outward to the tread.

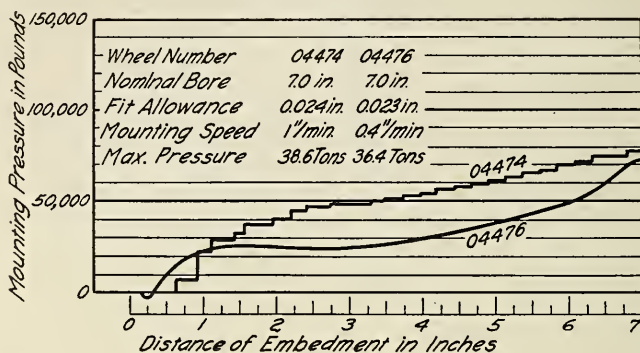


FIG. 19. AUTOGRAPHIC DIAGRAMS OF PRESSURES REQUIRED TO MOUNT TWO 33-IN. 740-LB. ARCH PLATE WHEELS ON AXLE

In Fig. 19 are given the fit allowance, the mounting speed, and the pressure required to force the axle into the wheels. It will be noticed that the mounting speed of wheel No. 04 474 is 2.5 times that of wheel No. 04 476. The results on the 725-lb. M. C. B. wheels suggested that the speed of mounting might be a factor in the magnitude of the final pressure. Although wheel No. 04 474 had the higher pressing speed, yet its higher recorded pressure can hardly be ascribed to this cause, as the slightly greater final pressure, 4.2 tons, may have been due to other factors, such as fit allowance, character of wheel material, etc. A series of tests with a greater variation in speed would probably be necessary to satisfactorily answer this question.

The corresponding simple stresses caused by forcing the wheels on the axle are shown in Figs. 20 and 21. The values of the stresses are based on the measured strains and the assumed stress-strain relation given by Fig. 6. If the stress at some particular radius is taken, it will be noticed in Figs. 20 and 21 that there may be a considerable variation between the maximum and the minimum values occurring on the several radial gage-lines *A* to *H*. As a result of this, the variations in the intensities of the stresses on one radial gage-line may be entirely different from the variations on another, although adjacent. To obtain a diagram of the stress distribution across the section of the wheel, more representative of the wheel as a whole and better for

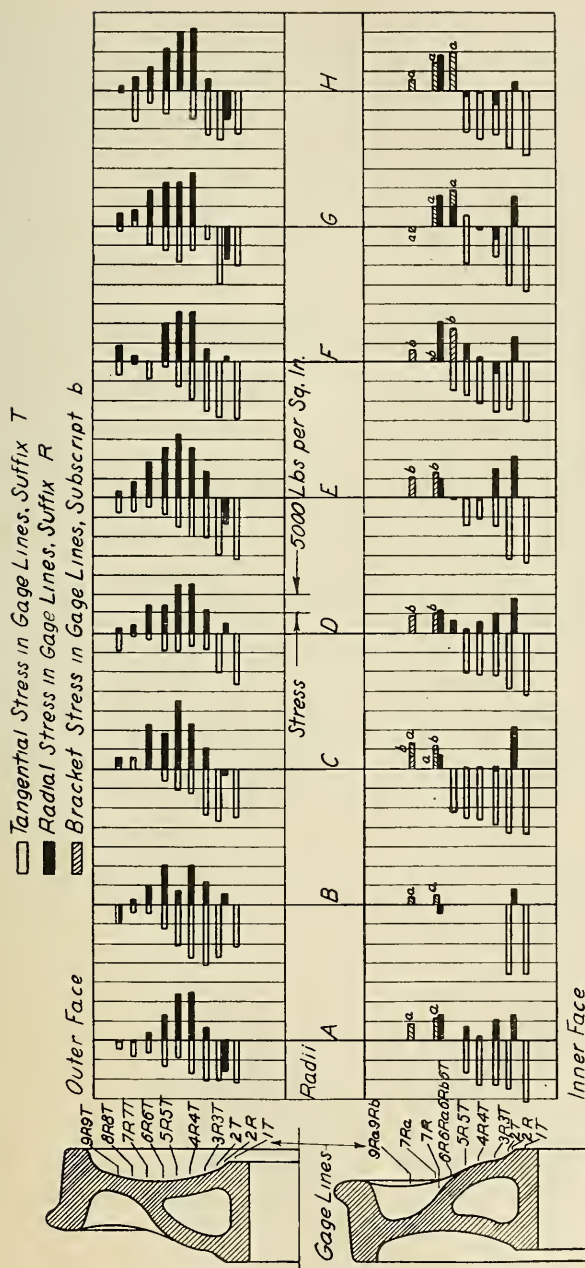
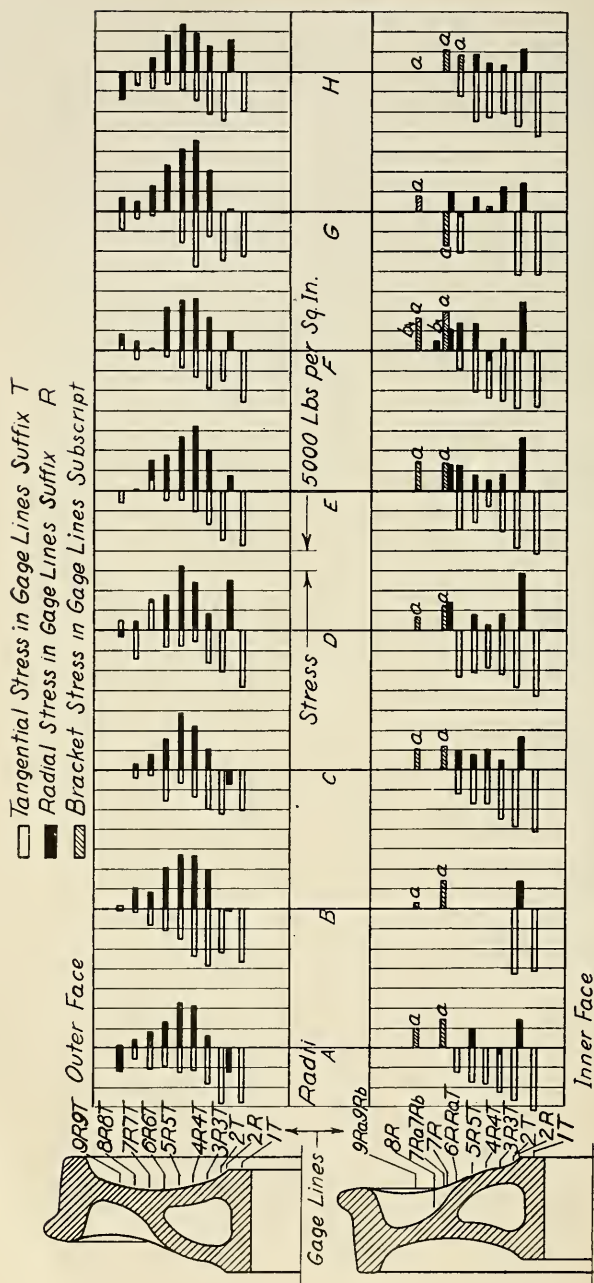


FIG. 20. CORRESPONDING SIMPLE UNIT STRESSES IN 33-IN. 740-LB. ARCH PLATE WHEEL No. 04 474 CAUSED BY MOUNTING ON AXLE



Stress-To Right of Base Line is Compression, To Left of Base Line is Tension

Maximum Mounting Pressure 36.4 Tons

FIG. 21. CORRESPONDING SIMPLE UNIT STRESSES IN 33-IN. 740-LB. ARCH PLATE WHEEL No. 04 476 CAUSED BY MOUNTING ON AXLE

purposes of comparison, the measured strains on the several radii at corresponding distances from the center of the wheel were averaged. The corresponding simple stresses for these average strains were then determined and plotted, as shown in Figs. 22 and 23. This method of determining the average stresses gives somewhat higher values than would be obtained by averaging the stresses corresponding to the measured strains.

Inspection of Figs. 22 and 23 immediately suggests the stress distribution across the section. The stresses on the tangential gage lines on both the inner and outer faces are tensile, largest near the bore and decreasing towards the tread. No such simple arrangement, however, exists in the stresses produced on the radial gage-lines. On these lines on the outer face when traversing the section from bore to tread a relatively small stress, either compression or tension, exists on gage-line $2R$, and the character of the stress then becomes compressive, reaching a maximum on either $4R$ or $5R$ —just previous to the joining of the inner and outer plates—after which the compressive stress again decreases. On the radial gage-lines $2R$ on the outer face of wheel No. 04 474, Fig. 20, the strain is an elongation, contrary to what might be expected. The reason for this condition is not apparent but it is perhaps due to a bending action at that point producing tension of a magnitude sufficient to change the compressive stress due to mounting into a tensile stress. On the outer face of both wheels the variation in the radial stresses in the regions represented by gage-lines $2R$ to $5R$ inclusive suggests a condition of bending combined with compression due to mounting.

On the inner face of wheel No. 04 474 the effect of bending is evident on the radial gage-lines. The stress on these lines is compressive and reaches a maximum at the bore. As the radial distance from the center increases, the stress decreases until a minimum is reached near $4R$, after which it increases up to $7R$. Beyond $7R$ no readings were obtained because of the interference of the wheel brackets with the strain gages, but it is reasonable to assume that a decrease in stress will develop soon after $7R$ is passed. The comparatively short radii of curvature in this portion of the wheel without doubt greatly affect any bending action which takes place and are largely responsible for the variation occurring in the magnitude of

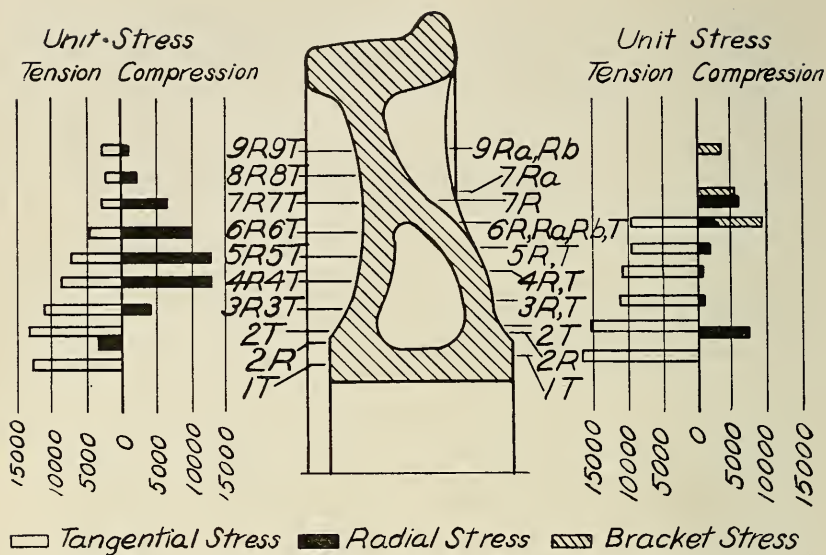


FIG. 22. AVERAGE UNIT STRESSES IN 33-IN. 740-LB. ARCH PLATE
WHEEL No. 04 474 CAUSED BY MOUNTING ON AXLE

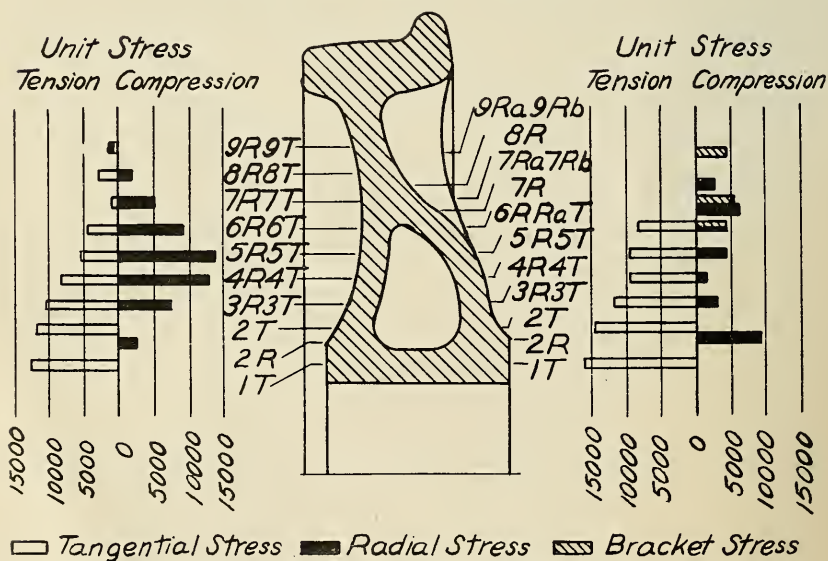


FIG. 23. AVERAGE UNIT STRESSES IN 33-IN. 740-LB. ARCH PLATE
WHEEL No. 04 476 CAUSED BY MOUNTING ON AXLE

the stresses. Similar conclusions may be reached with respect to the stresses on the radial gage-lines on the inner face of wheel No. 04 474 as shown in Fig. 22.

In wheel No. 04 474 the maximum average tensile and compressive strains are 0.00194 and 0.00084, located on tangential gage-lines $1T$ of the inner face, and radial gage-lines $5R$ of the outer face, respectively. The corresponding tensile stress is 16 800 lb. per sq. in. and the corresponding compressive stress 13 000 lb. per sq. in. In the other wheel, No. 04 476, the positions of the maximum average tension and compression are identical with those of wheel No. 04 474. The tensile stress, however, is 16 000 lb. per sq. in., i. e., slightly lower than in wheel No. 04 474, while the compressive stress is 13 700 lb. per sq. in., or 700 lb. higher than in wheel No. 04 474. The fit allowance on these two wheels differs by only 0.001 in. and a close agreement exists both as to magnitude and location of the maximum strains or stresses caused by mounting.

These wheels were also subjected to loads equalling 200 000 lb. per wheel, and the combined effects of mounting and the various static loads are shown in Figs. 24 to 27, inclusive. The corresponding numerical values are tabulated in Appendix B, Tables 4, 4a, 5, and 5a.

From the diagrams showing the combined effects of mounting and static loads, Figs. 24 to 27, inclusive, it is evident that the metal in a radial direction on the outer face of the wheel is in general in compression. An exception to this occurs on gage-line $2R$, where tension is indicated under several loads. Here again the tension is probably due to flexure in conjunction with the direct thrust of mounting. It will be noticed in addition that the intensity of the compressive stress on the outer face exceeds that on the inner face, this fact indicating that the static load is transmitted from rail to hub mainly through the outer plate, while the smaller portion of the load is carried through the inner plate. In general the magnitude of the tensile stress is slightly greater on the inner plate of the wheel than on the outer plate. In the curved portion bending assists in the production of tensile stress. The maximum compressive stress due to combined effects of mounting and static load may occur on either of the radial gage-lines $4R$ or $5R$ on the outer face, i. e., at a point in the outer plate just before it is joined with the inner plate. The

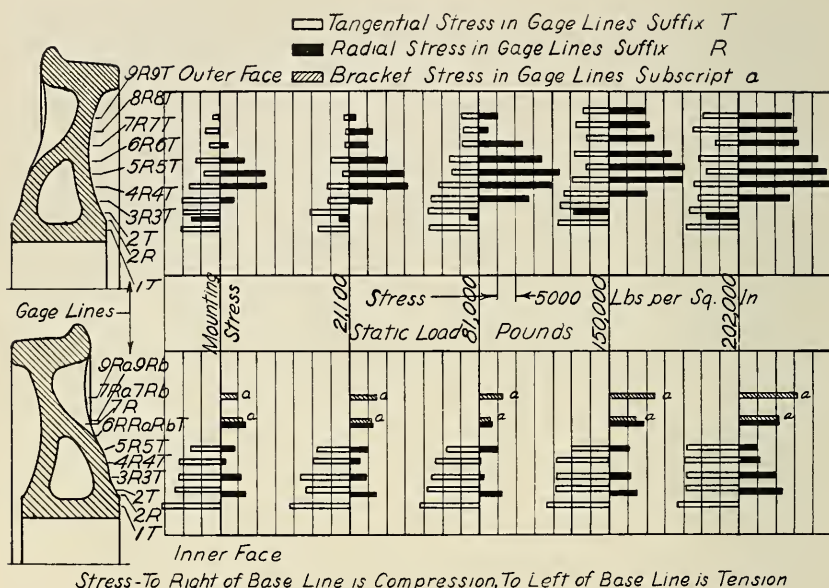


FIG. 24. CORRESPONDING SIMPLE UNIT STRESSES IN 33-IN. 740 LB ARCH PLATE WHEEL No. 04 474 DUE TO COMBINED EFFECTS OF MOUNTING AND STATIC LOADS. LOADS APPLIED ON A RADIAL

maximum tensile stress is found on either of the tangential gage-lines 1T or 2T on the inner face, that is, in the region of the hub. The conclusions to be drawn from the tests on these wheels are similar to those reached with regard to the 33-in. 725-lb. M. C. B. wheels—namely, that under load

- (1) the strain or stress due to static load is similar in kind to that of mounting, i. e., compressive on the radial gage-lines and tensile on the tangential gage-lines;
- (2) static load increases the magnitude of the compressive stresses on the radial gage-lines to a greater extent than it increases the magnitude of the tensile stresses on the tangential gage-lines;
- (3) for the normal service load the maximum strains or stresses are not greatly different from the maximum strains or stresses due to mounting;

(4) abnormally heavy wheel loads may be sustained in the absence of such other factors as side thrust, impact, etc., without greatly increasing the intensity of the tensile stresses over values already existent through pressing the wheel on the axle.

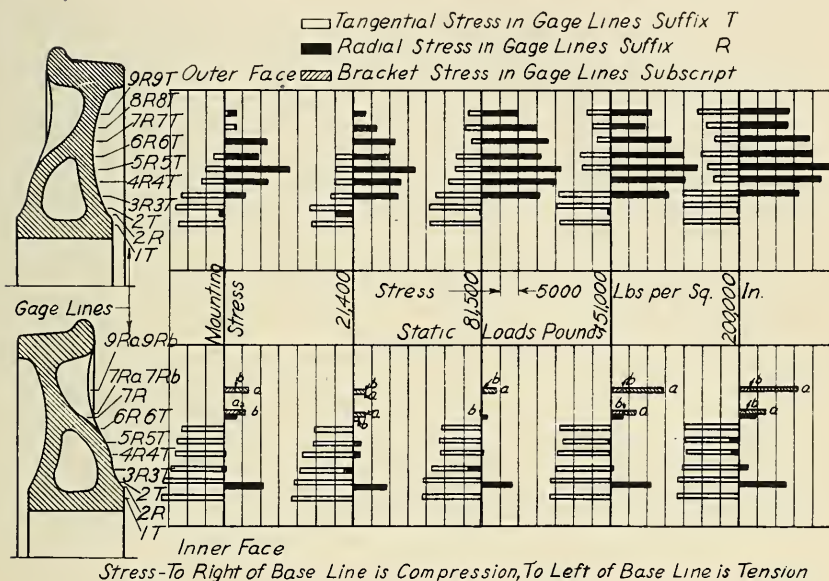


FIG. 25. CORRESPONDING SIMPLE UNIT STRESSES IN 33-IN. 740-LB. ARCH PLATE WHEEL No. 04 474 DUE TO COMBINED EFFECTS OF MOUNTING AND STATIC LOADS. LOADS APPLIED ON C RADIAL

It would have been desirable to examine the relative ability of the two types of wheels, as represented by the 725-lb. M. C. B. wheel and the 740-lb. Arch Plate wheel, to withstand the stresses produced by forcing the wheel on the axle and by the application of load. Due to their different fit allowances, however, and also to the relatively few strain measurements made on the 725-lb. wheels, no satisfactory comparison along these lines has been found possible.

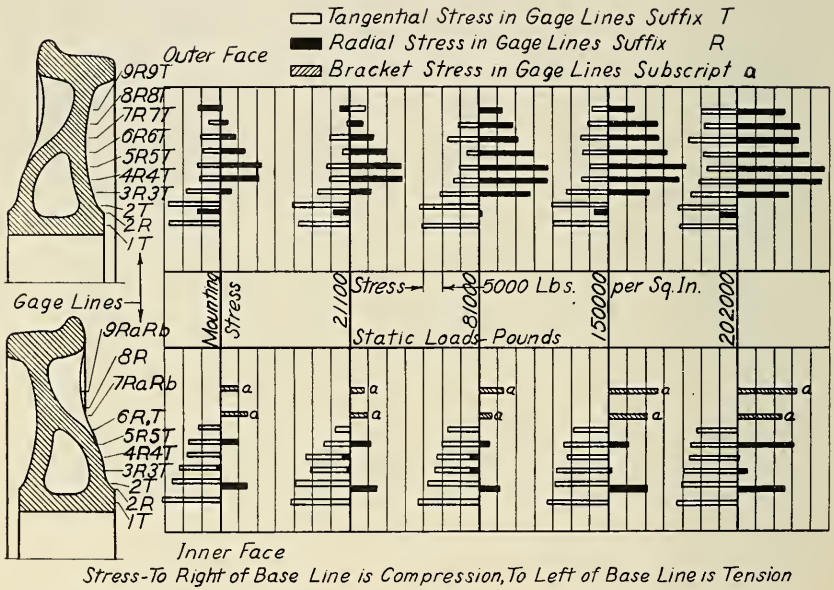


FIG. 26. CORRESPONDING SIMPLE UNIT STRESSES IN 33-IN. 740-LB. ARCH PLATE WHEEL NO. 04476 DUE TO COMBINED EFFECTS OF MOUNTING AND STATIC LOADS. LOADS APPLIED ON A RADIAL

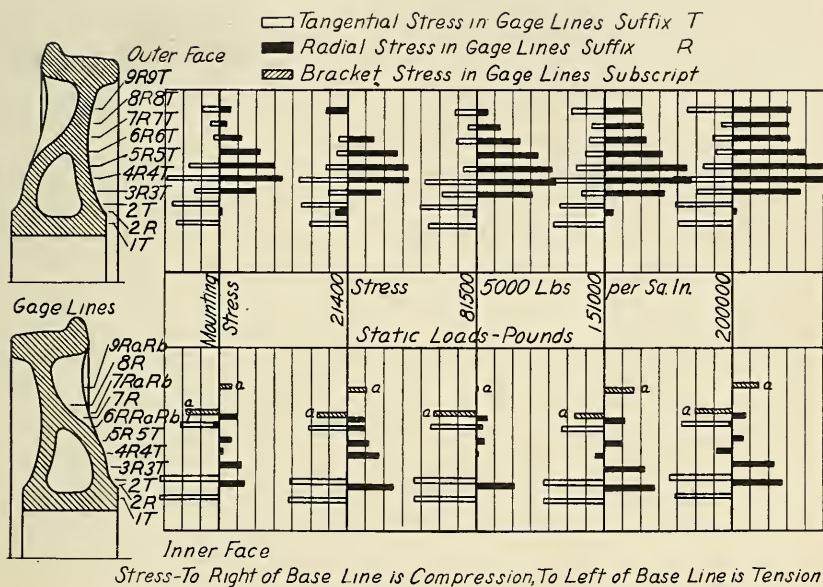


FIG. 27. CORRESPONDING UNIT STRESSES IN 33-IN. 740-LB. ARCH PLATE WHEEL NO. 04 476 DUE TO COMBINED EFFECTS OF MOUNTING AND STATIC LOADS. LOADS APPLIED ON G RADIAL

VI. RESULTS FROM ADDITIONAL MOUNTING TESTS

15. *Preparation of Wheels for Mounting.*—In order to determine in greater detail the effect of mounting, two 33-in. 625-lb. M. C. B., and two 33-in. 725-lb. M. C. B. wheels were subjected to this type of test. In forcing a wheel on an axle it is reasonable to expect higher stresses near the bore than at points distant from the bore. In the preceding tests it was impossible to take measurements at points less than $\frac{3}{4}$ in. from the bore. This condition arose through interference of the axle with the strain gage. To overcome this interference and thereby allow measurements to be taken in close proximity to the bore, the wheels were mounted on stub, or short, axles whose length equalled the thickness of the wheel at the bore. The location of the gage-lines on the several wheels is shown in Figs. 28 to 31, inclusive. The additional gage-lines in the hub region will be noticed. By the use of these stub axles it was possible to get measurements relating to tangential stresses within $\frac{3}{16}$ in. of the bore, and from these to get an estimate of the intensity of tangential or “hoop” stresses at the bore. To determine the intensity of stress in the region of the core holes on the inner face of the wheels, a few gage-lines were placed near certain core holes. These gage-lines are shown on the various figures. For a similar reason one of the wheels, No. 49 317, had gage-lines on the outer face in the vicinity of the chaplets.

In the case of the previously tested wheels the fit allowances were so nearly alike in each of the pairs of wheels tested that nothing as to the effect of fit allowance for a certain type of wheel could be determined. Accordingly, for the tests about to be described, certain fit allowances were chosen so that information might be obtained with regard to the effect of this factor.

16. *Corresponding Simple Stresses Due to Forcing Two 33-in. 625-lb. M. C. B. Wheels, with Different Fit Allowances, on Axles.*—The autographic diagrams taken while mounting these wheels are given in Fig. 32. Wheel No. 809 711 had a fit allowance of 0.009 in. and the maximum pressure was 14.4 tons. For this fit, the wheel was bored smooth with a tool of large radius, feed being approximately $\frac{1}{27}$ in. per revolution. The allowance in mounting wheel No. 822 269

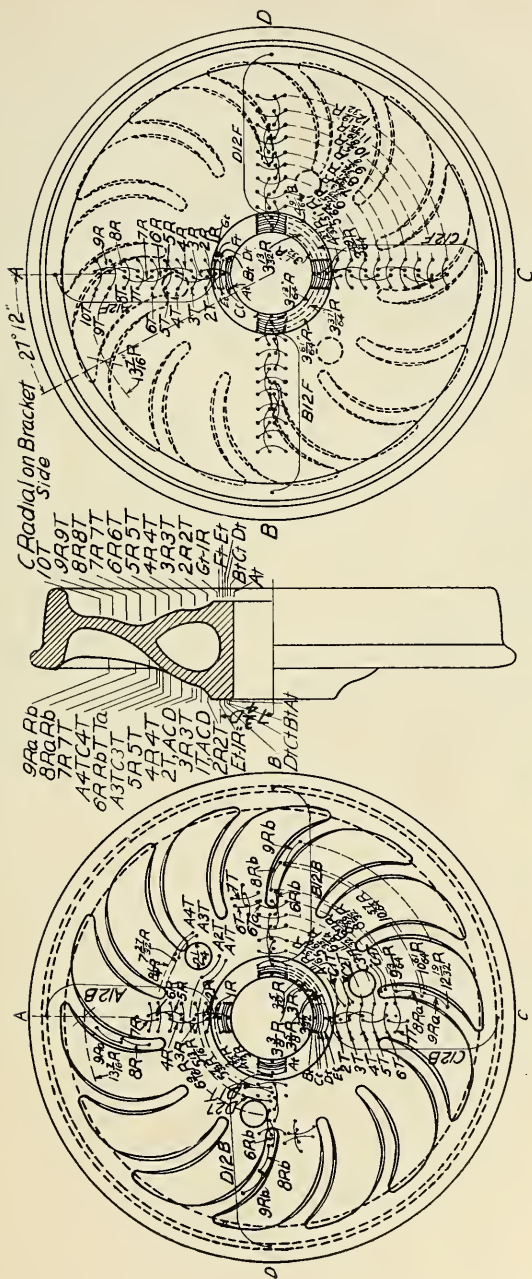


FIG. 28. LOCATION OF GAGE-LINES ON 33-IN. 625-LB. M. C. B. WHEEL NO. 809 711

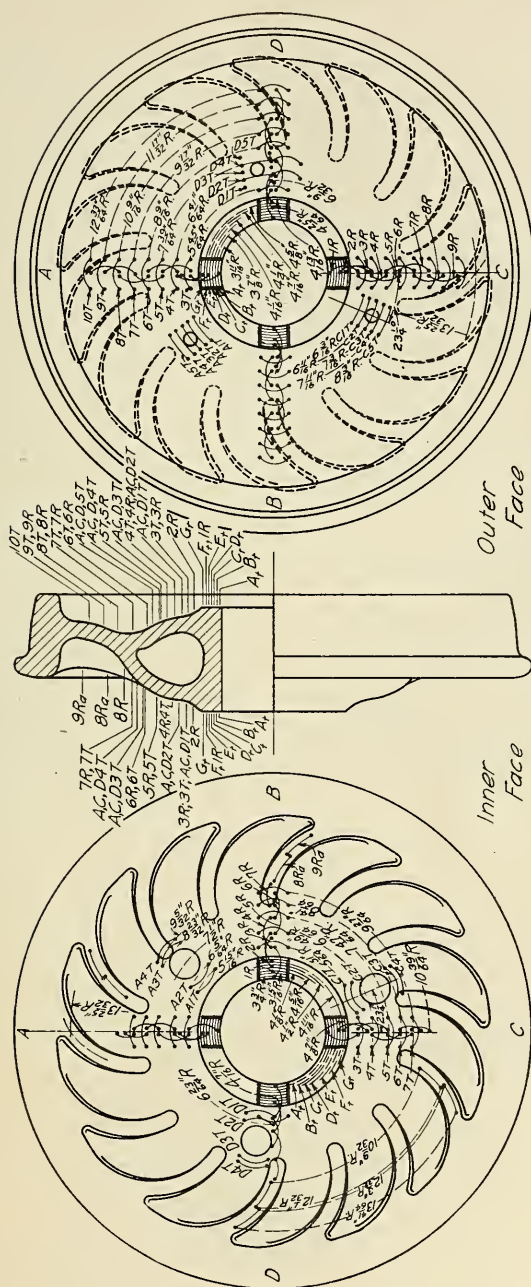


FIG. 30. LOCATION OF GAGE-LINES ON 33-IN. 725-LB. M. C. B. WHEEL No. 49 317

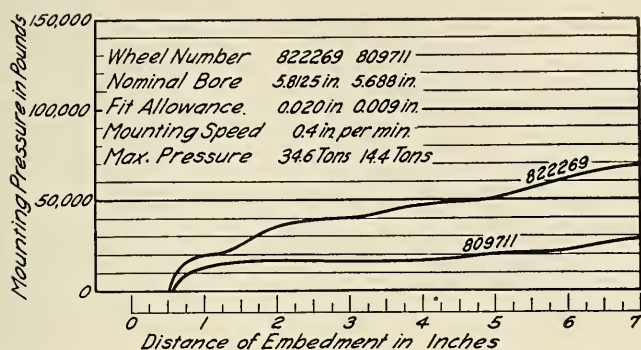


FIG. 32. AUTOGRAPHIC DIAGRAMS OF PRESSURES REQUIRED TO MOUNT TWO 33-IN. 625-LB. M. C. B. WHEELS ON AXLE

was 0.020 in., and the final pressure recorded was 34.6 tons. This wheel was also bored smooth and the axle showed no evidence of filing, the tool marks being plainly visible and indicating a feed of $1/14$ in. per revolution. The rate of mounting was 0.4 in. per min. in both cases.

The stresses set up on the various gage-lines of these two wheels are given in Figs. 33 and 34. The numerical results on which these figures are based are given in Appendix B, Tables 6 and 7. Here again variation exists in the intensity of the stress at uniform distances from the center, but on different radial lines. The stresses in wheel No. 822 269 are higher than those of wheel No. 809 711, a condition which would be expected, due to the fact that the former wheel had the larger fit allowance. In these two wheels the measurements were made on four radial lines. The four series of strain readings taken on the four radial lines were averaged and the corresponding stresses plotted in Figs. 35 and 36 in order to obtain a more representative diagram showing the variation of stress from point to point, and the effect of the different fit allowances. A comparison of these figures plainly shows the larger stresses in wheel No. 822 269 caused by the larger fit allowance. The trend of the variation in stress from bore to tread is not as clearly evident in Fig. 35 as in Fig. 36 on account of the relatively small values of the stresses shown in Fig. 35. Inspection of Fig. 36, however, suggests substantially the same conclusions as

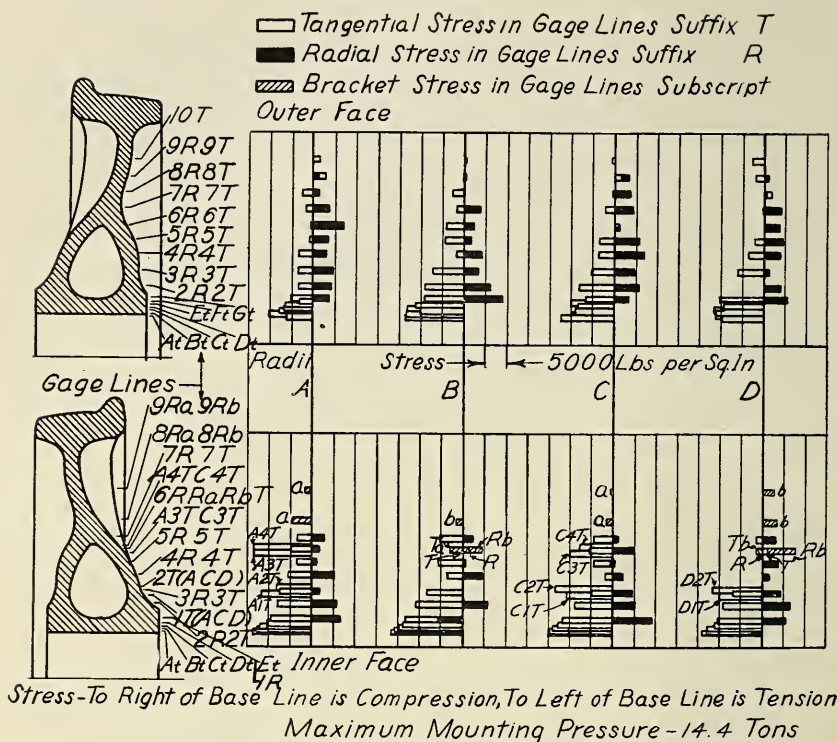


FIG. 33. CORRESPONDING SIMPLE UNIT STRESSES IN 33-IN. 625-LB. M. C. B. WHEEL NO. 809 711 CAUSED BY MOUNTING ON AXLE

were drawn in connection with the mounting tests previously discussed.

(1) The tangential or hoop stress on the outer face is tension and reaches a maximum at the bore; this stress decreases in intensity as the radial distance from the bore increases.

(2) In a radial direction on the outer face the stress is compression and similarly is a maximum at the bore, and, as the radial distance from the bore increases, its magnitude at first decreases, then increases up to a point near the intersection of the inner and outer plates, beyond which a decrease again occurs.

(3) The tangential stress on the inner face is tension and varies in a manner similar to that on the outer face, except that the decrease is not as uniform.

(4) In the radial direction on the inner face the stress is compressive, reaching a maximum at the bore, and as the radial distance from the bore increases the stress decreases until near the intersection of the inner and outer plates, where a tendency towards an increase, or an actual increase, occurs; beyond this region the stress again decreases.

(5) The tensile stresses on the tangential gage-lines adjacent to the core holes on the inner plate are almost equal in value to those on the tangential gage-lines nearest the bore. The stresses in the brackets caused by mounting are relatively insignificant.

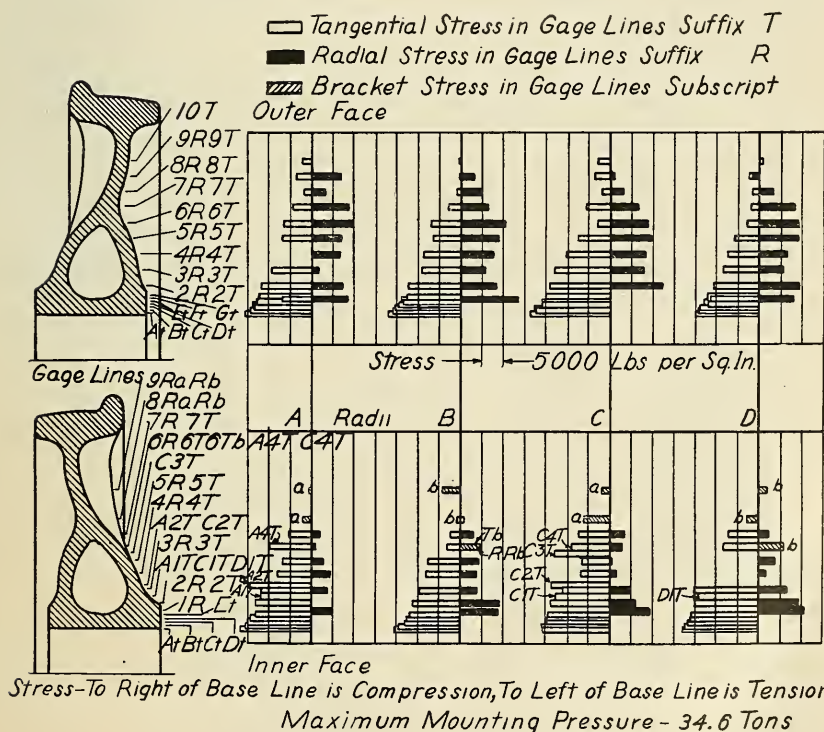


FIG. 34. CORRESPONDING SIMPLE UNIT STRESSES IN 33-IN. 625-LB. M. C. B. WHEEL No. 822 269 CAUSED BY MOUNTING ON AXLE

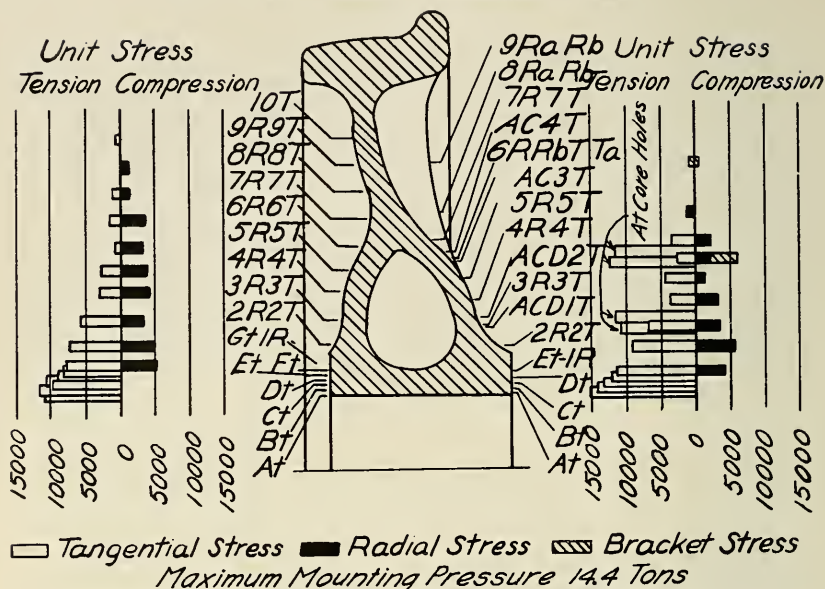


FIG. 35. AVERAGE UNIT STRESSES IN 33-IN. 625-LB. M. C. B.
WHEEL No. 809 711 CAUSED BY MOUNTING ON AXLE

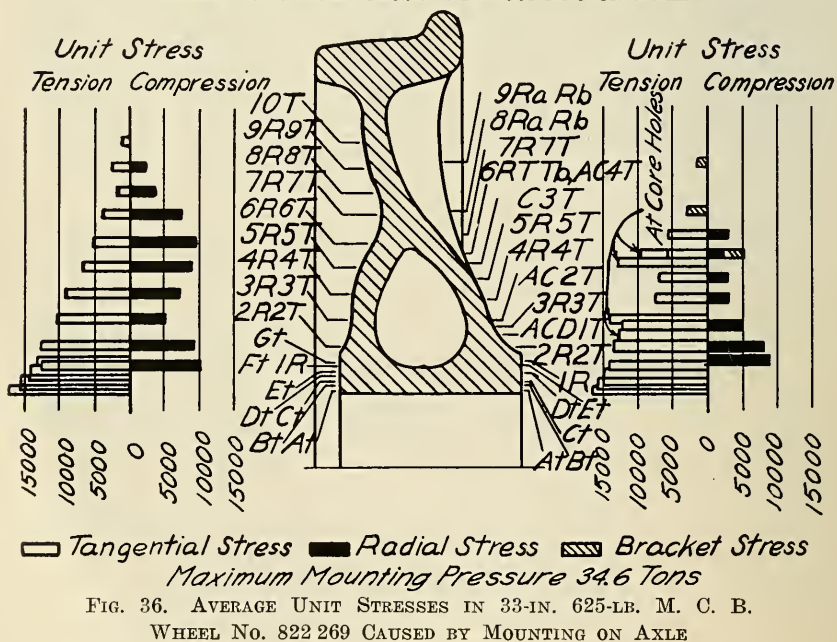


FIG. 36. AVERAGE UNIT STRESSES IN 33-IN. 625-LB. M. C. B.
WHEEL No. 822 269 CAUSED BY MOUNTING ON AXLE

17. *Corresponding Simple Stresses Due to Forcing Two 33-in. 725-lb. M. C. B. Wheels, with Different Fit Allowances, on Axles.*—For this test, wheels Nos. 49 317 and 49 305 had fit allowances of 0.0095 and 0.0136 in. respectively, while the corresponding final pressures were 39.9 and 43.0 tons. The rate of pressing was 0.4 in. per min. in each case. The mounting diagrams are shown in Fig. 37. As in the case of the 625-lb. wheels, one axle was filed smooth and the other retained the tool marks. As far as the stresses produced by mounting are concerned, there was nothing in these tests that would indicate the relative effects of a smooth or a rough axle. In Figs. 38 and 39 are shown the stresses corresponding to the measured strains caused by mounting. The corresponding results are presented in Appendix B, Tables 8 and 9. Appendix B also presents results for the average effects which are shown in Figs. 40 and 41. Although differences exist in the relative ability of the 625 and 725-lb. wheels to withstand the stresses produced by mounting, yet there is a marked similarity in the location of the maximum and minimum stresses. The conclusions to be drawn with reference to the 725-lb. wheel are in general identical with those for the 625-lb. type as given in Section 16, page 58 of this bulletin. In addition, Fig. 40 shows that the intensity of the tensile stress on the tangential gage-lines in the regions of the chaplets on the outer face nearly approaches that occurring at the bore.

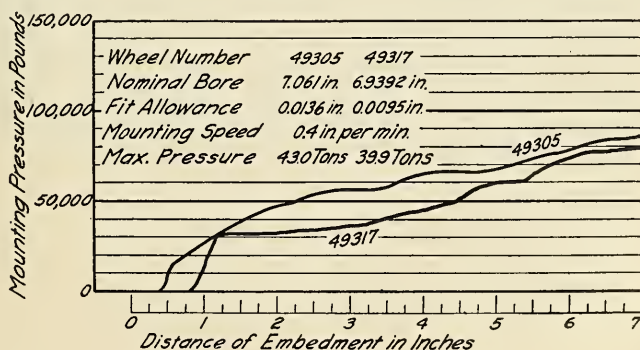
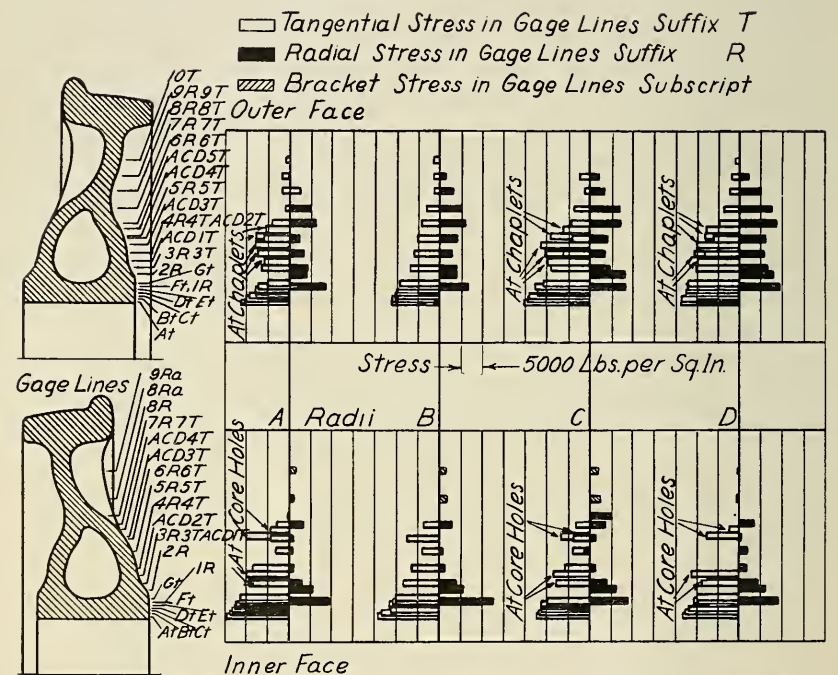
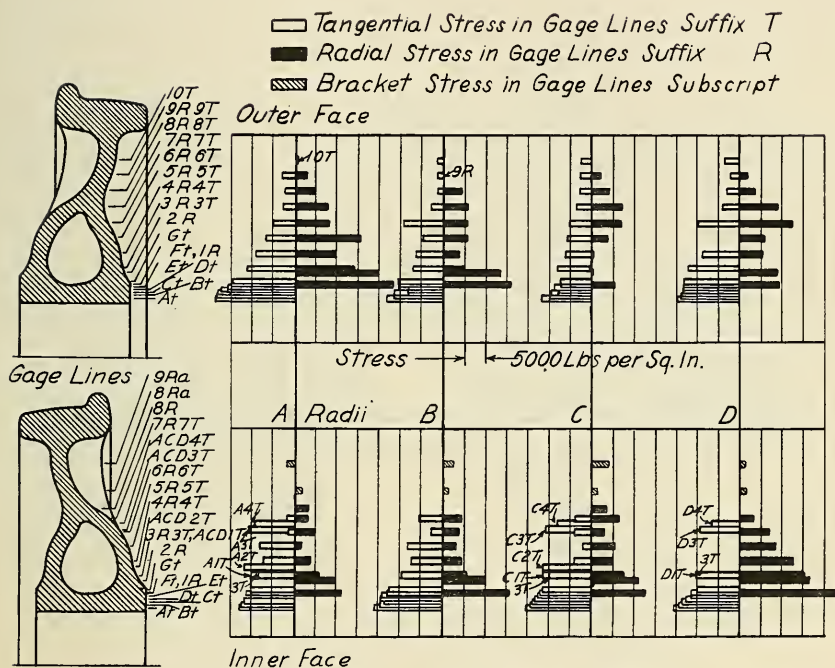


FIG. 37. AUTOGRAPHIC DIAGRAMS OF PRESSURES REQUIRED TO MOUNT TWO 33-IN. 725-LB. M. C. B. WHEELS ON AXLE



Stress-To Right of Base Line is Compression, To Left of Base Line is Tension
 Maximum Mounting Pressure 39.9 Tons

FIG. 38. CORRESPONDING SIMPLE UNIT STRESSES IN 33-IN 725-LB. M. C. B.
 WHEEL No. 49 317 CAUSED BY MOUNTING ON AXLE



Stress-To Right of Base Line is Compression, To Left of Base Line is Tension
 Maximum Mounting Pressure—43.0 Tons.

FIG. 39. CORRESPONDING SIMPLE UNIT STRESSES IN 33-IN 725-LB. M. C. B. WHEEL NO. 49 305 CAUSED BY MOUNTING ON AXLE

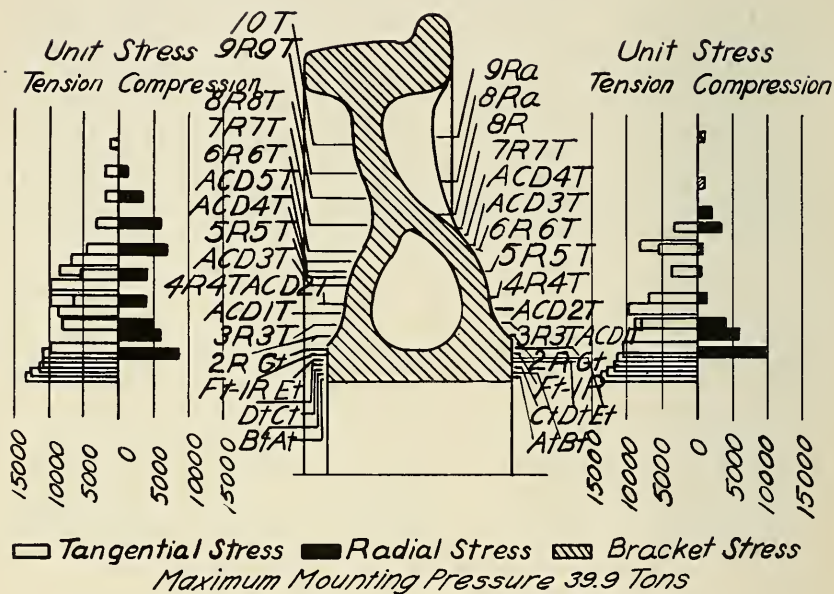


FIG. 40. AVERAGE UNIT STRESSES IN 33-IN. 725-LB. M. C. B.
WHEEL NO. 49 317 CAUSED BY MOUNTING ON AXLE

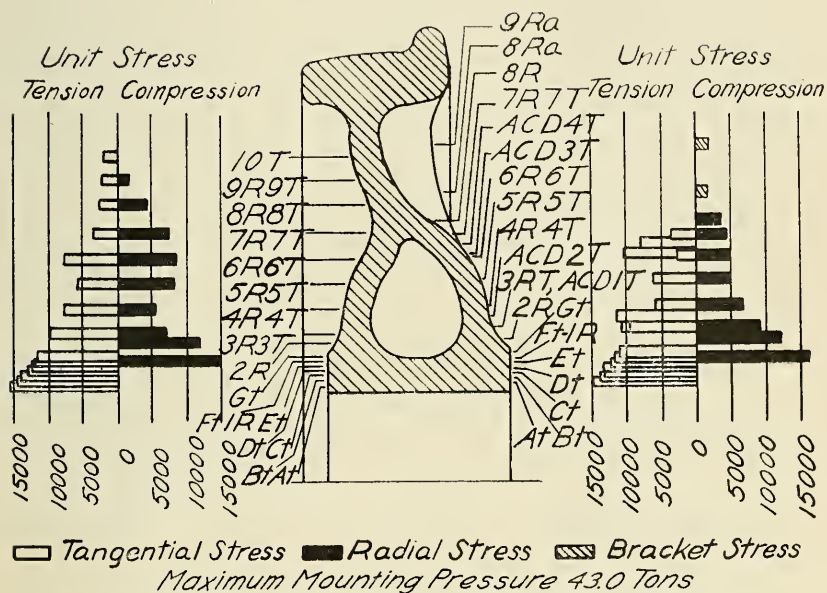


FIG. 41. AVERAGE UNIT STRESSES IN 33-IN. 725-LB. M. C. B.
WHEEL NO. 49 305 CAUSED BY MOUNTING ON AXLE

VII. SUMMARY OF CONCLUSIONS

The results recorded in the previous pages may be summarized as follows:

(1) The tensile strength of the metal taken from different parts of the plates of three wheels ranged from 23 300 to 32 800 lb. per sq. in., and the modulus of elasticity ranged from 14 to 28 million lb. per sq. in. It is probable that these variations may be explained by variations in chemical composition of the several specimens, and variations in the treatment of the wheels after casting. Whatever be the reason, however, they suggest that a study of the metallurgy of wheel irons offers possibilities of improvement by which the higher values obtained in these tests might be consistently maintained or possibly exceeded.

(2) No distinct relation was apparent between the ultimate strength of wheel iron and either the Brinell or the scleroscope hardness, nor could a constant relation be determined between the Brinell and scleroscope results.

(3) In forcing the 625-lb. and 725-lb. M. C. B. or Washburn type of wheel on an axle the maximum tensile strain or stress is a tangential or "hoop" strain or stress occurring at the bore, and it may be on either the inner or the outer face of the wheel.

On the outer face of this type of wheel in a radial direction the strains or stresses are compressive. They are a maximum at the bore and, in traversing the section of the wheel from the bore toward the tread, they decrease up to a point where the radius equals the mean radius of the core, beyond which an increase occurs up to a point at or near the intersection of the inner and outer plates, after which a decrease again occurs.

On the inner face in a radial direction the strains or stresses are likewise compressive and a maximum at the bore. If a similar traverse be made across the section, these strains or stresses decrease up to a point where the radius equals the mean radius of the core; they are then of approximately uniform intensity up to a point whose radius is equal to the outer radius of the core; beyond this point they again decrease.

(4) In pressing the 740-lb. Arch Plate wheels on the axle, the maximum tensile strain or stress was a tangential strain or

stress on the inner face and at the bore. In general the tensile stresses on either face of the wheel were a maximum at the bore and decreased toward the tread.

With respect to the strains or stresses in a radial direction on the outer face, no measurements were taken in close proximity to the bore, and accordingly nothing definite can be stated concerning their intensity in this region. However, the strain in a radial direction taken nearest to the bore was relatively small and was tension in one wheel and compression in the other. This fact may be due to bending action in conjunction with the thrust of mounting. As the tread was approached, the strains and hence the stresses became compressive and of increasing intensity, reaching a maximum at a point whose radius was equal to the mean radius of the core, beyond which they again decreased.

On the inner face over the region investigated, the strains or stresses in the radial direction were compressive, reaching the maximum nearest the bore and decreasing to a minimum at a point whose radius was equal to the mean radius of the core, after which they again increased.

(5) In the regions of the chaplets and core holes, pressing the wheel on the axle causes tensile strains in a tangential direction which are of lesser intensity than, but approach in magnitude to, those at the bore.

(6) The stresses and strains in the brackets which are produced in a radial direction by mounting are relatively insignificant.

(7) The strains caused by mounting the wheels on the axles, when mounting alone is considered, are greatest in the hub near the axles. These strains, although apparently high in the case of the greatest values recorded, are steady and not repeated as is the case with the majority of strains produced in service. Moreover, these highest strains extend through a comparatively thin layer of metal near the axle, and this strained layer is backed by other layers of less strained metal.

(8) In general the static load is transmitted from hub to rail, mainly through the outer plate, while the smaller portion of the load goes through the inner plate. This effect is more pronounced in the 740-lb Arch Plate than in the 725-lb. M. C. B. type of wheel. This division of the load seems desirable in that

the inner plate may be considered as affording reserve capacity for the purpose of absorbing the effect of side thrust on the flange when rounding curves.

(9) Pressing the wheel on the axle is much more effective in producing stress or strain within the wheel than the normal static load, and it therefore follows that the addition of the normal static load does not greatly add to or otherwise modify the more important of the existent strains caused by mounting.

(10) Abnormally heavy loads, in the absence of impact, side thrust, etc., may be sustained by wheels without increasing the normal strains, already existent, to such an extent as to seriously stress the wheel.

(11) The maximum strains reported, caused by the combined effects of mounting and static load, appear large when expressed in terms of the stress that would exist if the material were subjected to simple tension or simple compression. As previously stated, these strains are produced in the main by the mounting load, and the more important strains are those of tension, which in general are greatest near the bore of the wheel. The character of the strains, and the backing of the material most strained by less strained material, probably makes possible without injury to the material greater strains or deformations than would be allowable in the case of material not so supported and subjected to simple tensile stress. As previously stated, the problem is one of compound stress, and the method used in computing the stresses reported is thought to give the highest values for these that could be expected under such conditions. Any error in estimating, from the stress values determined in this way, when elastic failure might take place, would be upon the side of safety. In this connection it should also be remembered that the stresses reported are those produced by two forms of wheel loading only, and that the strains and stresses resulting from these two forms of loading (mounting and static) may be materially modified by additional stress-producing factors to which a wheel may be subjected in service.

APPENDIX A

A SHORT HISTORY OF THE CHILLED IRON CAR WHEEL WITH INFORMATION CONCERNING ITS MANUFACTURE AND SPECIAL PROPERTIES

If molten iron comes in contact with iron at room temperatures or is poured into a metallic mold it quickly solidifies. The sudden solidification of the metal prevents the combined carbon therein from precipitating out, thereby producing a metal in which all the carbon is in the combined form; that is, the so-called "white cast-iron" or "chilled iron" is produced. This principle was discovered in England about 1820 by a foundry workman who, quite by accident, noticed that when the excess metal remaining in the ladles after pouring the mold was emptied on the floors, the metal which came into contact with iron had quite different properties from that which solidified in contact with sand and air alone. The extreme hardness and resistance to wear of that portion of a casting chilled in this way were quite evident. In America the application of the principle of chilling to car wheel manufacture occurred about 1835. Three years later a Mr. Lobdell designed a wheel which marked the successful entrance of chilled iron into the car wheel field. In 1850 Mr. N. Washburn conceived a design of a chilled iron wheel which at that time weighed 500 lbs. The general features of this type of wheel, Figs. 42 and 43, are the merging of inner and outer plates into a single plate, the brackets and the "ogee" curve being formed on the outer face of the wheel. In 1904 the Master Car Builders Association adopted three designs of chilled wheels of the Washburn type weighing 600, 650, and 700 lbs., as recommended practice for cars of 30, 40, and 50 ton capacity. Slight variations in details of design persisted up to 1909, after which, reasonably strict adherence to the recommended M. C. B. standards was maintained. As a result of the maximum weight mentioned in the M. C. B. specifications, these three designs became commonly known as 625-lb., 675-lb., and 725-lb. wheels. In 1917 the 675-lb. Washburn type of wheel, generally spoken of herein as the M. C. B. type, for the 40-ton car was replaced by a 700-lb. pattern of different design. This design, generally called the Arch Plate type, differs from the Washburn type in that the three curves of the "ogee" curve on the outer face of the latter have been replaced

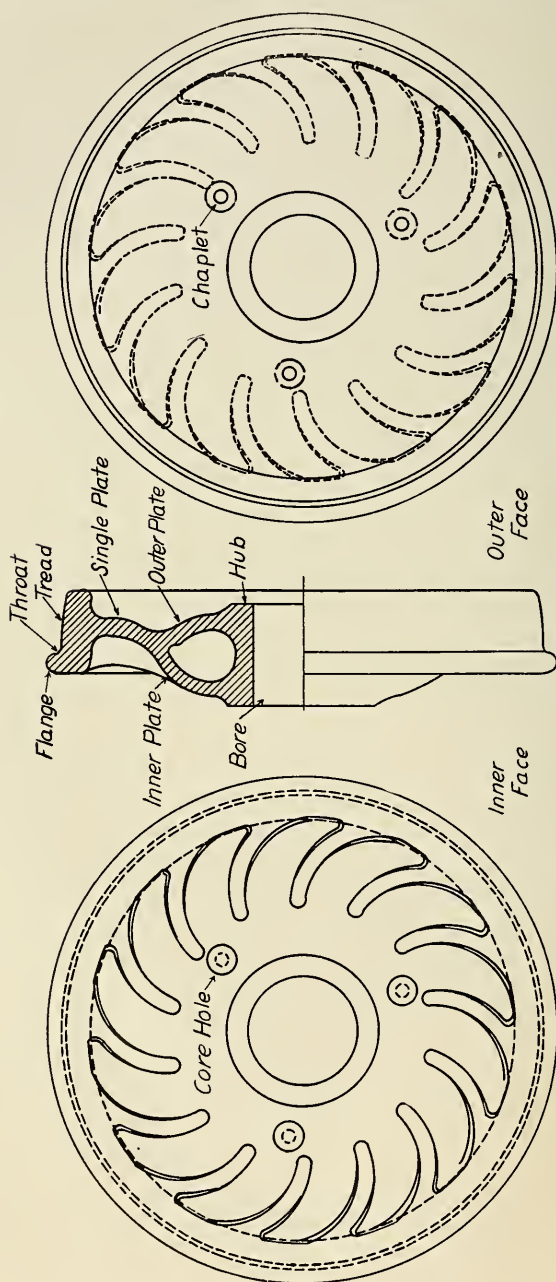
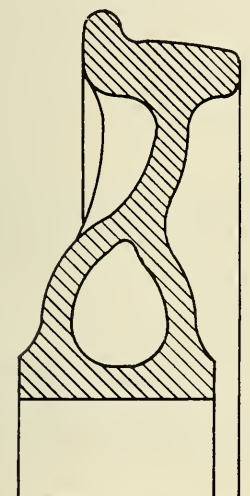
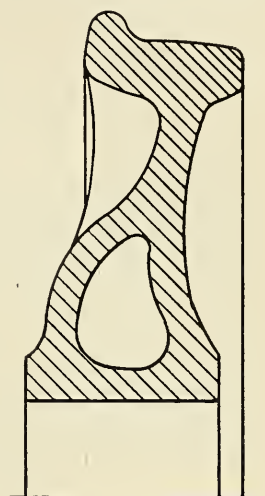


FIG. 42. M. C. B. OR WASHBURN TYPE OF CHILLED CAR WHEEL.

by a single curve of large radius. An additional wheel of the Arch Plate type weighing 850-lb. was also adopted as recommended practice for 70-ton capacity cars. In 1920 the Mechanical Section of the American Railroad Association adopted as recommended practice two more car wheels of the Arch Plate design with nominal weights of 650 and 750 lbs., these to take the place of the former 625 and 725-lb. wheels. Four 33-in. cast-iron wheels all of the Arch Plate design



M.C.B. or Washburn Type



Arch Plate Type

FIG. 43. SECTIONS OF CHILLED CAR WHEELS

weighing respectively 650, 700, 750, and 850 lbs. are now recommended practice for the American Railroad Association. The underlying reasons for substituting the Arch Plate for the Washburn pattern are due to both laboratory and service indications that the former, assuming equal weight of wheels in both cases, transmits the imposed loads with less internal strain, and as a corollary thereof, with a greater factor of safety. This very briefly covers the history of the chilled wheel to date.

The manufacture of chilled iron wheels is a rather complex problem and is carried on in a highly specialized manner. One of the staples of the iron markets in this country is old worn out

wheels and they represent from 40 to 60 per cent of the mixture used in the production of new wheels. The balance of the material consists of pig iron, special scrap, and alloys. To give the wheel certain properties, and to enable it to meet service conditions successfully and pass the rigid inspections imposed by the recommended M. C. B. specifications,* it is necessary to control the relative proportions of scrap wheels, pig iron, and alloys quite closely. In order to do this, chemical laboratories operated by responsible chemists are maintained

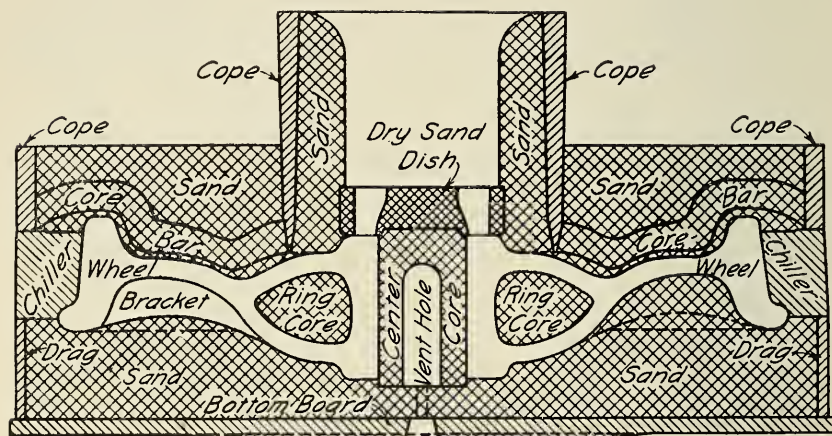


FIG. 44. CROSS SECTION OF A CAR WHEEL MOLD

as adjuncts to the foundries. These laboratories furnish both the necessarily accurate control of the material that goes into the cupola and an indication of the character of the finished product. The method of molding a cast-iron ear wheel is indicated by the cross-section of a car wheel mold shown in Fig. 44. Identification of the various parts of the mold is easily made by means of the figure. Attention, however, is specifically directed to the chill. This is a cast-iron ring of special chemical composition, whose inner face is machined to the desired contour of the tread and one half of the flange as shown. It rests on the novel and in turn supports the cope of the mold. After the mold is made up, everything is ready for pouring. The molten metal is taken from the bottom of the ladle so that the impurities

* Proc. M. C. B. Assn., Vol. 52, p. 490, 1918.

remain in the ladle. About 12 seconds is taken to fill a mold for a 725-lb. wheel. When the molten metal carrying carbon in the combined state comes in contact with the cold chill, the sudden solidification of the metal does not allow sufficient time for the precipitation of the carbon from the iron—thereby producing an extremely hard metal called “chilled iron” in that part of the wheel which comes into contact with the rail. The “chilled iron” exists to a depth of from $\frac{1}{2}$ to 1 in. The balance of the mold, which consists of specially tempered molding sand and two dry sand cores, allows a more gradual cooling or solidification of the metal in the other parts of the wheel. Consequently, the combined carbon in the molten metal has sufficient time to precipitate out during cooling, which in turn produces a gradation in the relative amounts of combined and graphitic carbon across the section of the wheel. This results in a wheel whose tread, or that part of the wheel which comes in contact with the rail, consists of an extremely hard metal capable of resisting both excessive deformation and wear, while the remainder of the wheel consists of relatively soft material which can be easily machined, and which is suited to resist the stresses put upon it in service.

After the wheel has been poured it is allowed to remain in the mold for about 5 min. or until it has solidified, after which it is removed and placed in a brick lined pit. After the pits are filled they are closed, and the temperature of the wheel at first becomes uniform and then gradually reduces over a period of about 3 to 5 days. This process is called annealing, and has for its purpose the removing of such strains as may have arisen because of the unequal conditions of cooling while pouring. Proper annealing is important, as wheels cooled in air are apt to crack and are subject to failure on account of excessive internal strains. If wheels containing high residual strains were placed in service a potential danger would exist, and might result in serious wheel failures. For this reason careful attention is given to the process of pitting or annealing.

After having been cooled in the pits the wheels are removed and cleaned of molding sand by means of a sand blast, and are then subjected to rigid inspection to see if the specifications of the buyer have been fulfilled. After having passed inspection the wheels are bored out at the center—the only machine work performed on the chilled car wheel during the process of manufacture. In machining,

the finished diameter of the bore is made slightly smaller—about 0.002 in. per in. of bore diameter—than the axle on which it is to be mounted. The slight difference between the size of the wheel bore and the axle requires a pressure of about 10 tons per in. of axle diameter in order to force the wheel on the axle; that is, a 6-in. wheel bore would require, approximately, the pressure of about 60 tons to mount the wheel on the axle. Mounting is accomplished in specially built hydraulic wheel presses, and the wheel is then ready to be placed in service.

APPENDIX B

TABULATED RESULTS

Tables 2 to 9, inclusive, present figures relating to the strain determinations which were made in connection with the eight wheels tested. In these tables the deformations measured by the strain gage have been reduced so that they are listed as unit strains. An absence of sign indicates tensile strain; whereas a minus sign denotes compressive strain.

TABLE 2

UNIT STRAINS PRODUCED BY MOUNTING WHEEL ON AXLE
33-in. 725-lb. M. C. B. Wheel No. 671 237

Strains Given in Table are Expressed in Hundred-thousandths
of Inches Per Inch

On Outer Face of Wheel								
Gage- Lines	Radials along which Gage-Lines Are Located							
	A	B	C	D	E	F	G	H
1R		—43		—28		13		—10
2R		—32		—109		—69		—82
3R		—102		—56		—127		—117
4R		—65		—24		—31		—12
5R		—105		—116		—32		—51
4T		222		66		39		53
5T		182		116		56		62
On Inner Face of Wheel								
5R				—62		—64		—4
5T		108		177		173		218

TABLE 2A

UNIT STRAINS PRODUCED BY COMBINED EFFECTS OF MOUNTING
AND STATIC LOADS

33-in. 725-lb. M. C. B. Wheel No. 671 237

Strains Given in Table are Expressed in Hundred-thousandths
of Inches Per Inch

Loads Applied at A

On Outer Face of Wheel						On Inner Face of Wheel					
Gage- Lines	Effect of Mount- ing Alone	Combined Effects of Mounting and Static Loads				Gage- Lines	Effect of Mount- ing Alone	Combined Effects of Mounting and Static Loads			
		Static Loads of						Static Loads of			
		21 000 lb.	80 500 lb.	150 000 lb.	200 000 lb.			21 000 lb.	80 500 lb.	150 000 lb.	200 000 lb.
B1R	—43	—49	—30	—56	—60						
B2R	—32	—36	—41	—44	—63						
B3R	—102	—99	—104	—118	—118						
B4R	—65	—68	—82	—72	—72						
B5R	—105	—131	—116	—97	—85						
B4T	222	215	213	227	217						
B5T	182	177	172	185	171	B5T	108	106	104	114	114
D1R	—28	—36	—37								
D2R	—109	—118	—121								
D3R	—56	—65	—49								
D4R	—24										
D5R	—116					D5R	—62	—50			
D4T	66										
D5T	116					D5T	177	161	159	182	177
F1R	13	5	16								
F2R	—69										
F3R	—127										
F4R	—31	—14	—30								
F5R	—32					F5R	—64	—76	—66		
F4T	39										
F5T	56					F5T	173	190	195		
H1R	—10	—22	—16	—9	—3						
H2R	—82	—99	—103	—99	—104						
H3R	—117	—125	—126	—125	—139						
H4R	—12	—11	0.0	—20	—15						
H5R	—51	—51	—43	—60	—68	H5R	—4	2	11	—7	—12
H4T	53	54	39	45	32						
H5T	62	56	53	60	65	H5T	218	210	213	221	226

TABLE 2a—Continued

UNIT STRAINS PRODUCED BY COMBINED EFFECTS OF MOUNTING
AND STATIC LOADS

33-in. 725-lb. M. C. B. Wheel No. 671 237

Strains Given in Table are Expressed in Hundred-thousandths
of Inches Per Inch

Loads Applied at C

Gage- Lines	Effect of Mount- ing Alone	Static Loads of	
		80 500 lb.	202 500 lb.
B1R	—43	—39	—35
B2R	—32		
B3R	—102	—124	—149
B4R	—65	—76	—89
B5R	—105	—104	—108
B4T	223	233	234
B5T	182	178	177
D1R	—28	—28	—45
D2R	—109	—118	—118
D3R	—56	—66	—92
D4R	—24	—34	
D5R	—116	—124	—121
D4T	66	44	37
D5T	117	110	115

TABLE 3

UNIT STRAINS PRODUCED BY MOUNTING WHEEL ON AXLE

33-in. 725-lb. M. C. B. Wheel No. 671 449

Strains Given in Table are Expressed in Hundred-thousandths
of Inches Per Inch

On Outer Face of Wheel								
Gage- Lines	Radials along which Gage-Lines Are Located							
	A	B	C	D	E	F	G	H
1R	-13	...	-52	-42	...	..	28	-25
2R	...	...	-67	...	...	..	-57	-59
3R	-72	...	-80	-75	...	..	-74	-116
4R	31	...	- 7	24	...	..	-49	-29
1T	20	...	41	...	...	..	5	23
2T	21	...	31	47	...	..	9	28
3T	48	...	26	37	...	..	39	44
4T	25	...	73	78	...	..	64	55
5T		103		115	...	92	..	149
On Inner Face of Wheel								
1R			-16					
1Rb	-27	-18			-29	-20		-6
2R							-14	
2Rb	-53	- 6			-74			-28
3R	-23			-25	-30	0.0		-17
4R	-16		-7	4	22	-10		0.0
5R			-31		-39		-32	
2T	...	...	36	..	..	..	24	..
3T	55	...	30	62	32	49	...	72
4T	110	...	57	51	15	79	...	59
5T	161	...	136	..	141	..	158	..

TABLE 3A

UNIT STRAINS PRODUCED BY COMBINED EFFECTS OF MOUNTING
AND STATIC LOADS

33-in. 725-lb. M. C. B. Wheel No. 671 449

Strains Given in Table are Expressed in Hundred-thousandths
of Inches Per Inch

Loads Applied at A

On Outer Face of Wheel						On Inner Face of Wheel					
Gage- Lines	Effect of Mount- ing Alone	Combined Effects of Mounting and Static Loads				Gage- Lines	Effect of Mount- ing Alone	Combined Effects of Mounting and Static Loads			
		Static Loads of						Static Loads of			
		21 000 lb.	80 500 lb.	150 000 lb.	200 000 lb.			21 000 lb.	80 500 lb.	150 000 lb.	200 000 lb.
A1R	—13	—19	—38	—83	—123	A1Rb	—27	—27	—37	—55	—81
A2R	...					A2Rb	—53	—53	—50	—61	—97
A3R	—72	—84	—140	—96	—160	A3R	—23	—28	—31	—34	—40
A4R	31	27	—15	4	—41	A4R	—16	— 9	— 7	—17	—22
A1T	20	26	42	62	95						
A2T	21	27	35	55	81		55	52	54	83	108
A3T	48	61	71	80	123	A3T	110	103	116	122	154
A4T	25	37	45	75	108	A4T	161	159	156	175	185
						A5T					

Loads Applied at G

		Static Loads of				Static Loads of	
		80 500 lb.	202 000 lb.			80 500 lb.	202 000 lb.
G1R	28	— 8	— 59	G2R	—14	2	—27
G2R	—57	—121	—197				
G3R	—74	—143	—267				
G4R	—49	— 75	—109				
G1T	5	29	82	G2T	24 158	37 164	84 184
G2T	9	27	74				
G3T	39	66	115				
G4T	64	92	132				
				G5T			

TABLE 4

UNIT STRAINS PRODUCED BY MOUNTING WHEEL ON AXLE

33-in. 740-lb. Arch Plate Wheel No. 04 474

Strains Given in Table are Expressed in Hundred-thousandths
of Inches Per Inch

On Outer Face of Wheel									
Gage- Lines	Radials along which Gage-Lines Are Located								Mean Experi- mental Strain
	A	B	C	D	E	F	G	H	
2R	48	-27	8	-14	38	-7	52	43	18
3R	-19	-34	-32	-35	-38	-18	-2	-17	-24
4R	-80	-71	-73	-82	-83	-82	-91	-109	-84
5R	-76	-20	-129	-80	-116	-83	-72	-98	-84
6R	-38	-61	-57	-42	-83	-60	-70	-66	-60
7R	-11	-29	-72	-42	-56	-2	-57	-35	-38
8R	2	-8	-17	-12	-23	-9	-24	-20	-14
9R	3	26	-17	-8	-9	-24	-18	-6	-7
1T	77	74	95	105	172	137	68	78	101
2T	68	113	109	67	142	127	143	97	108
3T	68	155	85	23	70	97	19	85	75
4T	52	115	36	17	65	63	35	45	54
5T	25	71	28	26	45	37	61	...	42
6T	38	35	16	23	23	7	36	34	26
7T	16	13	-3	8	13	26	27	18	15
8T	23	11	-14	3	18	2	-6	48	11
9T	11	26	-4	25	20	19	8	4	14

On Inner Face of Wheel

2R	-39	-23	-66	-52	-69	-38	-47	-13	-43
3R	-30		0.0	-29	-42	15	17	17	-7
4R	-8		-4	-16	4	-8	3	4	-4
5R	-21		-1	-6	1	-29	-15	7	-9
6Ra, b						-52	-58	-59	-56
6R				-18	2		-29		-15
7Ra	-34	-44	-23				-28	-42	-34
7R	-38	12	-19	-34	-29	-64	-47	-57	-34
7Rb			-33	-26	-38	-3			-25
9Ra	-26	-11	-39				8	-17	-17
9Rb			-17	-27	-32	-19			-24
1T	174	263	200	171	200	150	197	197	194
2T	95	282	187	128	164	91	149	139	154
3T	82		131	77	44	98	48	80	80
4T	81		105	74	30	72		96	76
5T	51		98	70	41	52	63	74	64
6T			84			42			63

TABLE 4A

UNIT STRAINS PRODUCED BY COMBINED EFFECTS OF MOUNTING
AND STATIC LOADS

33-in. 740-lb. Arch Plate Wheel No. 04 474

Strains Given in Table are Expressed in Hundred-thousandths
of Inches Per Inch

On Outer Face of Wheel						On Inner Face of Wheel					
Gage- Lines	Effect of Mount- ing Alone	Combined Effects of Mounting and Static Loads				Gage- Lines	Effect of Mount- ing Alone	Combined Effects of Mounting and Static Loads			
		Static Loads of						Static Loads of			
		21 100 lb.	81 000 lb.	150 000 lb.	202 000 lb.			21 100 lb.	81 000 lb.	150 000 lb.	202 000 lb.
A2R	48	16	14	59	56	A2R	-39	-42	-35	-45	-72
A3R	-19	-33	-90	-65	-89	A3R	-30	-25	-6	-35	-62
A4R	-80	-105	-157	-176	-249	A4R	-8	-15	-2	0.0	-30
A5R	-76	-96	-185	-167	-220	A5R	-21	-27	-27	-32	-27
A6R	-38	-61	-124	-125	-174	A7Ra	-34	-33	-15	-44	-65
A7R	-11	-29	-76	-80	-113	A7R	-38	-37	-18	-57	-65
A8R	2	-35	-14	-70	-109						
A9R	3	-9	-31	-60	-94	A9Ra	-26	-42	-35	-80	-107
A1T	77	54	113	110	127	A1T	174	175	179	189	203
A2T	68	76	117	102	109	A2T	95	89	130	127	135
A3T	68	34	104	88	76	A3T	82	110	105	120	131
A4T	52	36	75	64	91	A4T	81	67	90	123	142
A5T	25	11	42	33	56	A5T	51	57	58	67	125
A6T	38	27	48	56	74						
A7T	16	7	23	32	37						
A8T	23	7	22	52	93						
A9T	11	9	25	39	63						

		Static Loads of						Static Loads of			
		21 400 lb.	81 500 lb.	151 000 lb.	200 000 lb.			21 400 lb.	81 500 lb.	151 000 lb.	200 000 lb.
C2R	8	27	3	4	3	C2R	-66	-55	-49	-70	-83
C3R	-32	-78	-111	-109	-116	C3R	0.0	12	20	-5	-13
C4R	-73	-87	-155	-162	-194	C4R	-4	-10	-2	2	10
C5R	-129	-120	-179	-214	-251	C5R	-1	-12	0.0	5	13
C6R	-57	-61	-113	-131	-155	C7Ra	-23	-19	-1	-42	-42
C7R	-72	-71	-135	-115	-150	C7R	-19	-7	9	-19	-24
C8R	-17	-37	-99	-56	-89	C7Rb	-33	-6	3	-19	-9
C9R	-17	-18	-60	-70	-94	C9Ra	-39	-18	-22	-98	-112
						C9Rb	-17	-18	-3	-18	-16
C1T	95	81	128	123	149	C1T	200	197	180	190	205
C2T	109	87	130	145	147	C2T	187	169	171	182	200
C3T	85	36	100	127	145	C3T	131	132	118	141	159
C4T	36	29	57	...	40	C4T	105	116	82	133	133
C5T	28	27	41	47	53	C5T	98	75	84	106	136
C6T	16	29	39	51	68	C6T	84	73	76	122	142
C7T	-3	1	15	20	37						
C8T	-14	0.0	24	27	54						
C9T	-4	...	20	38	79						

TABLE 5

UNIT STRAINS PRODUCED BY MOUNTING WHEEL ON AXLE

33-in. 740-lb. Arch Plate Wheel No. 04 476

Strains Given in Table are Expressed in Hundred-thousandths
of Inches Per Inch

On Outer Face of Wheel									
Gage- Lines	Radials along which Gage-Lines Are Located								Mean Experi- mental Strain
	A	B	C	D	E	F	G	H	
2R	36	4	20	-81	-22	-29	-3	-46	-15
3R	-15	-61	-31	-23	-61	-49	-61	-36	-42
4R	-64	-88	-68	-77	-112	-86	-128	-58	-85
5R	-71	-90	-96	-114	-89	-82	-105	-74	-90
6R	-38	-65	-47	-53	-53	-67	-71	-53	-56
7R	-23	-24	-23	-45	-45		-35	-18	-30
8R	-11	-31	-9	-12	-3	-14	-12		-13
9R	34	3	0.0	8	-1	-23	-18	41	6
1T	123	114	68	129	114	102	83	68	100
2T	128	80	79	70	98	44	99	94	86
3T	59	137	67	51	53	62	37	75	68
4T	33	90	40	16	30	37	123	42	51
5T	36	46	19	22	13	22	49	26	29
6T	26	32	47	23	13	8		18	24
7T	29	23	8	-39	-14	-3	7	25	4
8T	15	5	11	43	-1	11	11	20	14
9T	-3	-4	2	-14	15	-7	27	2	2
On Inner Face of Wheel									
2R	-40	-41	-50	-99	-88	-80	-42	-32	-59
3R	7		-14	-23	-23	-18	-35	-10	-17
4R	-1		-30	-8	-15	11	-7	-12	-9
5R	-28		-22	-23	-24	-41	-21	-25	-26
6Ra								-23	
6R			-29		-37	-42	6		-26
7Ra	-42	-40	-35	-35	-41	-61	53	-32	-29
7R				-42	-39	-31	-28		-35
7Rb						-41			
8R						-15			
9Ra	-27	-8	-31	-19	-43	-48	-21	0.0	-25
9Rb						-37			
1T	174	167	160	195	166	123	176	180	168
2T	112	189	133	129	128	127	172	117	138
3T	84		97	78	68	98		70	82
4T	60		53	61	21	87		83	61
5T	55		54	74	47	68		81	63
6T	35		36	85	65	25	71	35	50

TABLE 5A

UNIT STRAINS PRODUCED BY COMBINED EFFECTS OF MOUNTING
AND STATIC LOADS

33-in. 740-lb. Arch Plate Wheel No. 04 476

Strains Given in Table are Expressed in Hundred-thousandths
of Inches Per Inch

Loads Applied at A

On Outer Face of Wheel						On Inner Face of Wheel					
Gage- Lines	Effect of Mount- ing Alone	Combined Effects of Mounting and Static Loads				Gage- Lines	Effect of Mount- ing Alone	Combined Effects of Mounting and Static Loads			
		Static Loads of						Static Loads of			
		21 100 lb.	81 000 lb.	150 000 lb.	202 000 lb.			21 100 lb.	81 000 lb.	150 000 lb.	202 000 lb.
A2R	36	23	— 4	21	26	A2R	—40	—41	—32	—66	—80
A3R	—15	—33	—92	—68	—92	A3R	7	5	14	— 5	—13
A4R	—64	—95	—138	—147	—200	A4R	— 1	12	14	— 2	— 1
A5R	—71	—94	—143	—169	—213	A5R	—28	—32	—15	—31	—104
A6R	—38	—62	—102	—111	—151						
A7R	—23	—38	—66	—88	—118	A7Ra	—12	—29	—19	—65	—76
A8R	—11	—20	—55	—87	—122						
A9R	34	13	—35	—41	—83	A9Ra	—27	—22	—38	—90	—111
A1T	123	116	156	145	163	A1T	174	178	202	196	198
A2T	128	150	181	155	173	A2T	112	144	123	134	151
A3T	59	54	72	83	97	A3T	84	74	91	122	140
A4T	33	30	41	43	71	A4T	60	92	86	79	107
A5T	36	35	29	28	50	A5T	55	43	67	93	101
A6T	26	10	18	41	59	A6T	35	23	53	62	81
A7T	29	30	51	45	109						
A8T	15	3	32	32	57						
A9T	—3	—25	2	26	65						

Loads Applied at G

		Static Loads of						Static Loads of			
		21 400 lb.	81 500 lb.	151 000 lb.	200 000 lb.			21 400 lb.	81 500 lb.	151 000 lb.	200 000 lb.
G2R	- 3	18	4	-11	- 6	G2R	-42	-82	-66	-90	-90
G3R	-61	-52	-103	-114	-137	G3R	-35	-51	-4	-69	-72
G4R	-128	-114	-177	-211	-262	G4R	- 7	- 5	- 4	12	23
G5R	-105	-111	-166	-186	-238	G5R	-21	-31	-14	-29	-16
G6R	-71	-87	-121	-106	-154	G6R	6	-25	-11	2	6
G7R	-35	-39	-75	-69	-107	G7Ra	53	51	82	50	69
G8R	-12	0.0	-38	-62	-104	G7R	-28	-25	-17	-32	-19
G9R	-18	35	-17	-58	-109	G9Ra	-21	-29	- 3	-48	-42
G1T	83	71	106	122	140	G1T	176	174	193	187	159
G2T	99	105	148	94	157	G2T	172	166	200	186	219
G3T	37	29	60	38	65	G6T	71	69	95	84	119
G4T	123	113	118	135	172						
G5T	49	35	19	49	48						
G7T	7	14	30	24	30						
G8T	11	-1	11	21	16						
G9T	27	6	32	63	94						

TABLE 6

UNIT STRAINS PRODUCED BY MOUNTING WHEEL ON AXLE

33-in. 625-lb. M. C. B. Wheel No. 809 711

Strains Given in Table are Expressed in Hundred-thousandths of Inches Per Inch

On Outer Face of Wheel						On Inner Face of Wheel						
Gage-Lines	Radials along which Gage-Lines Are Located				Mean Experimental Strain	Gage-Lines	Radials along which Gage-Lines Are Located				Mean Experimental Strain	
	A	B	C	D			A	B	C	D		
At	37	115	95	64	78	At	115	190	142	126	143	
Bt	67	109	..	83	86	Bt	100	163	137	102	126	
Ct	50	68	58	83	65	Ct	92	136	120	103	113	
Dt	40	108	65	86	75	Dt	69	118	100	81	92	
Et	37	81	51	60	57	Et	64	107	85	71	82	
Ft	18	..	67	65	50		..	..	..	..	..	
Gt	28	58	38	70	48		..	..	..	..	..	
2T	20	57	50	..	42	2T	50	75	53	58	59	
3T	20	43	28	35	32	3T	46	50	26	39	40	
4T	19	9	18	20	16	4T	30	20	9	..	20	
5T	5	25	18	11	15	5T	18	31	24	..	24	
6T	4	22	6	3	4	6T	27	2	28	6	13	
7T	9	9	11	1	8	6Tab	..	16	..	8	12	
8T	15	15	3	8	6	7T	18	28	23	8	19	
9T	14	1	6	11	2		..	..	..	..	..	
10T	6	1	4	15	3		..	..	..	..	..	
1R	20	53	17	30	30	1R	37	31	56	32	24	
2R	23	35	31	20	27	2R	31	33	29	38	33	
3R	26	18	27	5	19	3R	9	..	27	25	20	
4R	15	23	39	20	24	4R	29	24	13	7	18	
5R	17	10	33	25	21	5R	5	2	0.0	20	7	
6R	40	12	6	19	19	6R	10	8	28	1	11	
7R	19	19	24	21	21	6Rab	..	25	..	46	36	
8R	6	0.0	20	2	7	7R	15	13	6	16	12	
9R	5	1	17	5	7	8Rab	25	7	8	18	6	
						9Rab	9	..	3	13	0.0	
						Gage-Lines Adjacent to Core Holes						
						Between Radials A and B	Between Radials B and C	Between Radials C and D				
						1T	85	72	68			75
						2T	49	111	88			83
						3T	118	66				92
						4T	118	46				82

TABLE 7

UNIT STRAINS PRODUCED BY MOUNTING WHEEL ON AXLE

33-in. 625-lb. M. C. B. Wheel No. 822 269

Strains Given in Table are Expressed in Hundred-thousandths
of Inches Per Inch

On Outer Face of Wheel						On Inner Face of Wheel					
Gage- Lines	Radials along which Gage-Lines Are Located				Mean Experi- mental Strain	Gage- Lines	Radials along which Gage-Lines Are Located				Mean Experi- mental Strain
	A	B	C	D			A	B	C	D	
At	161	213	343	150	217	At	192	156	161	245	188
Bt	127	171	224	130	163	Bt	148	131	165	228	168
Ct	107	161	249	124	160	Ct	136	105	140	196	144
Dt	115	137	171	91	128	Dt	127	97	129	179	133
Et	97	115	174	106	123	Et	106	90	131	174	125
Ft	39	121	140	84	96		..	..	..	..	
Gt	95	99	140	94	107		..	..	..	..	
2T	86	93	128	76	96	2T	104	77	118	147	112
3T	60	59	105	53	69	3T	84	62	79	148	93
4T	..	54	71	45	57	4T	45	51	39	..	45
5T	40	37	44	33	38	5T	38	45	38	..	40
6T	37	41	16	16	28	6T	29	16	28	51	31
7T	26	16	31	12	21	6Tb	..	—21	..	..	
8T	9	6	11	9	9	7T	29	12	43	42	32
9T	20	2	22	14	14		..	..	..	..	
10T	11	3	15	4	6		..	..	..	..	
1R	—50	—88	..	—47	—62	1R	—29	—53	—58	—66	—52
2R	—42	—50	—77	—56	—56	2R	..	—56	—34	—54	—48
3R	—9	—33	—47	—31	—30	3R	—22	—20	—29	—38	—27
4R	—38	—44	—50	—36	—42	4R	—24	—20	—10	—9	—16
5R	—40	—54	—57	—55	—51	5R	—16	—24	—4	—21	—16
6R	—59	—64	—52	—55	—58	6R	—6	—24	—16	..	—15
7R	—52	—37	—37	—49	—44	6Rab	..	—24	..	—33	—28
8R	—18	—29	—16	—18	—20	7R	—11	—16	—20	—13	—15
9R	—41	—16	—4	9	—13	8Rab	10	0.0	34	15	15
						9Rab	4	24	10	—10	7
						Gage-Lines Adjacent to Core Holes					
						Between Radials A and B		Between Radials B and C		Between Radials D and A	
						1T	87	100	126	104	
						2T	130	118		124	
						3T		101		101	
						4T	65	57		61	

TABLE 8

UNIT STRAINS PRODUCED BY MOUNTING WHEEL ON AXLE
33-in. 725-lb. M. C. B. Wheel No. 49 317

Strains Given in Table are Expressed in Hundred-thousandths
of Inches Per Inch

On Outer Face of Wheel						On Inner Face of Wheel					
Gage- Lines	Radials along which Gage-Lines Are Located				Mean Experi- mental Strain	Gage- Lines	Radials along which Gage-Lines Are Located				Mean Experi- mental Strain
	A	E	C	D			A	B	C	D	
At	81	79	152	122	108	At	134	129	95	109	117
Bt	68	79	134	121	100	Bt	114	113	86	100	103
Ct	59	69	125	69	80	Ct	107	80	79	90	89
Dt	47	67	94	99	77	Dt	99	87	77	73	84
Et	48	61	88	94	73	Et	92	84	81	73	82
Ft	44	64	101	96	76	Ft	83	76	67	72	74
Gt	29	60	76	93	64	Gt	83	68	..	66	72
3T	38	40	56	70	51	3T	60	51	49	65	56
4T	28	34	36	51	37	4T	51	37	33	..	40
5T	34	28	24	36	30	5T	17	22	21	..	20
6T	22	24	26	28	25	6T	23	44	21	..	29
7T	5	17	21	21	16	7T	14	19	18	..	17
8T	10	7	8	13	10	..	..	..	..	..	..
9T	10	8	14	11	11	..	..	..	..	..	..
10T	6	8	3	7	6	..	..	..	..	..	..
1R	-51	-39	-49	-57	-49	1R	-57	-82	-54	-55	-62
2R	-22	-23	-47	-47	-35	2R	-30	-31	-35	-35	-33
3R	-23	-23	-26	-36	-27	3R	-17	-22	-22	-25	-22
4R	-17	-16	-19	-33	-21	4R	-6	-8	-7	-10	-8
5R	-14	-23	-22	-35	-24	5R	-4	-5	-7	-10	-3
6R	-35	-34	-40	-51	-40	6R	-6	-1	-7	-2	-4
7R	-28	-29	-39	-44	-35	7R	-19	-17	-20	-19	-19
8R	-13	-17	-17	-28	-19	8R	2	..	-27	..	-12
9R	1	-8	-11	-11	-7	8Ra	-6	-10	-13	4	-6
						9Ra	-8	-9	-10	2	-6
Gage-Lines Adjacent to Chaplets						Gage-Lines Adjacent to Core Holes					
	Between Radials						Between Radials				
	A and B	B and C	D and A				A and B	B and C	D and A		
1T	34	63	64	54		1T	51	45	54	50	
2T	45	70	75	63		2T	63	53	80	65	
3T	48	82	69	66		3T	63	40	47	50	
4T	47	60	51	53		4T	24	15	13	17	
5T	32	36	50	39							

TABLE 9

UNIT STRAINS PRODUCED BY MOUNTING WHEEL ON AXLE

33-in. 725-lb. M. C. B. Wheel No. 49 305

Strains Given in Table are Expressed in Hundred-thousandths
of Inches Per Inch

On Outer Face of Wheel						On Inner Face of Wheel					
Gage- Lines	Radials along which Gage-Lines Are Located				Mean Experi- mental Strain	Gage- Lines	Radials along which Gage-Lines Are Located				Mean Experi- mental Strain
	A	B	C	D			A	B	C	D	
At	297	104	87	139	157	At	97	167	149	156	142
Bt	268	82	67	116	133	Bt	91	132	125	126	118
Ct	240	79	61	113	123	Ct	83	114	116	110	106
Dt	208	47	49	110	104	Dt	..	129	107	95	110
Et	148	62	56	109	94	Et	74	119	96	80	92
Ft	134	68	44	110	89	Ft	76	100	89	80	86
Gt	128	68	44	109	87	Gt	65	94	83	65	77
3T	74	40	51	97	66	3T	48	61	65	65	60
4T	55	36	36	55	46	4T	39	42	22	..	34
5T	36	26	33	..	32	5T	46	48	11	0 0	35
6T	26	58	30	65	45	6T	6	14	21	..	14
7T	13	15	20	24	18	7T	8	30	20	..	19
8T	12	6	15	18	13	..	..	..	..	..	..
9T	14	7	15	12	12	..	..	..	..	..	..
10T	— 3	7	14	20	10	..	..	..	..	..	..
1R	—209	—114	—30	—56	—102	1R	—72	—110	—82	—197	—115
2R	—155	— 89	— 1	—52	— 74	2R	—59	— 60	—69	—115	— 76
3R	— 94	—28	2	..	— 40	3R	—35	—32	—44	—104	— 54
4R	— 65	—22	— 3	—30	—30	4R	—23	—25	—27	— 82	—39
5R	—108	—34	—20	—32	—48	5R	—10	—18	—30	— 51	—27
6R	— 50	—32	—40	—79	—50	6R	—31	—22	—16	—40	—27
7R	—47	—34	—41	—54	—44	7R	—20	—24	—36	—18	—24
8R	—29	—26	—22	—19	—24	8R	—21	..	—20	..	—20
9R	—17	— 1	—11	— 9	—10	8Ra	—12	— 8	— 8	— 8	— 9
						9Ra	7	—14	—22	— 7	— 9
						Gage-Lines Adjacent to Core Holes					
						Between Radials A and B	Between Radials B and C	Between Radials D and A			
						1T	65	82	69	72	
						2T	82	82	..	82	
						3T	70	73	60	68	
						4T	56	48	39	48	

APPENDIX C

AGREEMENT BETWEEN THE ASSOCIATION OF MANUFACTURERS OF
CHILLED CAR WHEELS AND THE UNIVERSITY OF ILLINOIS,
PROVIDING FOR A COÖPERATIVE INVESTIGATION
RELATING TO CHILLED CAR WHEELS

The following articles of agreement, prepared by the Engineering Experiment Station of the University of Illinois, were submitted to, and, on January 26, 1916, approved by Geo. W. Lyndon, President of, and acting for, the Association of Manufacturers of Chilled Car Wheels. The articles of agreement were likewise submitted to, and, on February 8, 1916, approved by the Board of Trustees of the University of Illinois.

(1) That the Engineering Experiment Station will undertake an investigation concerning the stresses and behavior of chilled iron car wheels, under prescribed conditions, the details of the work to be determined by the Station in conference with the Association's Consulting Engineer, Mr. F. K. Vial.

(2) That all results secured from such an investigation shall be the property of the Engineering Experiment Station. They may be published as Station bulletins or otherwise, as from time to time may be determined by the Director. Prior to any publication, however, the intention of the Director shall be formally communicated to the Association of Manufacturers of Chilled Car Wheels in order that said Association may at all times be informed as to the purpose of the Station. The results also shall at all times be open to the inspection and use of the representative of the Association of Manufacturers of Chilled Car Wheels, who may be assumed to act in an advisory capacity to the Station.

(3) That the funds of the Association of Manufacturers of Chilled Car Wheels are to be drawn upon for all costs of labor and supplies arising from the proposed investigation. If equipment is required in excess of that already available to the Station, it will be provided at the expense of the Association. The Station will supply the time of members of its staff for the direction of the work, and the use of such facilities as it may already possess.

(4) That upon the approval of these proposals the Association of Manufacturers of Chilled Car Wheels will at once transmit to the Business Office of the University the sum of \$500 as an initial deposit, and each month thereafter will transmit such sum or sums as may be called for by estimates of the costs of the investigation for the month next succeeding.

Funds thus supplied are to constitute a credit upon which the Director of the Engineering Experiment Station may from time to time draw to cover the costs of the proposed work. It will be the purpose to maintain, during the lifetime of this agreement, a balance in this account entirely sufficient to meet all charges that will need to be made against it.

(5) That this arrangement will continue in effect until canceled through the definite action of one or both of the parties in interest.

LIST OF
PUBLICATIONS OF THE ENGINEERING EXPERIMENT STATION

Bulletin No. 1. Tests of Reinforced Concrete Beams, by Arthur N. Talbot. 1904. *None available.*

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THE REHEATING OF COMPRESSED AIR

BY

C. R. RICHARDS

AND

J. N. VEDDER



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UNIVERSITY OF ILLINOIS-
ENGINEERING EXPERIMENT STATION

BULLETIN No. 130

JUNE, 1922

THE
REHEATING OF COMPRESSED AIR

BY

CHARLES RUSS RICHARDS

DIRECTOR OF THE ENGINEERING EXPERIMENT STATION AND
DEAN OF THE COLLEGE OF ENGINEERING

AND

JOHN N. VEDDER

RESEARCH ASSISTANT IN MECHANICAL ENGINEERING

ENGINEERING EXPERIMENT STATION

PUBLISHED BY THE UNIVERSITY OF ILLINOIS, URBANA

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THE REHEATING OF COMPRESSED AIR

I. INTRODUCTION

1. *Preliminary Statement.*—For many years compressed air has been employed as a medium for the transmission of power. The simplicity and relatively high efficiency of air compressors, the economy with which compressed air may be transmitted for comparatively long distances through pipe lines, and the variety of air motors and tools now available have led to the extensive use of compressed air in mining, quarrying, and tunneling, in various shop processes, and in the development of power.

Many devices operated by compressed air are run intermittently; and with such apparatus economy of operation is generally sacrificed for simplicity and minimum initial cost of equipment. Few air-driven tools use the air expansively, wherefore their thermal efficiency is small. When motors are operated continuously for considerable periods of time, however, economy of operation becomes a consideration, and it is essential that the air be expanded in the motor. Since the variations of volume and pressure of air during compression and expansion may be represented by the equation $pv^n = \text{a constant}$, air is heated during compression and cooled during expansion. In the compressed air plant the heat of compression is quickly dissipated by radiation from the air receivers and pipe lines, so that when the air reaches the motor its temperature is little if any higher than that of the atmosphere. Consequently the temperature of the air after expansion is so low that it causes serious operating difficulties, resulting both from the freezing of the moisture in the air and the consequent accumulation of ice in the exhaust pipe, and from the ineffectiveness of cylinder lubrication and resultant loss of mechanical efficiency. These difficulties may be overcome by heating the air before its delivery to the motor to an initial temperature such that the temperature of the exhaust air will be above the freezing point of water. This heating process is commonly designated as "reheating," and the term will therefore be employed in the discussion which follows.

It should be noted that when compressed air is used as a medium for the transmission of power the reheating process affords a means of increasing the total output of power per pound of air through

the expenditure of a small additional amount of heat energy, and that no other system of power transmission offers similar advantages.

2. *Purposes of Investigation and Significant Results.*—This investigation of the reheating of compressed air was undertaken to determine the ideal thermodynamic efficiencies resulting from the heat expended in the reheating process, the efficiency of external and internal combustion reheaters, the performance of an engine using air expansively under a wide variety of operating conditions, and the performance of the same engine operating with steam alone, and with mixtures of air and steam when steam is injected into the air pipe as a means of reheating the air.

The results of the investigation of reheaters show that in small external combustion reheaters maximum efficiencies of from 16.7 to 61.5 per cent may be secured, depending upon the type of reheater employed; and that in the internal combustion reheater of the type tested the efficiency varied from 69.4 per cent when 326 pounds of air per hour was heated, to 83.0 per cent when 1240 pounds of air per hour was heated.

The tests of the engine using air as the working medium show that under the most favorable conditions maintained the expenditure of coke per indicated horse power per hour gained through reheating was 0.98 pounds, and, per brake horse power per hour gained, 0.90 pounds. Under the least favorable conditions of operation with cold air the mechanical efficiency of the engine fell to 59.8 per cent, while under the best conditions of operation with reheated air the mechanical efficiency was 93.7 per cent.

In the tests of the engine using a mixture of air and steam it was found that the work done per pound of mixture was considerably in excess of that attainable by the separate use of the same weight of each ingredient in the mixture. The interest in air-steam mixtures developed during this investigation has led to further studies of the subject, the results of which will be presented in a later bulletin.

3. *Acknowledgments.*—The experimental work of this investigation was carried on in the Power Laboratory of the Department of Mechanical Engineering by JOHN N. VEDDER,* Research Assistant in Mechanical Engineering. During a part of the time, he was

* Now Associate Professor of Thermodynamics at Union College, Schenectady, New York.

assisted by WILLIAM E. MOSHER,[†] Research Graduate Assistant in Mechanical Engineering. Because of uncertainty regarding the accuracy of the method originally employed in calibrating the Venturi tube used as an air meter during the tests, the tube was recently recalibrated and all air weights recalculated by CRANDALL Z. ROSECRANS, Research Assistant in Mechanical Engineering. This recalibration was performed with the air weighing plant described on page 81 of Engineering Experiment Station Bulletin No. 120, entitled "Investigation of Warm Air Furnaces and Heating Systems."

The authors are indebted to the Ingersoll-Rand Company, the Sullivan Machinery Company, and the Kreutzburg Manufacturing Company for the loan of reheaters used during the investigation.

[†] Mr. Mosher died in Washington, D. C., on October 12, 1918. At that time he was Assistant Superintending Engineer of the U. S. Army Transport Service, in charge of marine refrigeration.

II. THERMODYNAMICS OF REHEATED AIR: IDEAL CYCLES

4. *Case I. No Expansion.*—Compressed air may be used in motors in which there is no expansion of the air, in those in which there is partial expansion, and in those in which there is complete expansion. The ideal efficiency resulting from the use of reheated air—that is, the ratio of the extra theoretical work secured through the use of reheated air, expressed in British thermal units (B.t.u.), to the heat added to reheat the air, in B.t.u.—is here developed for these three cases.

In Fig. 1 let $p_c = p_a = p_b =$ the initial pressure in pounds per square foot of air admitted to the cylinder of a motor whose clearance

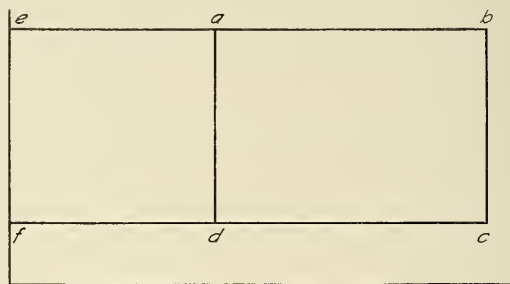


FIG. 1. WORK DIAGRAM FOR AIR. NO EXPANSION

volume is zero; and $p_f = p_d = p_c =$ the pressure of the atmosphere. Assume that $v_a =$ the volume in cubic feet of one pound of air whose absolute temperature is T_a . The theoretical work in foot-pounds per pound of air at T_a is represented by the area $e a d f$. If the air is heated at constant pressure to a temperature T_b its volume will become v_b and the theoretical work per pound of air is represented by the area $e b c f$. The difference between these areas, $a b c d$, represents the additional work per pound of air resulting from the heat added as "reheat."

$$\begin{aligned} \text{The area } a b c d &= (p_a - p_d)(v_b - v_a) \text{ ft. lbs.} \\ &= A (p_a - p_d)(v_b - v_a) \text{ B.t.u.} \end{aligned}$$

when $A = \frac{1}{778}$ = the reciprocal of the mechanical equivalent of heat.

The heat supplied = $c_p (T_b - T_a)$ B.t.u.

$$= c_p \frac{T_a}{v_a} (v_b - v_a) \text{ B.t.u.,}$$

since $\frac{T_a}{v_a} = \frac{T_b}{v_b}$

and c_p = mean specific heat of air at constant pressure.

The efficiency ratio then becomes

$$E = \frac{A (p_a - p_d) (v_b - v_a)}{c_p \frac{T_a}{v_a} (v_b - v_a)} = \frac{A v_a (p_a - p_d)}{c_p T_a}$$

$$\frac{p_a v_a}{T_a} = B, \text{ the air constant.}$$

If c_v = the mean specific heat of air at constant volume

$$B = \frac{c_p - c_v}{A}$$

and $\frac{v_a}{T_a} = \frac{B}{p_a} = \frac{c_p - c_v}{A p_a}$

Hence $E = \frac{k-1}{k} \left(1 - \frac{p_d}{p_a}\right) = 0.2884 \left(1 - \frac{p_d}{p_a}\right)$

since $k = \frac{c_p}{c_v} = 1.405$

5. *Case II. Complete Expansion.*—In Fig. 2 let $p_e = p_a = p_b$ = the initial pressure; $p_f = p_d = p_c$ = the atmospheric pressure in pounds per square foot; and v_a = the volume in cubic feet of one pound of air at a temperature T_a and pressure p_a . If in a cylinder without clearance the air be expanded to the atmospheric pressure p_d , the work in foot pounds for the cycle is represented by the area

$e a d f$. If the air is reheated at constant pressure from T_a to T_b , its volume is increased from v_a to v_b ; and the work of the cycle for complete expansion is represented by the area $e b c f$. The difference between the areas, $a b c d$, represents the additional work

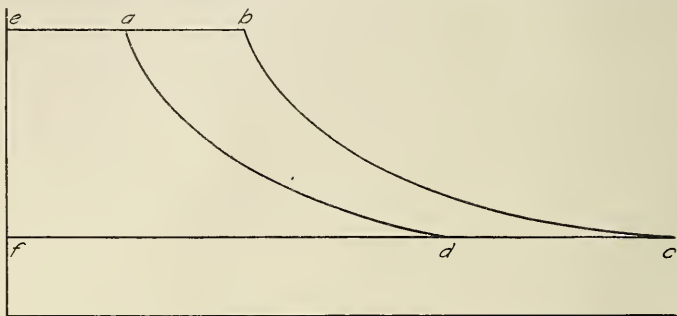


FIG. 2. WORK DIAGRAM FOR AIR. COMPLETE EXPANSION

per pound of air secured through the heat added. The efficiency of the reheating process is the ratio of the heat converted into work to the heat added. With complete expansion of the air, it is obvious that the efficiency is the maximum attainable.

$$\text{Since } p_a v_a^n = p_d v_d^n$$

$$\text{and } p_b v_b^n = p_c v_c^n$$

$$\text{the work area } e a d f = \frac{n}{n-1} p_a v_a \left[1 - \left(\frac{p_d}{p_a} \right)^{\frac{n-1}{n}} \right] \text{ ft. lbs.,}$$

$$\text{and the work area } e b c f = \frac{n}{n-1} p_b v_b \left[1 - \left(\frac{p_c}{p_b} \right)^{\frac{n-1}{n}} \right] \text{ ft. lbs.,}$$

$$= \frac{n}{n-1} p_a v_b \left[1 - \left(\frac{p_d}{p_a} \right)^{\frac{n-1}{n}} \right] \text{ ft. lbs.}$$

$$\text{The work gained } a b c d = \frac{n}{n-1} p_a (v_b - v_a) \left[1 - \left(\frac{p_d}{p_a} \right)^{\frac{n-1}{n}} \right] \text{ ft. lbs.,}$$

$$= \frac{n}{n-1} B (T_b - T_a) \left[1 - \left(\frac{p_d}{p_a} \right)^{\frac{n-1}{n}} \right] \text{ ft. lbs.}$$

$$\text{The "reheat" supplied} = \frac{1}{A} c_p (T_b - T_a) \text{ ft. lbs.}$$

The efficiency then becomes

$$\begin{aligned}
 E &= \frac{n}{n-1} \left[1 - \left(\frac{p_d}{p_a} \right)^{\frac{n-1}{n}} \right] \frac{A}{c_p} \frac{B}{c_p} \\
 &= \left(\frac{n}{n-1} \right) \left(\frac{k-1}{k} \right) \left[1 - \left(\frac{p_d}{p_a} \right)^{\frac{n-1}{n}} \right] \\
 &= \left(\frac{n}{n-1} \right) \left(\frac{k-1}{k} \right) \left[1 - \left(\frac{v_a}{v_d} \right)^{n-1} \right] \\
 &= \left(\frac{n}{n-1} \right) \left(\frac{k-1}{k} \right) \left[1 - \frac{T_d}{T_a} \right]
 \end{aligned}$$

If the air is expanded adiabatically n becomes equal to k and the efficiency is reduced to

$$E = 1 - \left(\frac{p_d}{p_a} \right)^{\frac{k-1}{k}} = 1 - \left(\frac{v_a}{v_d} \right)^{k-1} = 1 - \frac{T_d}{T_a}$$

6. *Case III. Partial Expansion.*—The least favorable case for reheated air used expansively is shown by the diagram, Fig. 3. As

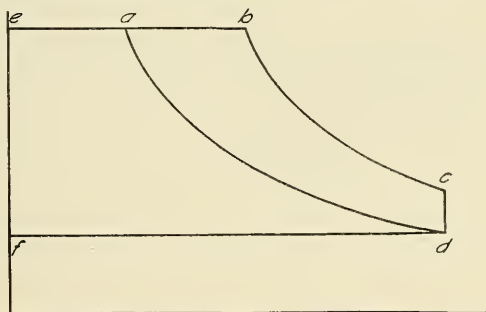


FIG. 3. WORK DIAGRAM FOR AIR. PARTIAL EXPANSION

in Case II, the area $e a d f$ represents the work in foot pounds per pound of air whose initial volume is v_a at a pressure p_a and temperature T_a . Assume that the temperature is increased to T_b at constant pressure, and that the volume v_c at the end of expansion is the same as the volume v_d attained after expansion of the unreheated air to atmospheric pressure. It is evident that the work area is the smallest possible when expansion is employed if the final volume is

not reduced below that attained when cold air is used. If the expansion of the reheated air is carried farther than shown, that is, if v_c is greater than v_d , the thermal efficiency will be somewhere between that of Case II and Case III.

In Fig. 3 the work resulting from the reheat supplied is represented by the area $a b c d$, which is the difference between areas $e b c d f$ and $e a d f$. Since

$$\text{area } e b c d f = \frac{n}{n-1} (p_a v_b - p_c v_c) + v_c (p_c - p_d)$$

$$\text{and area } e a d f = \frac{n}{n-1} (p_a v_a - p_d v_c)$$

$$\begin{aligned} \text{area } a b c d &= \frac{n}{n-1} \left[(p_a v_b - p_c v_c) - (p_a v_a - p_d v_c) \right] + \\ &\quad v_c (p_c - p_d) \\ &= \frac{n}{n-1} p_a (v_b - v_a) - \frac{1}{n-1} v_c (p_c - p_d) \text{ ft. lbs.} \end{aligned}$$

Substituting the appropriate values of $pv = BT$ in the preceding equation and multiplying by A ,

$$\text{the area } a b c d = \frac{AB}{n-1} \left[n (T_b - T_a) - (T_c - T_d) \right] \text{ B.t.u.}$$

The reheat supplied $= c_p (T_b - T_a)$; therefore the efficiency is

$$E = \frac{k-1}{k(n-1)} \left(n - \frac{T_c - T_d}{T_b - T_a} \right)$$

$$\text{Since } T_d = T_a \left(\frac{v_a}{v_d} \right)^{n-1}$$

$$T_b = T_a \left(\frac{v_b}{v_a} \right)$$

$$\text{and } T_c = T_b \left(\frac{v_b}{v_d} \right)^{n-1} = T_a \frac{v_b}{v_a} \left(\frac{v_b}{v_d} \right)^{n-1}$$

$$E = \frac{n(k-1)}{k(n-1)} \left[1 - \frac{1}{n} \left(\frac{v_a}{v_d} \right)^{n-1} \frac{\left(\frac{v_b}{v_a} \right)^n - 1}{\frac{v_b}{v_a} - 1} \right]$$

If adiabatic expansion is assumed, $n = k$ and the expression for efficiency reduces to

$$E = 1 - \frac{1}{k} \left(\frac{v_a}{v_d} \right)^{k-1} \frac{\left(\frac{v_b}{v_a} \right)^k - 1}{\frac{v_b}{v_a} - 1}$$

In this case the efficiency decreases as the volume v_b and temperature T_b increase.

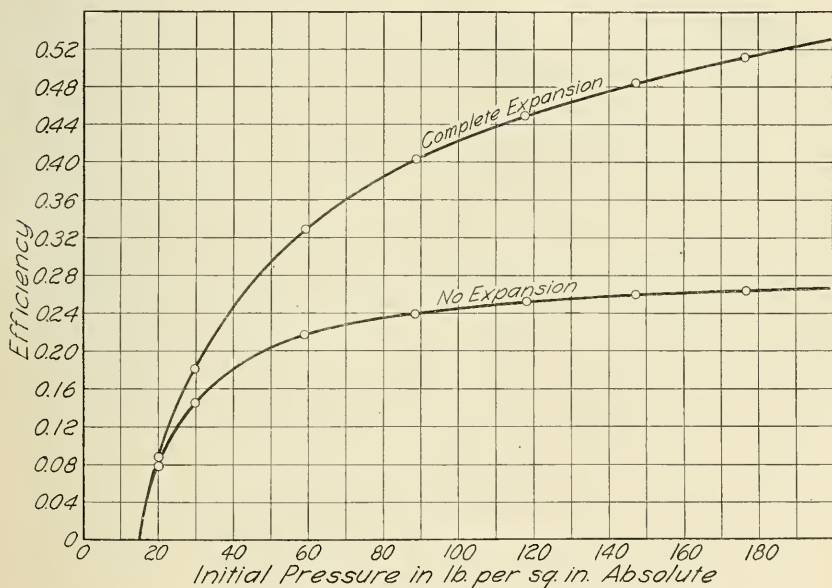


FIG. 4. THEORETICAL EFFICIENCIES OF AIR CYCLES

Reference to the efficiencies theoretically attainable in Cases I and II shows that they are independent of the reheat added and that they are functions of $\frac{p_d}{p_a}$ only. Fig. 4 shows the theoretical efficiencies for cycles having no expansion of the air and for those having complete adiabatic expansion.

III. TESTS OF REHEATERS

7. *Method of Testing External Combustion Reheaters.*—Most of the tests of external combustion reheaters had a duration of eight hours, all conditions during that period being maintained as nearly constant as possible. Since the reheaters tested and the weight of fuel consumed were small, the percentage of error in fuel weights due to starting and stopping the tests would have been increased with a reduction in time. With eight-hour tests, however, it was felt that this error would be very small.

Before starting a test a preliminary run of about one hour was made to insure the maintenance of standard conditions from the beginning of the test. When the compressor, piping system, and reheater had reached thermal equilibrium, the fire was quickly drawn and rebuilt with weighed kindling. This process required about three minutes, during which the air was closed off from the reheater and the output of the compressor escaped through the relief valve on the air receivers. As soon as the fire was properly started the air was turned on and the test begun.

The fuel used, a coke of good quality, was fired in approximately equal weights, and at equal intervals of time during each test. The weights in different tests varied with the weight of air heated; and, in general, the interval of time was 15 to 20 minutes, except in the case of the first two or three firings where the interval was shorter. During the preliminary run the depth of the fire necessary for the assigned "load" was determined; then, at the beginning of the test, the fire was brought to this depth in the shortest possible time by using maximum draft, until the flue gas temperature was 100 to 150 degrees above the normal for the test. This building up period lasted 15 to 20 minutes and the second and third charges were fired as soon as the previous charge showed red on top.

The constancy of the temperature of the reheated air leaving the reheater was the regulative factor controlling the tests. To maintain a constant temperature, a controllable furnace draft was found indispensable. The draft in the stack, to which the reheater was connected, was induced and controlled by a jet of compressed air from a one-eighth inch pipe. A valve controlling the jet of air made

it possible to obtain any draft from 0.05 to 0.40 inches of water. A thermometer inserted through a stuffing box directly in the stream of heated air made it possible to observe the temperature and to maintain it, with a variable draft, to within about 10 degrees Fahrenheit of the mean temperature.

A valve in the air pipe about ten feet from its entrance to the reheater was employed to maintain in the reheater a constant pressure of 100 to 110 pounds per square inch, gage, during most of the tests. The pressure in the receiver was 10 to 15 pounds higher. The flow of air was controlled by a second valve in the pipe leaving the reheater. The proper openings of these valves were determined by trial during the preliminary run, but slight readjustments were frequently necessary.

To assist in regulating the draft the temperature of the flue gases was determined by a Hoskins pyrometer. This temperature was kept as low as possible to permit the maintenance of the desired temperature of the reheated air. Samples of flue gas were taken in aspirator bottles over four-hour periods and analyzed during or immediately after the test by means of an Orsat apparatus filled with freshly prepared solutions.

After leaving the reheater the air passed through about 12 feet of uncovered pipe to a gage-box fitted with a standard orifice. A pressure of from 2 to 5 inches of water was maintained in the box and the discharge was computed by using the coefficients recommended by R. J. Durley.* As the relatively high temperatures might affect the coefficients of discharge, the Venturi tube, later used in metering the air for the engine tests, was connected in series with the reheater and gage-box, and several tests were run with air at different temperatures. As the ratio between the flow indicated by the meter and the gage-box followed the same law with and without the heater in operation, the use of Durley's coefficients for the measurement of air in the test was justified.

This same arrangement of apparatus was employed in calibrating the Venturi tube used in the tests. This tube is of a standard form manufactured by the Builders' Iron Foundry of Providence, Rhode Island. It is flanged for standard 2½-inch pipe connections and has

* Durley, R. J. "On the Measurement of Air Flowing into the Atmosphere through Circular Orifices in Thin Plates and Under Small Differences of Pressure." *Trans. A. S. M. E.*, v. 27, p. 193, 1906.

a throat diameter of 0.75 inches. The standard differential mercury manometer supplied by the manufacturers of the tube was used in all tests. In certain air-steam mixture tests, made at a later date than those described in this bulletin, the question of the accuracy of the calibration of the Venturi tube was raised and various attempts were made to recalibrate the apparatus by methods other than the one originally employed. No one of these attempts was successful until use was made of the air weighing plant* used to calibrate instruments employed in the investigation of warm air furnaces. A careful recalibration of the Venturi tube indicated that, with Durley's coefficients, the original calibration against standard orifices gave weights of air which were 3 per cent low. Accordingly all computations involving weights of air originally made for this bulletin have been corrected to agree with this new calibration. This merely involved the multiplication of the weights of air as originally calculated by 1.03. The determination by means of the Venturi tube of the weight of air flowing involves much laborious computation. In order to reduce the labor and at the same time secure greater accuracy, large scale graphical charts, prepared for use with the Venturi tube, were employed in making the calculations. It seems unnecessary to present this material here.†

During the tests the following observations were recorded every 10 minutes:

- (1) Temperature of air entering reheater
- (2) Pressure of air entering reheater
- (3) Temperature air leaving reheater
- (4) Flue gas temperature
- (5) Pressure in gage-box
- (6) Temperature in gage-box
- (7) Room temperature
- (8) Draft

Other observations and records were:

- (9) Barometric pressure (each hour)
- (10) Diameter of orifice

* For a description of the plant see "Investigation of Warm Air Furnaces and Heating Systems," Univ. of Ill. Eng. Exp. Sta., Bul. No. 120, 1921.

† The general theory of gaseous flow through Venturi tubes is presented in a paper, "The Flow of Fluids in a Venturi Tube," by Edgar P. Coleman, Trans. A. S. M. E., v. 28 p. 483.

- (11) Position of damper
- (12) Weight of kindling
- (13) Time of each firing
- (14) Net weight of fuel on scales after each firing
- (15) Weight of fuel fired each firing
- (16) Weight of residual fuel
- (17) Weight of ash and refuse
- (18) Flue gas analysis

Samples of each load of fuel placed on the scales were taken and sealed for analysis. Fig. 5 shows the general arrangement of testing apparatus, the figures referring to the numbered observations just listed. The fire was kept up to the standard condition until the end of the test. About two minutes before this time, the grates were thoroughly shaken and the ash and refuse collected, weighed, and preserved. Then at the end of the test the grates were dumped, and the fire quenched with the minimum quantity of water necessary. The residual fuel was then collected, weighed, and preserved.

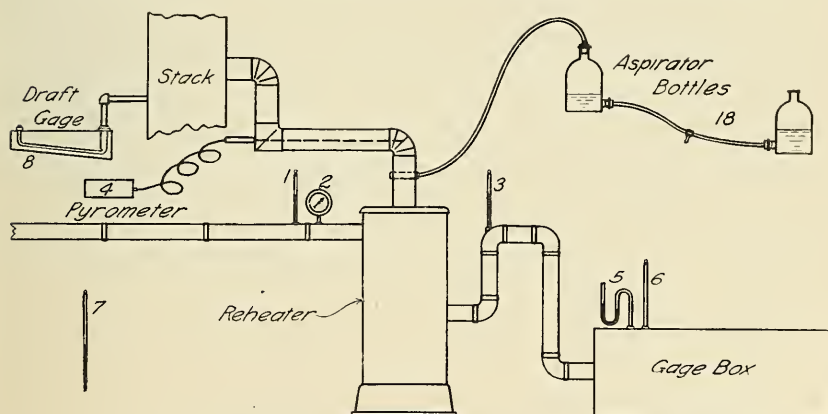


FIG. 5. GENERAL ARRANGEMENT OF APPARATUS FOR TESTING
EXTERNAL COMBUSTION REHEATER

8. *Definitions and Notation Employed.*—In rating compressed air equipment it is customary to express capacity in cubic feet of free air per minute. The term “free air” is loosely used, and without specific definition is meaningless. As employed in this bulletin the free air unit is one cubic foot of air at a temperature

of 62 degrees Fahrenheit and a pressure of 14.7 pounds per square inch absolute. The volume of a pound of air at 62 degrees Fahrenheit and 14.7 pounds per square inch absolute pressure is 13.147 cubic feet. The weight of a cubic foot of free air is 0.07608 pounds. One pound of air per hour is equivalent to 0.21906 cubic feet of free air per minute.

The specific heat of dry air at constant temperature, c_p , does not greatly vary within the temperature range employed in this investigation, but when moisture is present, the change of specific heat for small temperature changes is marked. According to Swann, the specific heat at constant pressure of dry air at 20 degrees centigrade (68 degrees Fahrenheit) is 0.24301. In computations made in this investigation the value of c_p for dry air was taken as 0.243, and it was assumed that the air before reheating was saturated with water vapor. By the theory of mixture of gases and vapors it is possible to calculate the value of c_p for such mixtures. Values of c_p thus calculated are given in the various tables presented.

The efficiency of a reheater is the ratio of the heat absorbed by the compressed air to the calorific value of the fuel charged against the reheater. The numerator of this fraction is the product of the weight of air heated in pounds, the specific heat of the air at constant pressure, and the rise in temperature in degrees Fahrenheit; the denominator is the calorific value of the total fuel fired minus the calorific value of the residual fuel either with or without the fuel in the ash—depending upon whether the efficiency of the grate is excluded or included. In internal combustion reheaters using liquid fuel the denominator is the calorific value of the liquid fuel consumed.

For convenience of reference the designations of all tests are explained as follows:

Reheater tests are designated by two numbers and a letter. The first number denotes the approximate quantity of free air in cubic feet per minute; the second, the approximate temperature of the air after reheating; and the letter is added only for the sake of distinction; e.g., 150-400-a.

Engine tests using air are designated by four numbers: first, a serial number; second, the gage pressure at the throttle; third, the temperature at the throttle; and fourth, the approximate cut-off in per cent of the engine stroke; e.g., 50-100-200-25.

Air-steam engine tests are denoted by the letters A-S followed by the gage pressure at the throttle and the approximate percentage of steam in the mixture; e.g., A-S-100-10.

Tests with steam alone are designated by the letter S, followed by the gage pressure at the throttle; e.g., S-80.

9. *Tests of Type I Reheater.*—This reheater consists of a cast iron fire-pot in the form of the frustrum of a cone, with base downward, and surrounded by a shell with an annular space through which the air circulates. A detachable grate at the bottom of the fire-pot made it possible to draw the fire in a few seconds. Since the air heating surface completely surrounds the fire-pot, the temperature of the exterior of the reheater is not greater than that of the reheated air; hence the loss by radiation is small. Type I reheater has an effective heating surface of 2.9 square feet, and, when reheating air to 400 degrees Fahrenheit, a capacity of about 200 cubic feet of free air per minute.

Coke was used as a fuel in all external combustion reheater tests. The samples of coke, residual fuel, ash, and refuse collected during the several tests of the Type I reheater were analyzed by the Department of Chemistry of the University of Illinois with the following results:

	B.t.u.
(1) Calorific value per pound of air-dry coke as fired, by the oxygen calorimeter	12 992
(2) Proximate analysis of air-dry coke:	
	Per Cent
Fixed carbon	88.76
Volatile matter	1.65
Moisture	0.55
Ash	9.04
	100.00
Moisture on drying	0.00
Sulphur determined separately	0.25
(3) Ultimate analysis of air-dry coke:	
Carbon	87.21
Hydrogen	0.74
Oxygen	1.34
Nitrogen	0.87
Sulphur	0.25
Ash	9.04
Moisture at 105 deg. C.	0.55
	100.00

(4) Proximate analysis of residual fuel:

Carbon	86.30
Earthy matter	13.40
Moisture	0.30
	100.00

(5) Proximate analysis of ash and refuse:

Carbon	30.92
Earthy matter	68.93
Moisture	0.15
	100.00

It was assumed that each pound of wood used as kindling had a calorific value of 0.4 pound of coke, and that each pound of oil used in kindling the fire had a calorific value of 1.47 pound of coke. The calorific value of carbon is taken as 14 540 B.t.u. per pound. From these assumptions and the data from the several analyses the various items presented in Table I were computed. In test 100-300-b the weight of ash and refuse was not recorded. Since they were obtained by computing the ash and refuse from the average grate efficiency, items 18 and 20 for this test are enclosed in brackets to indicate that their accuracy is open to some question.

Table 2 presents various data and the economic results of the tests of Type I reheater. The several numbered columns of the table present average recorded data or computed results. The methods of computing the various items are either indicated or obvious without further explanation. The efficiencies shown under items 14 and 15 are of chief interest. The efficiencies, exclusive of grate, vary from 48.0 to 61.5 per cent. The best efficiencies were obtained when from 100 to 150 cubic feet of free air per minute were reheated.

10. *Tests of Type II Reheater.*—In the Type II reheater the compressed air flowed through a series of cast iron coils set in a casing so that the coils were 12 to 15 inches above the normal level of the fire. No baffling was provided to force the heated gas over the coils, and the area of the furnace and casing exposed to the outer air was large; consequently the loss of heat by radiation was great. The heating surface of the coils was 18.6 square feet. The tests of this reheater were made in exactly the same manner as has been described for the Type I reheater.

TABLE 1
RESULTS OF TESTS OF TYPE I REHEATER

Designation	Date	Gage-box Data					Gross Fuel per Test			
		1	2	3	4	5	6	7	8	10
		Length of Test (hrs.)	Diameter of Orifice (in.)	Pressure in Gage-box (in. Water)	Temperature in Gage-box (deg. F.)	Barometer (in. Mercury)	Gage-box Coefficient (Durley's Curves)	Coke (lb. per Test)	Wood (lb. per Test)	Oil (lb. per Test)
50-400-a	6-10-12	8.00	1.5	3.56	241	29.5	0.603	24.0	3.00	0.30
100-400-a	6-11-12	8.00	2.0	4.86	297	29.4	0.601	42.5	3.00	0.25
150-400-a	6-12-12	8.00	2.5	4.39	316	29.2	0.598	52.0	3.00	0.25
200-400-a	6-14-12	8.00	3.5	2.04	308	28.8	0.597	70.0	4.00	0.30
275-200-b	5-30-12	6.33	3.5	3.88	203	29.3	0.595	39.5	3.00	0.66
275-200-c	6-1-12	8.50	3.5	4.02	223	29.1	0.595	50.0	3.00	0.60
100-250-b	6-3-12	8.00	2.5	2.15	220	29.2	0.599	23.0	3.00	0.40
200-300-b	6-13-12	4.00	3.5	2.16	287	29.1	0.597	33.0	3.00	0.30
100-300-b	5-27-12	5.00	2.5	2.08	225	28.8	0.599	20.0	1.25	0.10
100-300-c	5-29-12	8.00	2.5	2.06	243	29.1	0.599	37.5	3.00	0.30
100-400-b	6-5-12	8.00	2.5	2.10	291	29.3	0.599	44.0	3.00	0.30

TABLE 1 (CONTINUED)
RESULTS OF TESTS OF TYPE I REHEATER

Designation	Fuel Equivalents					Net Fuel				
	11	12	13	14	15	16	17	18	19	20
	Ash and Refuse (lb. per Test)	Residual Fuel (lb. per Test)	Coke Equivalent of Wood and Oil (lb. per Test)	Total Equivalent Coke (lb. per Test) (Item 8 + Item 13)	Coke Equivalent of Ash and Refuse (lb. per Test)	Coke Equivalent of Residual Fuel (lb. per Test)	Net Equivalent Coke Charged against Reheater (lb. per Test) (Item 14 — Item 16)	Net Equivalent Coke Consumed (lb. per Test) (Item 15)	Net Equivalent Coke Charged against Reheater (lb. per hr.) (Item 17 ÷ Item 2)	Net Equivalent Coke Consumed (lb. per Test) (Item 18 ÷ Item 2)
50-400-a	1.50	4.50	1.64	25.64	0.52	4.36	21.28	20.76	2.66	2.60
100-400-a	2.50	11.00	1.56	44.06	0.87	10.70	33.36	32.49	4.17	4.06
150-400-a	4.50	8.25	1.56	53.56	1.56	7.94	45.62	44.06	5.71	5.51
200-400-a	7.75	12.00	2.03	72.03	2.71	11.66	60.37	57.66	7.55	7.21
275-200-b	4.42	13.11	2.16	41.66	1.54	12.72	28.94	27.40	4.56	4.33
275-200-c	4.50	9.75	2.07	52.07	1.56	9.49	42.58	41.02	5.02	4.82
100-250-b	1.75	3.00	1.78	24.78	0.61	2.92	21.86	21.25	2.74	2.66
200-300-b	2.00	9.50	1.64	34.64	0.70	9.23	25.41	24.71	6.35	6.18
100-300-b	...	7.88	0.65	20.65	...	7.66	13.00	[12.55]	2.60	[2.51]
100-300-c	2.71	11.23	1.64	39.14	0.94	10.93	28.21	27.27	3.53	3.41
100-400-b	2.50	11.00	1.64	45.64	0.87	10.70	34.94	34.67	4.37	4.26

TABLE 2
RESULTS OF TESTS OF TYPE I REHEATER

1	2	3	4	5	6	7	8	9	10	11	12	
Designation	Length of Test (hr.)	Pressure of Air Entering Reheater (lb. per sq. in. Gage)	Temperature of Air Entering Reheater (deg. F.)	Temperature of Air Leaving Reheater (deg. F.)	Rise of Temperature	Air Reheated (lb. per hr.)	Air Reheated in cu. ft. of Free Air (62° F., 14.7 lb. Abs.)	Specific Heat at Const. Press. Air Saturated Entering Reheater	B. t. u. Transmitted to Compressed Air (per hr.)	R. t. u. Transmitted per sq. ft. of Heating Surface	Total B. t. u. Supplied per hr.	Grate Loss in B. t. u. per hr.
50-400-a	8.00	109.7	98.1	402.8	304.7	218	47.7	0.244	16170	5590	34559	844
100-400-a	8.00	109.9	111.5	409.2	297.7	432	95.0	0.244	31440	10860	54250	1406
150-400-a	8.00	109.6	121.0	397.2	276.2	644	141.0	0.245	43750	15110	73665	2530
200-400-a	8.00	117.4	132.7	385.9	252.3	838	183.5	0.245	51810	17890	98112	4363
275-200-b	6.33	99.4	128.1	225.0	96.9	1253	275.0	0.246	29810	10290	58675	2388
275-200-e	8.50	100.1	139.5	247.3	107.8	1232	271.0	0.246	33130	11420	67530	2380
100-250-b	8.00	108.0	111.2	278.7	167.5	471	103.0	0.244	19250	6640	35345	984
200-300-b	4.00	108.5	126.3	343.8	217.2	880	192.5	0.245	46750	16140	82627	2250
100-300-b	5.00	95.7	108.8	287.0	178.2	459	100.0	0.244	19960	6880	33820	[1210]
100-300-e	8.00	101.4	101.6	324.4	222.8	454	99.4	0.244	24620	8520	45896	1525
100-400-b	8.00	106.9	110.1	401.5	291.4	444	97.5	0.244	31570	10870	56805	1406

TABLE 2 (CONTINUED)
RESULTS OF TESTS OF TYPE I REHEATER

Designation	13	14	15	16	17	18	19	20	21	Flue Gas Analysis		
										Carbon Dioxide (CO ₂)	Oxygen (O ₂)	Carbon Monoxide (CO)
	B. t. u. Liberated per hr. by Combustion	Efficiency Including Grate (Per cent)	Efficiency Excluding Grate (Per cent)	Net Equivalent Coke Charged Against Reheater (lb. per hr.)	Pounds Coke Required to Raise 1000 lb. Air 100° F. (Item 16) $\div$ $\left(\frac{\text{It. 5} \times \text{It. 6}}{100} \right)$	Draft At Base of Stack in in. of Water	Room Temperature (deg. F.)	Barometer (in. Mercury)	Temperature of Flue Gases (deg. F.)			
50-400-a	33715	46.8	48.0	2.66	4.00	0.09	80	29.5	568	8.9	10.8	0.0
100-400-a	52844	57.9	59.4	4.17	3.24	0.15	84	29.4	611	9.1	10.1	0.0
150-400-a	71135	59.4	61.5	5.71	3.20	0.24	87	29.2	581	10.8	8.1	0.0
200-400-a	93749	52.8	55.2	7.55	3.57	0.35	84	28.8	601	10.5	8.4	0.0
275-200-b	56287	50.6	53.0	4.56	3.75	0.04	75	29.3	572	12.8	5.7	0.0
275-200-c	65150	49.1	50.9	5.02	3.77	0.03	85	29.1	561	13.2*	5.0	0.0
100-250-b	34561	54.4	55.6	2.74	3.47	0.08	83	29.2	606	6.5	12.7	0.5
200-300-b	80377	56.6	58.1	6.35	3.32	0.30	80	29.1	632	9.5	9.5	0.0
100-300-b	[32610]	59.0	[61.1]	2.60	3.18	0.05	86	28.8	475			
100-300-c	44371	52.4	55.6	3.53	3.50	0.05	73	29.1	607	11.3	7.7	0.0
100-400-b	55399	55.6	57.0	4.37	3.38	0.14	81	29.3	629	12.7	6.8	0.5

Analyses of the fuel, residual fuel, ash, and refuse gave the following results:

	B.t.u.
(1) Calorific value per pound of air-dry coke as fired, by the oxygen calorimeter	12 718
(2) Proximate analysis of air-dry coke:	
	Per Cent
Fixed Carbon	87.25
Volatile matter	2.24
Ash	0.50
Moisture	10.01
	100.00
Moisture on drying	0.00
Sulphur separately determined	0.66
(3) Ultimate analysis of coke:	
Carbon	83.95
Hydrogen	1.20
Oxygen	2.68
Nitrogen	1.00
Sulphur	0.66
Ash	10.01
Moisture at 105 deg. C.	0.50
	100.00
(4) Proximate analysis of residual fuel:	
Carbon	58.64
Earthy matter	41.22
Moisture	0.14
	100.00
(5) Proximate analysis of ash and refuse:	
Carbon	18.62
Earthy matter	81.38
Moisture	0.00
	100.00

Tables 3 and 4 present all the essential data and results of the tests of this reheater. The efficiencies obtained were low, due to excessive loss of heat by radiation and in the flue gases. With small loads the fire was necessarily thin and the excess air large; with larger loads the loss from excess air was reduced, although the flue gas temperature was abnormally high, and the loss of heat in the

TABLE 3
RESULTS OF TESTS OF TYPE II REHEATER

Designation	Gage-box Data						Gross Fuel per Test			
	1	2	3	4	5	6	7	8	9	10
Date	Length of Test (hrs.)	Diameter of Orifice (in.)	Pressure in Gage-box (in. Water)	Temperature in Gage-box (deg. F.)	Barometer (in. Mercury)	Gage-box Coefficient (Durley's Curves)	Coke (lb. per Test)	Wood (lb. per Test)	Oil (lb. per Test)	
50-400-d	6-22-12	8.00	1.5	3.58	242	29.5	0.602	65.0	10.00	0.25
100-400-d	6-24-12	8.00	2.0	4.80	301	29.3	0.600	100.0	6.00	0.75
150-400-d	10-23-12	8.00	3.0	2.19	311	29.4	0.598	103.5	8.60	1.50
200-400-d	10-22-12	8.00	3.0	3.84	308	29.2	0.597	133.5	9.50	1.30
250-400-d	10-24-12	8.00	3.5	3.27	325	29.4	0.597	145.5	8.75	1.50
300-400-d	10-25-12	8.00	3.5	4.75	341	29.3	0.595	174.5	9.50	1.50
350-400-d	10-26-12	8.00	4.0	3.96	348	29.3	0.595	189.5	9.50	1.50

TABLE 3 (CONTINUED)
RESULTS OF TESTS OF TYPE II REHEATER

Designation	Fuel Equivalents					Net Fuel				
	11	12	13	14	15	16	17	18	19	20
	Ash and Refuse (lb. per Test)	Residual Fuel (lb. per Test)	Coke Equivalent of Wood and Oil (lb. per Test)	Total Coke Equivalent (lb. per Test) (Item 8 + Item 13)	Coke Equivalent of Ash and Refuse (lb. per Test)	Coke Equivalent of Residual Fuel (lb. per Test)	Net Coke Charged against Reheater (lb. per Test) (Item 14 - Item 16)	Net Coke Consumed (lb. per Test) (Item 17 - Item 15)	Net Coke Charged against Reheater (lb. per hr.) (Item 17 ÷ Item 2)	Net Coke Consumed (lb. per hr.) (Item 18 ÷ Item 2)
50-400-d	4.75	10.00	4.36	69.36	1.01	6.71	62.65	61.64	7.83	7.71
100-400-d	6.40	14.70	3.49	103.49	1.35	9.88	93.61	92.26	11.70	11.53
150-400-d	6.50	8.50	5.63	109.12	1.38	5.70	103.42	102.04	12.93	12.76
200-400-d	8.00	11.00	5.69	139.19	1.70	7.39	131.80	130.10	16.48	16.26
250-400-d	6.50	14.00	5.68	151.18	1.38	9.41	141.77	140.39	17.72	17.55
300-400-d	8.00	21.50	5.98	180.48	1.70	14.42	166.06	164.36	20.76	20.55
350-400-d	10.00	15.00	5.98	195.48	2.12	10.03	185.45	183.33	23.18	22.92

TABLE 4
RESULTS OF TESTS OF TYPE II REHEATER

Designation	1	2	3	4	5	6	7	8	9	10	11	12
	Length of Test (hr.)	Pressure of Air Entering Reheater (lb. per sq. in. Gage)	Temperature Air Entering Reheater (deg. F.)	Temperature Air Leaving Reheater (deg. F.)	Rise of Temperature	Air Reheated (lb. per hr.)	Air Reheated in cu. ft. of Free Air per min. (62° F., 14.7 lb. Abs.)	Specific Heat at Const. Press. Air Saturated Entering Reheater	B. t. u. Transmitted to Compressed Air (per hr.)	B. t. u. Transmitted per sq. ft. of Heating Surface	Total B. t. u. Supplied per hr.	Grate Loss per hr. in B. t. u.
50-400-d	8.00	82.3	98.3	390.6	292.3	232	51	0.242	16397	882	99602	1606
100-400-d	8.00	82.8	105.4	406.0	300.6	428	94	0.243	31286	1682	148860	2166
150-400-d	8.00	104.4	109.1	405.6	296.5	661	145	0.243	47640	2561	164425	2200
200-400-d	8.00	102.3	118.8	380.2	261.4	868	190	0.245	55604	2989	209556	2716
250-400-d	8.00	106.6	122.2	392.4	270.2	1078	236	0.245	71353	3337	225425	2200
300-400-d	8.00	106.7	136.1	399.5	263.4	1279	280	0.246	82844	4454	263994	2766
350-400-d	8.00	106.5	146.4	398.0	251.6	1513	333	0.247	94060	5058	294775	3387

TABLE 4 (CONTINUED)
RESULTS OF TESTS OF TYPE II REHEATER

Designation	13	14	15	16	17	18	19	20	21	22		
										Flue Gas Analysis		
	B. t. u. Liberated per hr. by Combustion	Efficiency Including Grate (Per cent)	Efficiency Excluding Grate (Per cent)	Net Equivalent Coke Charged Against Reheater (lb. per hr.)	Pounds Coke Required to Raise 1000 lb. Air 100° F. (Item 16 $\div$ $\frac{It.6}{100} \times 1000$)	Draft At Base of Stack (in. Water)	Room Temperature (deg. F.)	Barometer (in. Mercury)	Temperature of Flue Gases (deg. F.)	Carbon Dioxide (CO ₂)	Oxygen (O ₂)	Carbon Monoxide (CO)
50-400-d	97996	16.5	16.7	7.83	11.50	0.14	89	29.5	415	4.6	16.3	0
100-400-d	146694	21.0	21.3	11.70	9.10	0.14	100	29.3	495	6.0	14.0	0
150-400-d	162225	29.0	29.4	12.93	6.60	0.18	67	29.4	574	5.6	14.2	0
200-400-d	206850	26.5	26.9	16.48	7.27	0.35	73	29.2	599	4.5	15.5	0
250-400-d	223225	31.7	32.0	17.72	6.08	0.21	70	29.4	691	8.3	11.6	0
300-400-d	261288	31.4	31.7	20.76	6.15	0.20	76	29.3	735	9.7	10.6	0
350-400-d	291388	31.9	32.3	23.18	6.09	0.24	75	29.3	777	11.4	8.8	0

flue gas large. While the efficiencies might have been somewhat improved by substituting a long flaming coal for coke, the general design of the reheater is not conducive to fuel economy.

11. *Tests of Type III Reheater.*—This reheater is of the internal combustion type using gasoline as fuel. Its capacity is about 250 cubic feet of free air per minute, heated to a temperature of 400 degrees Fahrenheit. The sectional view of the reheater, Fig. 6, shows

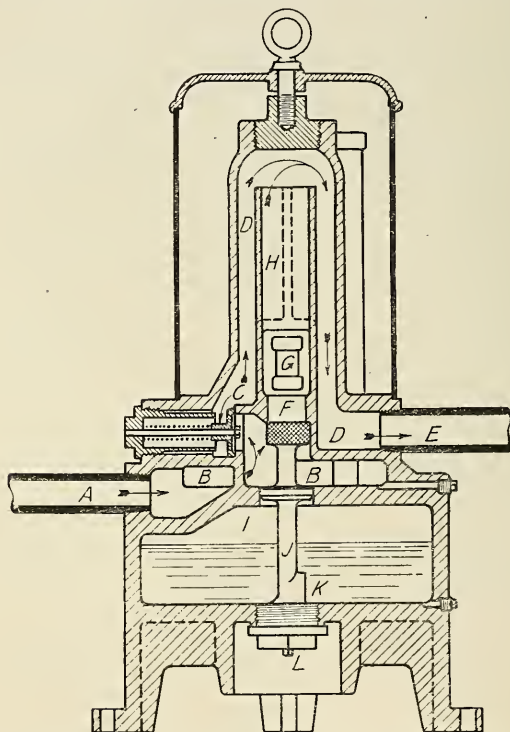


FIG. 6. SECTIONAL VIEW OF INTERNAL COMBUSTION REHEATER

the method of operation. Cold air enters the reheater at *A* and the reheated air leaves at *E*. The incoming air enters the space *B* and its pressure is exerted on the surface of the gasoline, *I*, in the base of the reheater. *C* is an adjustable spring loaded valve set to cause a slight reduction of pressure in the air, which flows through the

valve into the space *D* and out at *E*. A tube, *J*, having an opening, *K*, from the gasoline reservoir leads to a needle valve and atomizer at *H'*, adjustable at *L*; above the atomizer a combustion tube opens directly into the space *D*. Since the air pressure on the surface of the gasoline is slightly above that in space *D* and tube *H*, gasoline is forced through the atomizing tube upon the electrical ignition coil, *G*. The gasoline mixture burns in *H* and the products of combustion pass out with the reheated air.

In actual operation it was found difficult to secure and maintain the proper adjustment of the differential valve, *C*, and the atomizer. Considerable difficulty with the electrical ignition device was also experienced. Despite these difficulties, however, this reheater was used throughout the various engine tests described in a later section, and in general it proved satisfactory and convenient. It is decidedly easier to maintain constancy of reheat with this reheater than with the external combustion type. The slightest change in the position of the needle valve controlling the flow of gasoline affected at once the temperature of the out-going air.

The tests of this reheater were relatively simple. The Venturi tube already described was employed to measure the air. Its arrangement is shown in Fig. 7, and that of the reheater in Fig. 8.

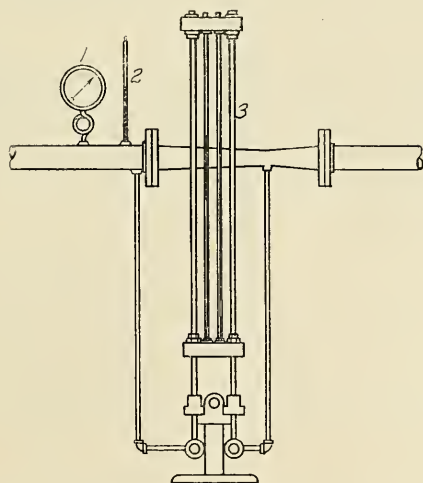


FIG. 7. APPARATUS FOR TESTING INTERNAL COMBUSTION REHEATER;
GENERAL ARRANGEMENT OF VENTURI TUBE

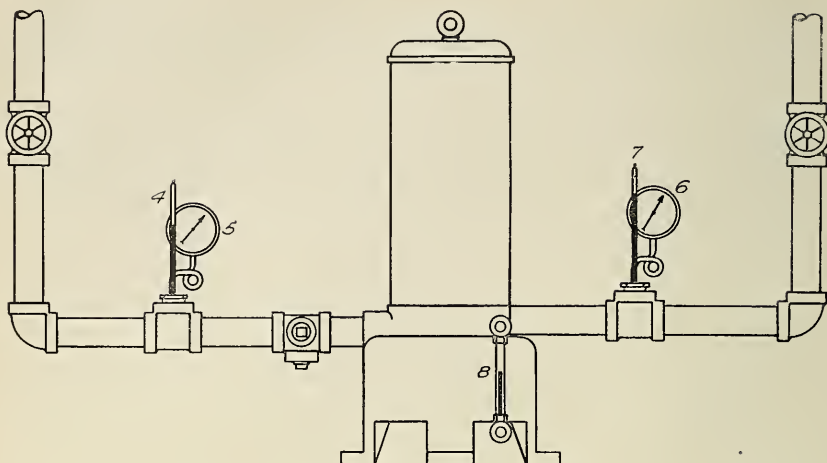


FIG. 8. APPARATUS FOR TESTING INTERNAL COMBUSTION REHEATER;
GENERAL ARRANGEMENT OF REHEATER

The numbers shown in these two figures refer to the various observations necessary, as follows:

- (1) Upstream pressure on tube (every 10 minutes)
- (2) Upstream temperature (every 10 minutes)
- (3) Manometer (every 2 minutes)
- (4) Temperature of air entering reheater (every 10 minutes)
- (5) Pressure of air entering reheater (every 10 minutes)
- (6) Pressure of air leaving reheater (every 10 minutes)
- (7) Temperature of air leaving reheater (every 5 minutes)
- (8) Gasoline gage at beginning and end of test
- (9) Room temperature (once for each test)
- (10) Barometer (once for each test)

With three exceptions the duration of each test was one hour; two tests were continued for $1\frac{1}{2}$ hours, and one—an overload test designated as 370-400-f—was stopped after 30 minutes because of the excessive load on the compressor. The gasoline used in the tests of this reheater had a specific gravity of 0.732 and a calorific value of 20 309 B.t.u. per pound.

Fig. 9 shows the drop of pressure through the reheater at different rates of air flow. During test 370-400-f, the compressor was unable to deliver the air at the normal pressure of 100 pounds gage;

in fact the average pressure was only 48 pounds gage. This probably had an effect on the loss of pressure with an air flow of 370 cubic feet of free air per minute.

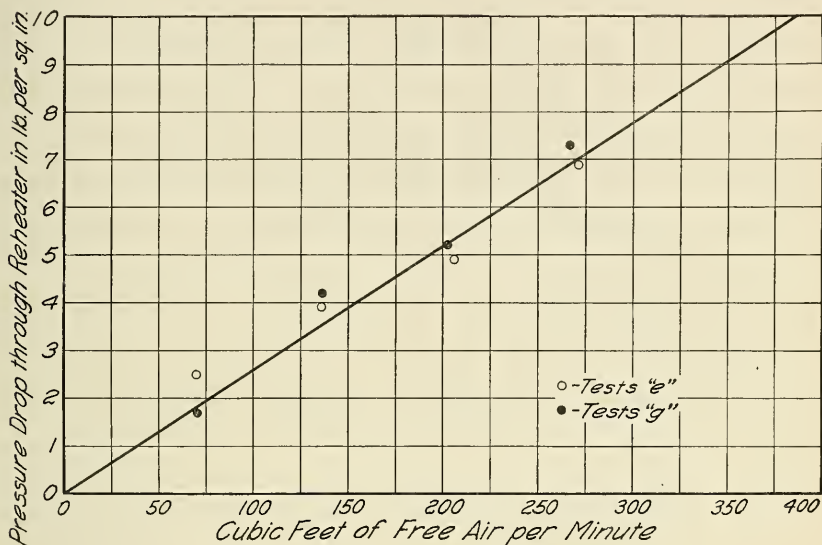


FIG. 9. DROP OF PRESSURE IN REHEATER

Table 5 presents the various observed quantities and the computed results. Item 17, the efficiency of the reheater, shows that due to more perfect vaporization and combustion of the gasoline the efficiency is better for the larger loads and higher temperatures of the reheated air. During some of the engine tests described in a later section, the temperature of the reheated air was maintained at 200 degrees Fahrenheit, and in some instances with comparatively small rates of flow. Under these conditions the combustion was so poor that there was a strong odor of unburned gasoline in the engine exhaust. Since the loss of heat by radiation could not have been large during any of the tests, it is evident that under the most favorable conditions maintained the loss from incomplete combustion was excessive.

TABLE 5
RESULTS OF TESTS OF TYPE III REHEATER

Designation	1	2	3	4	5	6	7	8	9
	Length of Test (hr.)	Pressure of Air Entering Reheater (lb. per sq. in. Gage)	Pressure of Air Leaving Reheater (lb. per sq. in. Gage)	Pressure Drop Through Reheater	Temper- ature Air Entering Reheater (deg. F.)	Temper- ature Air Leaving Reheater (deg. F.)	Rise of Temper- ature	Specific Heat at Const. Press. Air Saturated Entering Reheater	Venturi Meter Data
									Temper- ature Upstream (deg. F.)
									Pressure Upstream (lb. per sq. in. Gage)
70-400-e	1.5	99.3	96.8	2.5	99.0	402.0	303.0	0.244	112
140-400-e	1.0	99.5	95.6	3.9	112.3	403.0	290.7	0.245	132
200-400-e	1.0	99.7	94.8	4.9	125.3	400.0	274.7	0.247	154
260-400-e	1.5	100.8	93.9	6.9	140.0	405.0	265.0	0.250	168
370-400-f	0.5	48.0	34.5	13.5	156.0	404.0	248.0	0.264	180
70-300-g	1.0	99.1	97.4	1.7	110.0	300.2	190.2	0.244	122
140-300-g	1.0	100.5	96.3	4.2	113.1	302.1	189.0	0.245	133
200-300-g	1.0	100.0	94.8	5.2	128.4	302.2	173.8	0.247	148
260-300-g	1.0	99.3	92.0	7.3	137.4	308.2	170.8	0.250	165

TABLE 5 (CONTINUED)
RESULTS OF TESTS OF TYPE III REHEATER

Designation	9	10	11	12	13	14	15	16	17	18
	Venturi Meter Data			Air Reheated (cu. ft. of Free Air per min., 62° F., 14.7 lb. Abs.)	B. t. u. Absorbed by Com- pressed Air (per hr.)	Fuel Used (lb. per hr.)	lb. of Gasoline Required to Raise 1000 lb. of Air 100° F.*	Coke Equiv- alent of Item 14 (calorific Value 13 000 B. t. u. per lb.)	B. t. u. in Fuel Used (per hr.)	Efficiency (Per cent)
70-400-c	0.35	29.2	69.7	24215	1.57	1.58	2.46	31884	75.9	92
140-400-c	1.32	29.2	137.0	44455	2.70	1.49	2.33	54834	81.1	97
200-400-c	3.21	29.2	206.0	63807	3.72	1.44	2.25	75549	84.5	99
260-400-c	5.91	29.3	271.9	82159	4.73	1.44	2.25	96062	85.5	98
370-400-f	26.72	29.3	378.0	113020	6.80	1.58	2.46	138100	81.9	98
70-300-g	0.35	29.1	71.4	15105	1.04	1.68	2.62	21121	71.5	83
140-300-g	1.30	29.1	136.2	28760	1.77	1.50	2.35	35947	80.0	83
200-300-g	3.05	29.1	201.8	39532	2.55	1.59	2.49	51788	76.3	83
260-300-g	5.81	29.1	266.0	51853	3.47	1.67	2.61	70492	73.5	83

$$*(\text{Item 13}) \div \left(\frac{\text{Item 7}}{100} \right) \times \left(\frac{\text{Item 10}}{1000} \right)$$

IV. TESTS OF ENGINE USING AIR AS WORKING FLUID

12. *Purposes of Tests.*—A series of tests, during which air at various initial temperatures was used expansively as the working fluid in a small Corliss engine, was made to determine first, what proportion of the heat applied in the reheating process is utilized in the development of indicated work within the cylinder, and as effective or “brake” work delivered by the engine; and second, the efficiencies of the actual engine using reheated air in comparison with those theoretically obtainable. To secure the desired results it was necessary to establish the efficiencies of the engine under various conditions of operation without reheat, and to compare these results with those obtained under similar conditions of operation with the air heated to the several temperatures, specified by the range of the investigation. Since the injection of steam in the air pipe has occasionally been employed as a means of reheating the air in order to overcome operating difficulties with the motor, the experimental investigation was extended to determine the effectiveness of air-steam mixtures in comparison with steam and unreheated air used separately.

13. *Description of Experimental Plant.*—Air for the tests was supplied by an Ingersoll-Sergeant, two-stage, cross compound compressor, having an approximate capacity of 375 cubic feet of free air per minute compressed to 100 pounds per square inch gage. From the compressor the air was delivered to a receiver having a capacity of 65 cubic feet. From there it passed through the Venturi tube previously described, to a second receiver—capacity, 17 cubic feet—employed to reduce the pulsations of pressure in the Venturi tube, and from thence through the internal combustion reheater, described in Section 11, to the engine. During the tests using air only the exhaust air from the engine was allowed to escape into the laboratory through a short exhaust pipe; during the tests of air-steam mixtures the exhaust pipe was connected to a Dean surface condenser having a tube surface of 100 square feet. Above the engine throttle valve the air line was provided with an effective mixing chamber and water separator for use when steam was injected into the air.

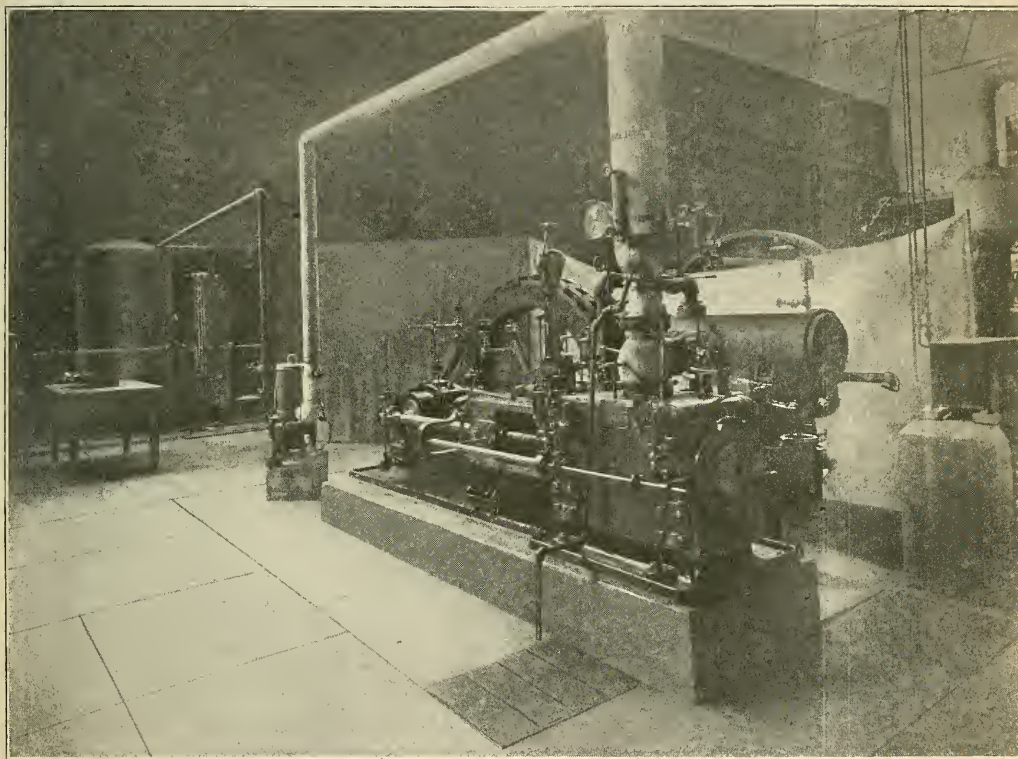


FIG. 10. GENERAL ARRANGEMENT OF EXPERIMENTAL PLANT

The engine used in this investigation was of the simple Corliss type built by the Murray Iron Works of Burlington, Iowa. Its principal dimensions are as follows:

	Inches
Diameter of Cylinder	8.00
Length of Stroke	18.05
Diameter of Piston Rod	1.87
Diameter of Steam Pipe	2.50
Diameter of Exhaust Pipe	3.00

Cylinder clearance in per cent of the total piston displacement is:

	Per Cent
For the Head End of the Cylinder	5.48
For the Crank End of the Cylinder	5.63

A specially designed indicator reducing motion of great accuracy was permanently attached to the engine, which was also equipped with a positive revolution counter and a Prony brake of excellent design and unusual steadiness. A pair of Tabor outside spring indicators were used in all tests.

Wherever necessary, pressures were determined by carefully calibrated standard test gages graduated in one pound increments. Temperatures were determined by standardized thermometers, those at the throttle and in the steam chest being secured by thermometers immersed through stuffing boxes directly in the stream of air. The temperature of the exhaust was secured by a pentane thermometer graduated in degrees centigrade, and placed in a mercury well in the exhaust pipe as close to the cylinder as possible. These temperatures are not an accurate measure of the actual temperatures in the cylinder during the exhaust stroke, since heat was rapidly absorbed from the surrounding atmosphere by the exhaust air in the pipe. During the earlier tests a hand pump was employed for lubricating the cylinder; in the later, a forced feed lubricator was attached to the engine. Fig. 10 shows the general arrangement of the plant.

14. *Method of Conducting Test.*—To insure a condition of thermal equilibrium in the piping and engine, the plant was operated for a period of one and one-half to two hours at the load to be maintained during the test. The cylinder of the engine was slow in reaching uniform temperature conditions, particularly when the point of cut-off was less than one-quarter of the stroke. When the difference between the temperatures at the throttle valve and in the steam chest

became nearly constant it was possible to begin the actual test. As soon as thermal equilibrium was obtained, the predetermined conditions of pressure, air temperature, cut-off, etc., to be maintained during the test were established and the test was started. The order in which tests were undertaken was so chosen as to require the least possible change in conditions between tests, for any change involved the lapse of considerable time between tests to re-establish thermal equilibrium. The tests using air were continued for thirty minutes, during which the following measurements were secured:

- (1) Upstream temperature of air (every 10 minutes)
- (2) Temperature entering reheater (every 10 minutes)
- (3) Temperature leaving reheater (every 10 minutes)
- (4) Temperature in steam chest (every 10 minutes)
- (5) Temperature of exhaust (every 10 minutes)
- (6) Temperature of room (every 10 minutes)
- (7) Upstream pressure (every 10 minutes)
- (8) Manometer reading of Venturi meter (every 2 minutes)
- (9) Indicator diagrams (every 5 minutes)
- (10) Height of barometer (once for each test)
- (11) Height of gasoline in reservoir of reheater at beginning and end of test

The pressure and temperature of the air at the throttle and the load on the brake were adjusted at the beginning of the test and maintained constant during the test.

When the reheater was not in use the air reached the engine from the compressor at about 100 degrees Fahrenheit; therefore, this was established as the basic temperature of the unreheated air. It was originally intended to cool the air in the receiver to room temperature by a spray of water, but this plan was abandoned when it was found that with air at 100 degrees Fahrenheit, great trouble was experienced through the formation of ice in the exhaust ports and pipe.

15. *Classification of Air Tests.*—For convenience the meter, indicator, brake horse power, and reheater data for all the tests in which air alone was used are given in Table 6. Of the 63 tests reported those numbered 1 to 16, inclusive, are tests in which the expansion of the air was complete to atmospheric pressure; those numbered 17 to 22, inclusive, are the first tests with variable cut-off; those numbered

TABLE 6
OBSERVED VALUES FOR ENGINE TESTS WITH REHEATED AIR

Designation	Venturi Meter			Indicator Data				b. h. p. Data			Reheater			
	1	2	3	4	5	6	7	8	9	10	11	12	13	14
Date	Tem- pera- ture of Air Up- stream (deg. F.)	Pres- sure of Air Up- stream (lb. per sq. in. Gage)	Barom- eter (in. Mer- cury)	Venturi Manom- eter (in. Mer- cury)	m. e. p. Head End of Cylinder	m. e. p. Crank End of Cylinder	i. h. p. Head End of Cylinder	i. h. p. Crank End of Cylinder	Revol- utions per min.	Weight on Scales	Tem- pera- ture Enter- ing Reheater (deg. F.)	Tem- pera- ture Leaving Reheater (deg. F.)	Rise of Tem- pera- ture (deg. F.)	
1-100-400-15.....	2-19-13	132	103	29.0	0.83	28.0	29.8	6.80	6.86	106.1	116	104	452	348
2-100-300-15.....	2-19-13	135	103	29.0	0.91	27.7	28.9	6.73	6.65	106.1	111	103	328	225
3-100-200-15.....	2-19-13	115	103	29.0	1.04	27.6	29.7	6.71	6.83	106.1	106	97	215	118
4-100-100-15.....	2-20-13	130	103	29.2	1.54	28.6	31.2	6.94	7.17	105.9	101			
5-80-400-20.....	2-24-13	101	82	29.4	0.87	26.5	27.4	6.40	6.26	105.4	106	80	446	366
6-80-300-20.....	2-24-13	90	82	29.4	1.00	26.3	26.9	6.35	6.16	105.5	101	74	338	264
7-80-200-20.....	2-21-13	126	83	29.0	1.07	25.9	26.5	6.27	6.08	105.8	86	92	227	135
8-80-100-20.....	2-20-13	116	83	29.2	1.80	27.7	28.3	6.68	6.47	105.3	90			
9-60-400-25.....	2-24-13	94	62	29.4	0.93	23.0	23.8	5.50	5.39	104.4	91	82	453	371
10-60-300-25.....	2-21-13	105	63	28.7	1.11	23.0	23.1	5.51	5.24	104.6	86	88	325	237
11-60-200-25.....	2-21-13	123	62	29.0	1.34	23.2	23.4	5.57	5.33	104.9	81	93	232	139
12-60-100-25.....	2-20-13	137	62	29.2	1.93	22.7	23.7	5.45	5.39	104.8	71			
13-40-400-35.....	2-24-13	93	43	29.4	1.08	18.5	19.6	4.40	4.41	103.7	72	81	472	391
14-40-300-35.....	2-21-13	102	42	28.9	1.22	17.7	18.4	4.21	4.14	103.9	64	85	328	243
15-40-200-35.....	2-21-13	107	43	29.0	1.49	18.7	18.8	4.46	4.25	104.2	56	89	246	157
16-40-100-35.....	2-20-13	130	41	29.2	1.92	17.7	18.3	4.22	4.14	104.2	48			
17-100-400-15.....	3-11-13	138	103	29.3	0.90	30.3	30.3	7.33	6.94	105.6	123	102	443	341
18-100-400-17.5.....	2-25-13	118	103	29.3	1.00	33.0	33.5	7.99	7.68	105.7	136	98	447	349
19-100-400-20.....	3-15-13	150	105	28.8	1.30	37.9	37.4	9.13	8.54	105.2	153	114	437	323
20-100-400-20.....	2-25-13	119	103	29.3	1.26	39.1	38.6	9.42	8.80	105.2	156	98	443	345
21-100-400-25.....	2-25-13	119	104	29.4	1.65	45.1	44.7	10.82	10.16	104.7	181	98	438	340
22-100-400-25.....	3-15-13	153	104	28.8	1.74	43.8	45.8	10.45	10.36	104.2	181	112	435	323

TABLE 6 (CONTINUED)
OBSERVED VALUES FOR ENGINE TESTS WITH REHEATED AIR

Designation	Venturi Meter					Indicator Data				b. h. p. Data		Reheater		
	1	2	3	4	5	6	7	8	9	10	11	12	13	14
Date	Tem- pera- ture of Air Up- stream (deg. F.)	Pres- sure of Air Up- stream (lb. per sq. in. Gage)	Barom- eter (in. Mer- cury)	Venturi Meter Manom- eter (in. Mer- cury)	m. e. p. Head End of Cylinder	m. e. p. Crank End of Cylinder	i. h. p. Head End of Cylinder	i. h. p. Crank End of Cylinder		Revolu- tions per min.	Weight on Scales	Tem- pera- ture Enter- ing Reheater (deg. F.)	Tem- pera- ture Leaving Reheater (deg. F.)	Rise of Tem- pera- ture (deg. F.)
2-25-13	120	104	29.4	1.93	48.5	48.7	11.59	11.04		104.4	200	93	445	352
3-11-13	124	105	29.3	2.15	49.3	51.4	11.74	11.60		104.0	209	100	439	339
3-15-13	151	104	28.8	2.21	50.9	50.9	12.12	11.49		104.0	206	107	435	328
3-11-13	137	106	29.3	2.78	57.1	56.1	13.55	12.61		103.6	236	103	439	336
3-19-13	135	104	29.1	2.00	43.2	45.5	10.30	10.28		104.1	176	106	331	225
4- 9-13	139	104	29.1	1.95	42.5	45.8	10.14	10.36		104.2	176	104	323	219
3-20-13	138	105	29.0	2.52	44.3	47.2	10.58	10.68		104.3	169	108	213	105
4- 9-13	124	105	29.1	2.32	42.7	45.9	10.22	10.39		104.3	171	96	209	113
4- 3-13	135	105	28.9	2.97	41.6	45.3	9.92	10.22		104.1	166			
3-15-13	141	84	28.8	1.38	33.6	35.7	8.03	8.09		104.4	136	102	448	346
4- 9-13	126	83	29.1	1.28	32.2	34.8	7.68	7.86		104.1	130	101	437	336
3-19-13	109	83	29.1	1.59	33.1	35.6	7.88	8.03		104.0	132	92	334	242
4- 7-13	114	83	29.4	1.55	32.1	34.2	7.65	7.72		104.1	131	84	321	237
3-20-13	125	84	29.0	1.92	32.8	34.8	7.83	7.87		104.2	119	102	215	113
4- 7-13	103	84	29.4	1.84	32.6	34.4	7.77	7.77		104.1	131	81	218	137
3-26-13	139	84	29.2	2.31	31.0	32.6	7.41	7.38		104.4	122			
4- 3-13	129	84	28.9	2.34	32.0	34.8	7.62	7.85		104.0	122			
3-19-13	95	62	29.3	0.95	23.3	23.9	5.55	5.40		104.1	82	83	458	375
3-28-13	106	62	29.5	0.96	22.7	23.6	5.41	5.40		104.1	93	84	452	368
3-19-13	101	62	29.3	1.06	22.6	23.9	5.39	5.33		104.1	82	87	327	240
3-28-13	103	63	29.5	1.07	22.8	23.8	5.43	5.38		104.1	93	83	347	264
3-28-13	115	63	29.5	1.33	22.9	24.0	5.46	5.42		104.0	90	81	206	125
4- 7-13	96	63	29.4	1.31	22.9	23.7	5.47	5.36		104.3	80	80	220	140
3-26-13	132	63	29.2	1.39	22.0	23.9	5.25	5.41		104.3	72			
4- 3-13	122	63	28.9	1.60	22.6	24.2	5.39	5.47		104.1	78			

TABLE 6 (CONTINUED)
OBSERVED VALUES FOR ENGINE TESTS WITH REHEATED AIR

Designation	Venturi Meter				Indicator Data					b. h. p. Data		Reheater		
	1	2	3	4	5	6	7	8	9	10	11	12	13	14
Date	Temperature of Air Upstream (deg. F.)	Pressure of Air Upstream (lb. per sq. in. Gage)	Barometer (in. Mercury)	Venturi Manometer (in. Mercury)	m. e. p. Head of Cylinder	m. e. p. End of Cylinder	i. h. p. Head of Cylinder	i. h. p. End of Cylinder	i. h. p. Crank End of Cylinder	Revolutions per min.	Weight on Scales	Temperature Entering Reheater (deg. F.)	Temperature Leaving Reheater (deg. F.)	Rise of Temperature (deg. F.)
48-100-400-25..... 49-100-300-25..... 50-100-200-25..... 51-100-100-25..... 52-80-400-25..... 53-80-300-25..... 54-80-200-25..... 55-80-100-25..... 56-60-400-25..... 57-60-300-25..... 58-60-200-25..... 59-60-100-25..... 60-100-400-15..... 61-100-400-20..... 62-100-400-30..... 63-100-400-35.....	5-14-13	138	104	29.0	1.55	43.0	45.5	10.26	10.29	104.2	181	116	432	316
	5-15-13	138	105	29.0	1.81	42.2	45.0	10.07	10.17	104.2	176	116	313	197
	5-20-13	127	104	29.0	2.18	43.9	44.8	10.45	10.11	104.0	171	107	216	109
	5-28-13	137	105	29.1	2.80	42.0	45.2	9.98	10.18	103.8	166			
	5-19-13	116	83	29.3	1.14	32.7	34.1	7.80	7.71	104.2	133	96	433	337
	5-19-13	110	84	29.3	1.36	32.6	34.1	7.77	7.70	104.1	133	97	324	227
	5-20-13	119	84	29.1	1.67	33.0	33.5	7.86	7.56	104.1	131	99	215	116
	5-28-13	115	84	29.1	2.18	32.2	35.6	7.67	8.03	104.0	114			
	5-19-13	110	62	29.3	0.81	22.5	23.7	5.37	5.36	104.2	86	93	445	352
	5-19-13	110	63	29.3	1.03	23.2	24.3	5.53	5.48	104.0	83	94	330	236
	5-20-13	106	63	29.1	1.25	23.7	24.7	5.64	5.58	104.0	82	93	216	123
	5-28-13	120	64	29.1	1.58	23.2	25.3	5.53	5.71	104.0	72			
	5-21-13	126	103	29.0	0.76	29.3	30.6	7.07	6.99	105.3	120	114	439	325
	5-21-13	135	104	29.0	1.11	36.9	39.1	8.85	8.88	104.7	152	118	427	309
	5-21-13	145	105	29.0	1.97	49.7	51.0	11.82	11.50	103.9	206	122	425	303
	5-21-13	147	107	29.0	2.56	56.7	56.3	13.41	12.62	103.3	235	124	436	312

23 to 47, inclusive, are the first tests with incomplete expansion; and those numbered 60 to 63, inclusive, are later tests with variable cut-off. Obviously this classification is overlapping. Thus the variable cut-off tests are also incomplete expansion tests. The object in view in tests 17 to 22, inclusive, was the determination of the effects of cut-off when the temperature and pressure were maintained constant at 400 degrees Fahrenheit and 100 pounds per square inch gage, respectively.

16. *Complete Expansion Tests.*—In these tests the load was adjusted to give a cut-off such that the release was at atmospheric pressure and substantially at the end of the stroke; in other words the "toe" of the indicator diagram came to a well defined point and was never rounded or looped. As shown by the designation, these tests were run at four different gage pressures, including 40, 60, 80, and 100 pounds per square inch. Each pressure was combined with four different temperatures, including 100, 200, 300, and 400 degrees Fahrenheit.

It was in the complete expansion tests that freezing of the exhaust first caused trouble. To expand from 100 pounds gage to atmospheric pressure requires a ratio of expansion of about 5.3. The cut-off at this pressure is shown by the indicator diagrams to be at about 15 per cent of the stroke, the release pressure about 1 pound above atmosphere, and the drop of pressure at cut-off about 12 per cent of the initial gage pressure. Then, if subscript 1 denotes conditions at cut-off and subscript 2 those at release, we have the formula:

$$\frac{p_1}{p_2} = \left(\frac{v_2}{v_1} \right)^n = \left(\frac{100 + \text{clearance per cent}}{\text{cut-off per cent} + \text{clearance per cent}} \right)^n$$

Taking atmospheric pressure at 14.5 pounds, which was about the average condition for the tests, we have:

$$p_1 = 100 - 12 + 14.5 = 102.5$$

$$p_2 = 14.5 + 1 = 15.5$$

$$\frac{102.5}{15.5} = \left(\frac{105.5}{20.5} \right)^n \text{ or } 6.61 = 5.15^n$$

$$n = \frac{\log. 6.61}{\log. 5.15} = \frac{0.821}{0.713} = 1.15$$

which checks very well with the value of n obtained by plotting the indicator diagrams taken under these conditions on logarithmic cross-section paper. This value of n shows a considerable heat exchange between the working medium and the cylinder walls. From the formula

$$\frac{T_1}{T_2} = \left(\frac{v_2}{v_1} \right)^{n-1} = 5.15^{0.15}$$

we find that for air at 100 degrees at cut-off $T_1=560$ and $T_2=438$, that is, the release temperature was -22 degrees Fahrenheit. It is probable, however, that at cut-off the temperature of the air was at least 20 degrees below the throttle temperature, as the drop in temperature between the throttle and steam chest is 5 degrees for tests of this kind. This would make the temperature at release 37 degrees below zero Fahrenheit. It is not to be inferred that the value of n found above is constant under all conditions. With cut-off at 25 per cent of the stroke, or more, the average value is about $n=1.275$. In test 4-100-100-15 the temperature registered by the thermometer in the exhaust pipe was -11 degrees Fahrenheit. With a later cut-off and consequently larger amount of air this same thermometer registered 10 to 15 degrees lower than this (see test 31-100-100-25). It might be expected that the lowest temperature would be obtained with earliest cut-off. This would occur if the value of n remained constant under all conditions of operation, but with quantities of air as small as those admitted when the cut-off is at only 15 per cent of the stroke the heat absorbed during the stroke has a greater relative warming effect. In consequence the value of n increases; conversely, the expansion becomes more nearly adiabatic as the cut-off is made later.

Calculating the release temperature when $n=1.275$, the formula becomes:

$$\frac{T_1}{T_2} = \left(\frac{100 + 5.5}{25 + 5.5} \right)^{0.275}$$

For $T_1=560$, this gives $T_2=398$, or 62 degrees below zero Fahrenheit. The fall of temperature between throttle and steam chest is not so great in this case as with the 15 per cent cut-off. We may therefore

assume that the temperature at cut-off is nearer the throttle temperature but somewhat below it. It is safe to say that the temperature of the air in the cylinder just as the valve is on the point of releasing is between 35 and 50 degrees below zero when the cut-off is at from 15 to 20 per cent of the stroke, and 60 to 70 degrees below zero when the cut-off is at 20 to 25 per cent of the stroke, the temperature at the throttle being in all cases 100 degrees Fahrenheit. After reaching the value 1.275 at 25 per cent cut-off n changes very little, so that the release temperature now depends only on the ratio of expansion, and as this diminishes with later cut-off the temperature in the exhaust again rises as the cut-off becomes later than 25 per cent.

With such temperatures prevailing in the cylinder and exhaust pipe, any moisture in the air is certain to cause operating difficulties in a few minutes. The exhaust ports and exhaust pipe become coated with ice, resulting in an increase in the back pressure which may in time prevent operation of the engine. During the tests, vigorous pounding of the exhaust pipe was frequently necessary to clear it of ice. This was done as soon as the indicator diagrams gave evidence, through an increase in the back pressure, of such ice accumulation.

In addition to the trouble from the formation of ice, the low temperature in the cylinder seriously interfered with effective lubrication. The increased viscosity of the lubricant resulting from the low temperature in the cylinder caused a large internal loss by friction. This difficulty was particularly pronounced when a mixture of cylinder oil and graphite was used as the cylinder lubricant. These two effects of the cold caused the brake horse power and mechanical efficiency of the engine to show considerable irregularities.

Various expedients were adopted to eliminate the lubrication difficulties but without complete success in any instance. Figure 11 shows the effect of variable lubrication produced through the infrequent injection of oil into the cylinder. When oil was thus injected it was noticed that a surprisingly large addition of weight was required to keep the brake scales in equilibrium immediately after the injection. This method of feeding oil was therefore immediately abandoned. In Figure 11 the heavy line is the indicator diagram taken just before lubrication; the dotted line is that taken after lubrication. The interval between the two diagrams was less than 10 minutes. No lubricant had been injected for more than one hour prior to taking the first of these diagrams.

At times the combustion in the reheater was extremely imperfect, in which case a third factor affecting the mechanical efficiency was introduced, one whose effect was entirely unmeasurable. When large quantities of unburned carbon with possibly some free gasoline passed

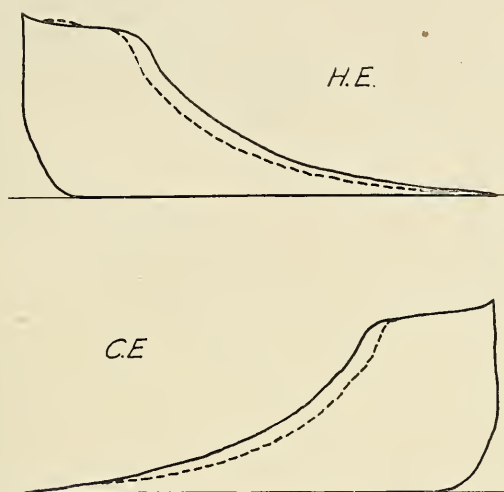


FIG. 11. INDICATOR DIAGRAMS SHOWING EFFECT OF CYLINDER LUBRICATION

into the cylinder, it is probable that the cylinder lubricant became more viscous because of the presence of the carbon, or its lubricating qualities may have been partially destroyed by the presence of free gasoline. To reduce these lubrication difficulties, the engine was finally equipped with a force feed lubricator set to feed 9 drops per minute. With this device cylinder lubrication was improved although it was never entirely satisfactory.

The effect of these disturbing conditions can readily be seen by examination of Items 6 and 7 in Table 7, or from the curves of mechanical efficiency, Fig. 12. The maximum and minimum mechanical efficiencies obtained under similar conditions are tabulated together in Tables 9 and 10.

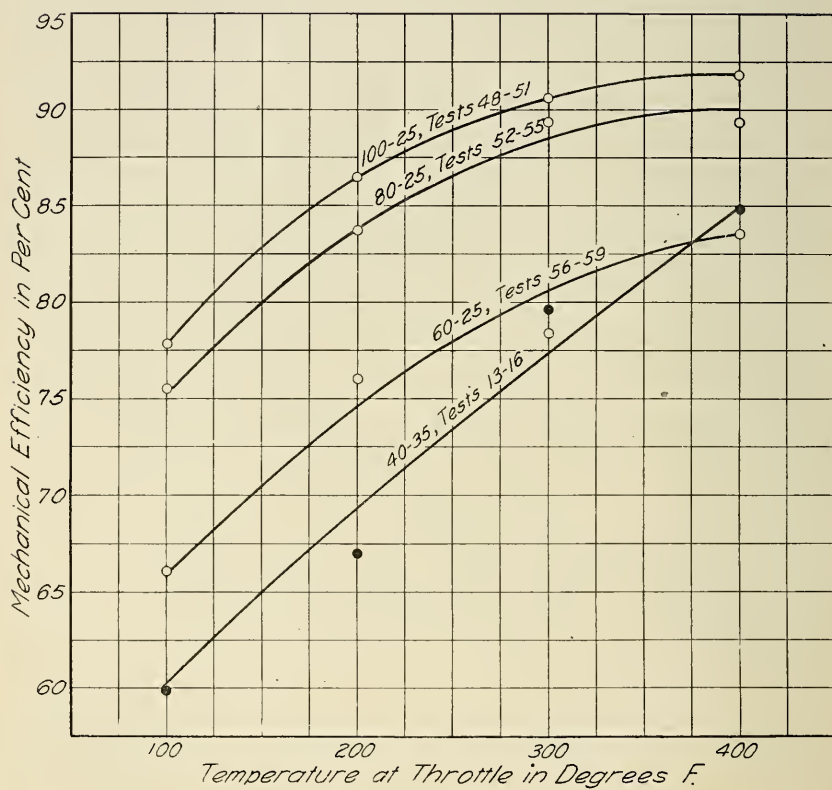


FIG. 12. EFFECT OF CHANGES IN LUBRICATION CONDITIONS

TABLE 7.
OBSERVED VALUES AND COMPUTED RESULTS FOR ENGINE TESTS WITH REHEATED AIR

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																	
Designation	Pressure at Throttle (lb. per sq. in. Gage)	Temp. at Throttle (deg. F.)	Total Air		Air Rate						Temperature (deg. F.)				Barom-eter (in. Mercury)	B. t. u. Req. to Raise Air (Item 3) from 100° F. to Throttle Temp.	lb. Air Req. per i. h. p. hr. at 100° F. and Pressure in Item 1	i. h. p. hr. Obtainable from Air (Item 3) at 100° F. and Pressure in Item 1	i. h. p. hr. Gained (Item 5 Less Item 19)	B. t. u. Gained (Item 17 ÷ Item 20)	Thermal Efficiency (2546 ÷ Item 21)	Ideal Efficiency (See Note)	Efficiency Ratio (Item 22 ÷ Item 23)	lb. Air Req. per b. h. p. hr. at 100° F. and Pressure in Item 1	b. h. p. hr. Obtainable from Air (Item 3) at 100° F. and Pressure in Item 1	b. h. p. hr. Gained (Item 6 Less Item 26)	B. t. u. Req. per b. h. p. hr. Gained (Item 17 ÷ Item 27)	i. h. p. hr. Gained per 1000 B. t. u. Added	b. h. p. hr. Gained per 1000 B. t. u. Added	13 000 B. t. u. Fuel Applied at 60% Efficiency		20 000 B. t. u. Fuel Applied at 80% Efficiency		Designation																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																	
			cu. ft. Free Air per min. (60° F., 14.7 lb. per sq. in. Abs.)	i. h. p.	b. h. p.	Mech. Efficiency	lb. per hr. per i. h. p.	cu. ft. Free Air per min. (92° F., 14.7 lb. per sq. in. Abs.)	lb. per hr. per b. h. p.	cu. ft. Free Air per min. (62° F., 14.7 lb. per sq. in. Abs.)	Average Cut-off Per Cent of Stroke	In Steam Chest	At Exhaust	Room																lb. per hr. per i. h. p. hr. Gained	lb. per hr. per b. h. p. hr. Gained	lb. per hr. per i. h. p. hr. Gained	lb. per hr. per b. h. p. hr. Gained																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																		

TABLE N
COMPARISON OF RESULTS OBTAINED IN TESTS IN MARCH AND MAY RESPECTIVELY

Designation	1	2	3	4	5	6	7	8	9	10
	Date	Cut-off, % of Stroke	Total Air (lb. per hr.)	i. h. p.	b. h. p.	Mech. Efficiency	Air Rate (lb. per i. h. p. hr.)	Air Rate (lb. per b. h. p. hr.)	Exhaust Temp. (deg. F.)	Room Temp. (deg. F.)
17-100-400-15	3-11-13	15.7	514	14.27	12.98	91.0	36.7	40.6	66	69
60-100-400-15	5-21-13	14.1	487	14.06	12.64	89.2	34.6	38.5	71	87
19-100-400-20	3-15-13	20.0	622	17.67	16.09	91.1	35.2	38.6	64	78
61-100-400-20	5-21-13	19.2	586	17.73	15.92	89.8	32.9	36.7	70	87
22-100-400-25	3-15-13	24.8	715	20.81	18.87	90.6	34.3	37.9	65	73
48-100-400-25	5-14-13	23.9	683	20.55	18.87	91.8	33.3	36.2	65	93
25-100-400-30	3-15-13	29.7	802	23.61	21.43	90.7	34.0	37.5	64	68
62-100-400-30	5-21-13	28.7	765	23.32	21.40	91.8	32.9	35.9	71	87
26-100-400-35	3-11-13	34.4	913	26.16	24.47	93.5	34.9	37.3	67	69
63-100-400-35	5-21-13	33.5	874	26.03	24.40	93.7	33.6	35.9	71	87

TABLE 9
EXTREME VARIATIONS OF MECHANICAL EFFICIENCY WITH
SIMILAR PRESSURE AND TEMPERATURE CONDITIONS

Cut-off at 25 per cent of stroke

400 deg.		300 deg.		200 deg.		100 deg.	
High	Low	High	Low	High	Low	High	Low
100 lb. Gage							
91.8	90.3	90.6	89.1	86.5	82.9	85.4	77.8
80 lb. Gage							
89.3	87.2	89.4	86.3	87.8	79.0	86.2	75.5
60 lb. Gage							
89.5	78.0	89.5	78.4	86.0	76.0	74.7	66.1

TABLE 10
EXTREME VARIATIONS OF MECHANICAL EFFICIENCY WITH
SIMILAR PRESSURE AND TEMPERATURE CONDITIONS

Various Cut-offs*

400 deg.		300 deg.		200 deg.		100 deg.	
High	Low	High	Low	High	Low	High	Low
100 lb. Gage							
93.7	90.2	90.6	88.1	86.5	82.9	85.4	75.7
33.5	14.2	24.5	14.8	25.2	25.0	25.5	15.8
80 lb. Gage							
89.3	87.2	89.4	85.1	87.8	73.7	86.2	72.0
23.8	25.3	23.8	18.6	25.1	17.5	24.2	21.1

TABLE 10 (CONTINUED)

EXTREME VARIATIONS OF MECHANICAL EFFICIENCY WITH
SIMILAR PRESSURE AND TEMPERATURE CONDITIONS

Various Cut-offs*

60 lb. Gage

89.5 25.2	78.0 24.9	89.5 25.1	78.4 25.0	86.0 24.9	76.0 24.4	74.7 24.2	66.1 24.4
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40 lb. Gage

84.8 34.7	79.6 32.7	67.0 34.0	59.8 32.5
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* The lower figures under each pressure are the actual points of cut-off in per cent of the stroke.

17. *Variable Cut-off Tests.*—Tests 17 to 22 and 60 to 63 inclusive were made for the purpose of determining the air economy of the engine with different ratios of expansion, other conditions remaining constant. All these tests were therefore made with the same pressure and temperature conditions at the throttle, namely 100 pounds per square inch gage pressure and 400 degrees Fahrenheit. The cut-off was increased from test to test by about 5 per cent, starting with 15 per cent in the first test of each series. At the assigned initial pressure this is the earliest cut-off that could be maintained, since this gives atmospheric pressure at release and any earlier cut-off would have caused a loop of negative work on the diagram.

18. *Incomplete Expansion Tests.*—Of the 3 tests made on compressed air 34 were incomplete expansion tests, the cut-off in each case being at 25 per cent of the stroke. Since the cut-off was kept constant during these tests the lowest initial pressure which could be maintained without the formation of a loop of negative work on the indicator diagram was 60 pounds per square inch gage.

As shown by Table 7, these tests include three initial pressures, 100, 80, and 60 pounds per square inch gage, each combined with four temperatures at the throttle, 400, 300, 200, and 100 degrees Fahrenheit, respectively. With two exceptions the tests were run under identical conditions; the exceptions being those tests made

with a pressure of 100 pounds gage and air temperatures of 400 degrees and 100 degrees Fahrenheit. Tests 27 to 47 inclusive were made during March and April, while tests 48 to 63 inclusive were made during the latter part of May. This repetition of tests under the same conditions seemed desirable when it was observed that there was a perceptible decrease in the air consumption per indicated horse power as the season advanced. The curves of air rate per indicated horse power per hour plotted against temperature for tests made during one month fell slightly below those made during the preceding month in spite of every effort to make all conditions the same. The May tests gave an air rate per indicated horse power per hour of from 1 to $1\frac{1}{2}$ pounds less than those made in March. Table 8 shows these variations. With a cut-off earlier than 25 per cent the variations in air consumption were even greater than those with 25 per cent cut-off.

Since the engine employed in these tests was new, the gradual reduction in air consumption was largely brought about by improvement in the condition of the cylinder and valves, resulting from wear; and to a lesser extent by the increasing room temperature as the outdoor temperature increased. Thus, during the tests made in May the room temperature was about 20 degrees higher than that maintained in March.

Table 7 presents certain observed and all computed values for the whole series of air-engine tests. While the derivation of the various items is generally obvious, it seems desirable to indicate the source of the data or the method of computation of each item listed.

Items 1 and 2 are average values of data from tests.

Item 3 is computed from the Venturi meter data given in Table 6, using the general method described in the paper mentioned on page 20.

Item 4=Item 3 $\times$ 0.219 (see Section 8).

Item 5 is computed from the engine constants, L = stroke in feet and A = mean area of piston in square inches, the average speed in revolutions per minute, N , and the average mean effective pressure, P , from the various indicator diagrams, by the formula

$$\text{I. h. p.} = \frac{2 \, PLAN}{33000}$$

Item 6 is computed from the radius of brake-arm in feet, R , the load on the scales in pounds, W , and the average revolutions per minute, N , by the formula

$$\text{B. h. p} = \frac{2 \pi R W N}{33000}$$

Item 7 = Item 6 divided by Item 5.

Item 8 = Item 3 divided by Item 5.

Item 9 = Item 4 divided by Item 5.

Item 10 = Item 3 divided by Item 6.

Item 11 = Item 4 divided by Item 6.

Item 12 is obtained from the indicator diagrams.

Items 13, 14, 15, and 16 are average data from the tests.

Item 17 = Item 3 multiplied by the difference between Items 2 and 1 and by the specific heat of air at constant pressure (0.24), the values recorded being the nearest tens.

Item 18 is selected from the curves representing the air consumption per indicated horse power for various throttle temperatures, Figs. 13 and 14, since these values are possibly somewhat more consistent than the results of the individual test at 100 degrees temperature at the throttle; wherefore these values do not agree exactly with the items given under Item 8.

Item 19 = Item 3 divided by Item 18.

Item 20 = Item 5 minus Item 19.

Item 21 = Item 17 divided by Item 20.

Item 22 = the heat equivalent of one horse power per hour (2546 B.t.u.) divided by Item 21.

$$\text{Item 23} = 1 - \left(\frac{14.7}{\text{Item 1} + 14.7} \right)^{0.286}$$

No attempt has been made to determine the ideal efficiency of the tests based upon incomplete expansion of the air. For some of the tests the value of Item 23 is therefore somewhat high.*

Item 24 = Item 22 divided by Item 23. For the reason stated under Item 23, the value of Item 24 is in some cases slightly smaller than would have been the case had the ideal efficiency for incomplete expansion been computed.

* See Section 5, Case II, for complete adiabatic expansion, which represents the maximum attainable thermal efficiency.

Item 25 is obtained from the curves representing the air consumption per brake horse power per hour for the various throttle temperatures. These values were selected for the reason stated under Item 18.

Item 26 = Item 3 divided by Item 25.

Item 27 = Item 6 minus Item 26.

Item 28 = Item 17 divided by Item 27.

Item 29 = Item 20 multiplied by 1000 and divided by Item 17.

Item 30 = Item 27 multiplied by 1000 and divided by Item 17.

Item 31 = 1000 divided by the product of 13 000 and 0.6 and Item 29.

Item 32 = 1000 divided by the product of 13 000 and 0.6 and Item 30.

Item 33 = 1000 divided by the product of 20 000 and 0.8 and Item 29.

Item 34 = 1000 divided by the product of 20 000 and 0.8 and Item 30.

The steam chest temperatures, Item 13, show a surprising drop between the temperature at the throttle and the point in the steam-chest where these temperatures were taken. The amount of drop decreases rapidly with the lowering of the throttle temperature. As the steam chest thermometer was placed in a thermometer well, it is probable that in the first twenty-one tests these differences are too great. This well was later removed and replaced by a stuffing-box through which the thermometer was passed, thus giving the temperature by direct contact between the bulb and the heated air as it passed to the steam ports of the engine. The drop of temperature is greater the earlier the cut-off, and decreases with later cut-off. For cut-off at 25 per cent of the stroke a difference of 40 to 50 degrees may be expected between throttle and steam chest temperatures when the throttle conditions are 100 pounds gage pressure and 400 degrees Fahrenheit. Exhaust temperatures, Item 14, are particularly unsatisfactory. The thermometer could not be placed near enough to the exhaust port of the engine to give the reading there and the temperature in the exhaust pipe was affected both by the room temperature and the exhaust pipes of other engines.

The heat required to raise the weight of air used in a particular test from 100 degrees Fahrenheit to the temperature maintained at the throttle, Item 17, does not include the loss of heat from the air

pipe between the reheater and the throttle. Due to its low specific heat and heat content, the drop in temperature of air flowing in a pipe is large. If the reheater is located l feet from the throttle valve, the temperature, t_r , required at the reheater to maintain a temperature, t_t , at the throttle may be computed by the following equation:

$$\log_e (t_r - t_o) = \frac{\pi D k l}{12 M c_p} + \log_e (t_t - t_o)$$

in which

t_o = temperature of the freezing point, 32 degrees Fahrenheit;

d = diameter of pipe in inches;

l = length of pipe in feet;

m = weight of air flowing per hour;

c_p = specific heat of air at constant pressure, (0.24);

k = coefficient of heat loss in B.t.u. per square foot of surface per degree difference between the air inside the pipe and that outside; approximately 3 for uncovered pipe and 0.6 for covered pipe.

The solution of a specific problem by means of the foregoing equation will show the great importance of locating the reheater as near the engine as possible, and of carefully insulating the connecting pipe and fittings.

The air-rates per indicated horse power per hour for various throttle temperatures are shown in Figs. 13 and 14. The former includes the data from tests 1 to 16 inclusive and 22 to 47 inclusive; the latter, tests 48 to 59 inclusive, made after the engine was well "worn in" and with a room temperature 20 degrees higher than in the earlier tests. The air rate in pounds per indicated horse power per hour for various throttle temperatures and gage pressures is shown in Fig. 15. The air rates per brake horse power per hour for various throttle temperatures are shown in Fig. 16. Some of these data are shown in an even more interesting manner in Fig. 12 in which the various pressure curves are drawn with mechanical efficiencies as ordinates and throttle temperatures as abscissæ.

It was expected that there would be some improvement in the mechanical efficiency with increasing air temperatures, but the extent of this improvement was such as to astonish everyone concerned with the tests. With a throttle temperature of 400 degrees Fahrenheit and

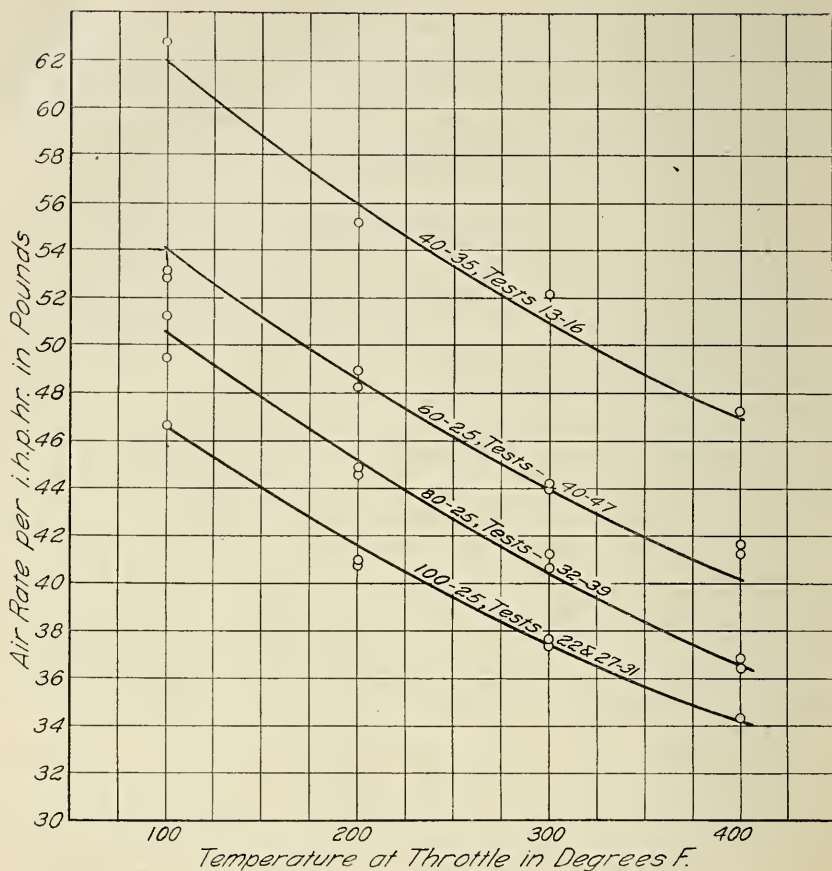


FIG. 13. AIR CONSUMPTION PER INDICATED HORSE POWER PER HOUR
FOR VARIOUS THROTTLE TEMPERATURES

a pressure of 100 pounds gage, mechanical efficiencies of 89.2 to 93.7 per cent were obtained; and with a throttle temperature of 100 degrees Fahrenheit and the same pressure, the mechanical efficiencies varied from 75.7 to 85.4 per cent. The lowest recorded mechanical efficiency is that of test 16, its value being only 59.8 per cent. It should be noted that in nearly every test the gain in brake horse power through reheating is greater than the gain in indicated horse power. Tables 9 and 10 present the high and low mechanical efficiencies obtained under various conditions of operation.

Items 31 and 32 (Table 7) present the computed weights of fuel consumed per hour for each indicated and brake horse power gained by reheating. Because of conditions discussed in the preceding paragraph, the weight of fuel per brake horse power per hour is *less* than that required per indicated horse power. While the weight of fuel required per brake horse power per hour fluctuated considerably for different tests, it should be noted that in several tests values of less than one pound of fuel were obtained. Few devices for converting heat into work can show equally satisfactory results.

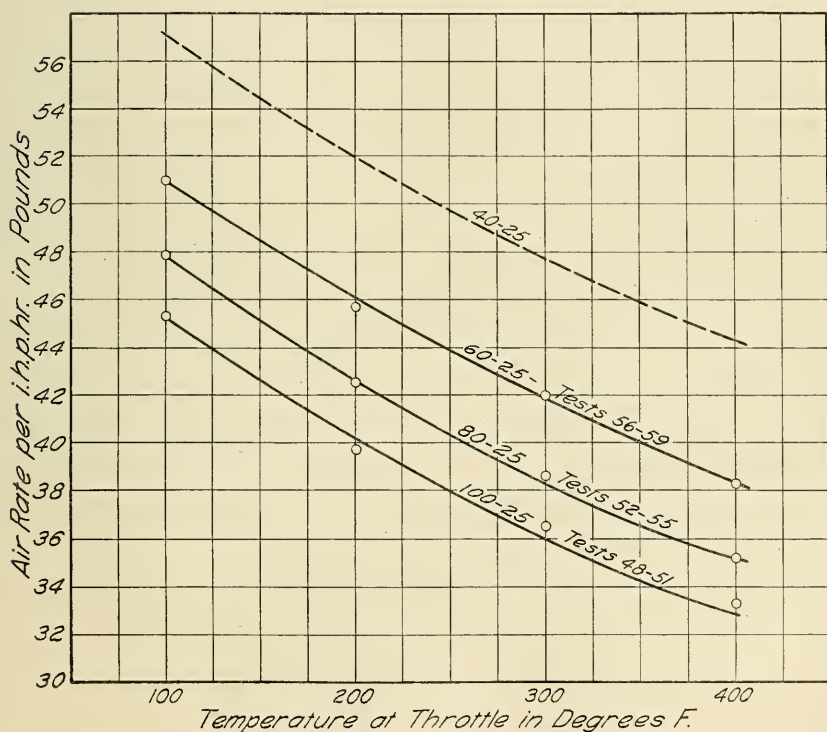


FIG. 14. AIR CONSUMPTION PER INDICATED HORSE POWER PER HOUR FOR VARIOUS THROTTLE TEMPERATURES

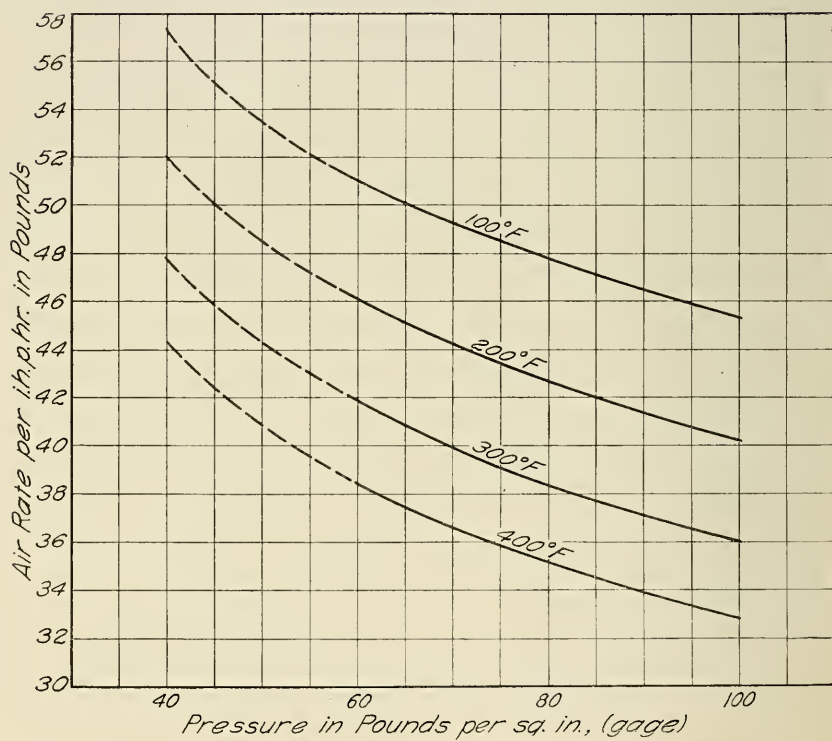


FIG. 15. AIR CONSUMPTION PER INDICATED HORSE POWER PER HOUR FOR VARIOUS THROTTLE TEMPERATURES AND GAGE PRESSURES

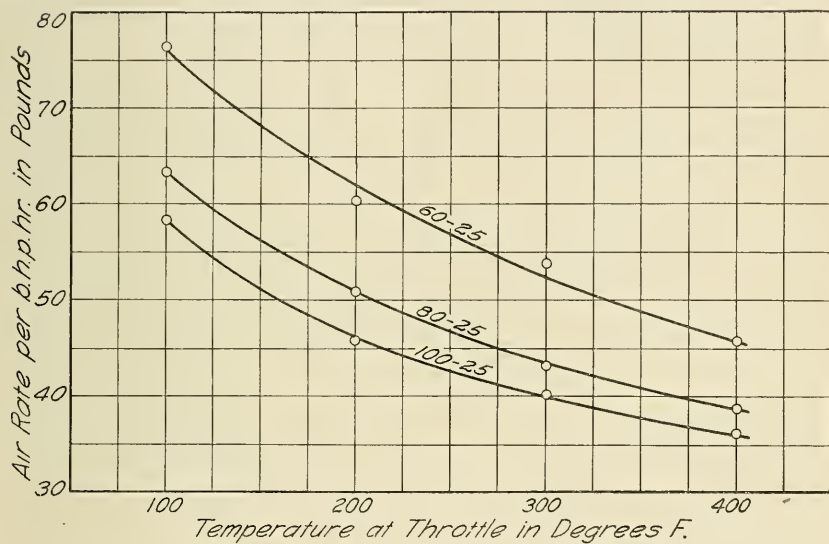


FIG. 16. AIR CONSUMPTION PER BRAKE HORSE POWER PER HOUR FOR VARIOUS THROTTLE TEMPERATURES

V. TESTS OF ENGINE USING MIXTURES OF AIR AND STEAM AS THE WORKING FLUID

19. *Purposes of Tests.*—The primary purpose of this series of tests of air-steam mixtures was the determination of the efficiency resulting from the use of steam as a medium for reheating compressed air; the secondary purpose was the devising of methods of testing which might be applicable in a more complete study of the subject, to be undertaken after the present general investigation.

20. *Method of Conducting Tests.*—While the plant described in Section IV was employed in these tests, with the condenser in service, the use of steam necessitated several changes in the methods of testing from those employed with air. The air from the compressor was delivered to the mixing chamber immediately above the throttle valve at a temperature which was maintained as nearly as possible at 100 degrees Fahrenheit; at this point steam was injected into the air until the required conditions of pressure and ratio of air to steam were obtained. To secure these conditions the running of from one to three preliminary tests was necessary. Before making these trial tests, an estimate was made to determine the approximate weight of air and steam required for a given mixture with the engine cut-off at 25 per cent of the stroke. The valve in the air line was regulated until the Venturi meter showed the correct flow of air, when steam was admitted in sufficient quantity to give the required pressure at the throttle. The plant was operated with these valve adjustments until the condenser was discharging steadily and other conditions were constant. When the weight of condensate showed the desired ratio of air and steam the test was started and continued for one hour.

In this test series much less time was required for securing thermal equilibrium in the cylinder and steam chest than in the air tests. The thermometers at the throttle, in the steam chest, and in the exhaust pipe quickly reached definite constant temperatures which were maintained as long as conditions were kept uniform. Thus, despite the fact that considerable time was required for the adjustment of the mixture, the preparatory run was generally no longer than when air alone was used.

The pressure at the throttle and the load on the brake were kept constant by frequent adjustment throughout the test. The following observations were taken as indicated:

- (1) Temperature at throttle (every 10 minutes)
- (2) Temperature in steam chest (every 10 minutes)
- (3) Revolutions per minute (every 10 minutes)
- (4) Pressure in mixing chamber (every 10 minutes)
- (5) Temperature in mixing chamber (every 10 minutes)
- (6) Temperature of air before entering mixing chamber (every 10 minutes)
- (7) Temperatures in throttling steam calorimeter (every 10 minutes)
- (8) Venturi manometer (every 2 minutes)
- (9) Pressure of air before entering mixing chamber (every 10 minutes)
- (10) Temperature in exhaust pipe (every 10 minutes)
- (11) Temperature of condensate (every 10 minutes)
- (12) Room temperature (taken twice per test)
- (13) Barometric pressure (taken once per test)
- (14) Weight of condensate from condenser
- (15) Weight of condensate from mixing chamber

The total steam admitted is determined in three parts: first, that which is condensed in the mixing chamber and there separated from the remaining mixture; second, that which is condensed in the condenser; and third, that which is contained in the air saturated with water vapor leaving the condenser at atmospheric pressure and the temperature maintained in the condenser. With an ample supply of cold condensing water and a consequent low exhaust temperature, the actual weight of moisture in the exhaust from the condenser is small. For great precision in measuring the steam supplied, the weight of vapor in the air entering the mixing chamber should be subtracted from the total weight of steam determined as described above. Because of difficulty in determining the exact condition of the compressed air in the air main, and the fact that the error in weight of steam should be small even though the air was initially saturated, the air for the sake of simplicity was assumed to be dry. With the air supply at 75 pounds per square inch absolute and 100 degrees Fahrenheit, the error due to this assumption is approximately

2 per cent of the weight of vapor in the exhaust air at 200 degrees Fahrenheit and a small fraction of one per cent of the total weight of steam used in the process.

21. *Discussion of Observed and Calculated Data.*—The heat content per pound of steam added was determined from the calorimeter data Items 7 and 8, Table 11, and recorded as Item 10; this multiplied by the total condensate, Item 15, of the same table, which is the sum of the condensate from the condenser, the condensate from the separator, and the moisture carried away in the air, gives the total heat energy in the steam added. While it may seem that Item 4 of Table 12, which gives the total steam in terms of equivalent dry steam at the throttle pressure, should be obtained by dividing the total heat energy in the steam by the heat content of dry steam at throttle pressure, this is not the case. Although in all of these computations the air was supposed to be delivered to the mixer at 100 degrees Fahrenheit, Item 6, Table 11, shows that this was not actually true. A correction was therefore necessary. If the air entered the mixer below 100 degrees Fahrenheit more steam was actually used than if it had entered at the standard temperature; hence the heat required to raise the air used in the test from the temperature entering the mixer to 100 degrees Fahrenheit was subtracted from the total heat of the steam used. Similarly when the air entered the mixer at a temperature above 100 degrees Fahrenheit less steam was actually used than when the air temperature was 100 degrees; hence the correction was added. The total heat thus corrected and divided by the heat content of dry steam at throttle pressure gives Item 4 of Table 12. By means of this correction it is possible in subsequent calculations to disregard the actual temperature of the air entering the mixer. From the previous air tests, air-rates per indicated horse power and brake horse power are known.

The manometer readings, Item 4, Table 11, are very small when the per cent of steam in the mixture is large. When the air consumption was small the mercury in the manometer was replaced by water which clearly showed small differences of pressure. In the table these pressures were converted from the observed heights of the column of water to heights of a column of mercury. From Item 6 of Table 12 it is seen that after all corrections were made the ratio of the steam used to the total mixture often deviated considerably

TABLE 11
OBSERVED VALUES FOR ENGINE TESTS WITH AIR-STEAM MIXTURES

Designation	Venturi Meter				Throttling Calorimeter					
	1	2	3	4	5	6	7	8	9	10
Date	Temperature of Air Upstream (deg. F.)	Pressure of Air Upstream (lb. per sq. in. Gage)	Venturi Meter Manometer (in. Mercury)	Barometer (in. Mercury)	Temperature of Air Entering Mixer (deg. F.)	Pressure at Calorimeter (lb. per sq. in. Gage)	Temperature at Calorimeter (deg. F.)	Quality of Steam	Heat Content of Steam B. t. u.	
A-S-100-10	5-5-13	128.6	101.3	1.42	29.31	106.7	121.4	250.4	0.972	1168
A-S-100-20	5-5-13	137.4	101.0	0.91	29.30	113.4	122.1	251.0	0.972	1168
A-S-100-30	5-5-13	136.0	102.9	0.64	29.30	113.0	120.1	249.9	0.971	1167
A-S-100-50	5-6-13	101.5	101.2	0.35	29.45	87.2	120.4	249.3	0.972	1167
A-S-100-75	5-6-13	102.3	100.5	0.12	29.45	89.2	120.6	249.9	0.972	1167
A-S-100-80	5-6-13	97.0	100.0	0.03	29.45	91.2	121.9	249.9	0.971	1167
S-100	5-6-13				29.45		100.0	255.6	0.980	1171
A-S-80-10	4-25-13	106.9	81.9	1.32	29.35	89.4	110.5	249.3	0.977	1170
A-S-80-20	4-25-13	96.1	81.1	0.90	29.35	80.2	110.0	249.7	0.977	1170
A-S-80-22.3	4-21-13	104.8	82.4	0.83	29.41	83.1	114.2	248.2	0.973	1167
A-S-80-30	4-28-13	98.0	80.7	0.60	29.21	82.3	110.6	249.5	0.975	1170
A-S-80-50	4-28-13	99.5	80.7	0.32	29.21	84.3	109.9	250.1	0.976	1170
A-S-80-75	4-28-13	93.4	80.1	0.13	29.21	84.3	110.2	249.3	0.975	1169
A-S-80-80	4-30-13	91.0	77.1	0.04	29.37	87.5	110.1	251.1	0.975	1169
S-80	4-25-13				29.35		80.0	248.6	0.980	1167
A-S-60-10	4-15-13	118.7	61.6	0.99	29.20	85.1	113.9	254.6	0.977	1170
A-S-60-20	4-15-13	106.1	62.7	0.62	29.20	76.9	110.0	256.4	0.979	1171
A-S-60-30	4-15-13	101.4	61.8	0.46	29.20	82.2	111.3	255.5	0.978	1170
A-S-60-50	4-16-13	106.1	62.1	0.26	29.14	84.8	114.4	248.1	0.972	1166
A-S-60-75	4-16-13	106.3	61.0	0.11	29.14	87.2	109.9	247.3	0.972	1166
A-S-60-80	4-21-13	83.4	60.4	0.03	29.46	74.8	106.3	243.2	0.971	1165
S-60	4-21-13				29.46		60.0	231.4	0.978	1160

TABLE 11 (CONTINUED)
OBSERVED VALUES FOR ENGINE TESTS WITH AIR STEAM MIXTURES

Designation	Condensate						i. h. p. Data				
	11	12	13	14	15	16	17	18	19	20	21
	Temperature of Condensate (deg. F.)	Condensate Weighed (lb. per hr.)	Moisture Carried Away in Saturated Exhaust Air of Mixture (lb. per hr.)	Condensate in Separator Due to Heating Air (lb. per hr.)	Total Wet Steam (lb. per hr.)	m. e. p. Head End of Cylinder	m. e. p. Crank End of Cylinder	i. h. p. Head End of Cylinder	i. h. p. Crank End of Cylinder	Revolutions (per min.)	Net Weight on Scales
A-S-100-10	71.4	96.6	9.3	6.9	112.8	46.0	46.5	10.97	10.51	104.2	191
A-S-100-20	82.6	162.5	10.4	5.7	178.6	45.4	46.7	10.85	10.57	104.4	191
A-S-100-30	83.6	207.7	8.2	6.6	222.5	45.7	47.0	10.91	10.65	104.4	191
A-S-100-50	85.1	290.0	7.2	6.2	303.4	45.4	48.0	10.82	10.84	104.1	196
A-S-100-75	88.9	423.6	4.8	4.6	433.0	47.3	49.6	11.26	11.18	104.0	199
A-S-100-80	90.4	500.8	2.7	1.8	505.3	47.8	48.7	11.39	11.00	104.1	199
A-S-100	96.0	630.4	0.0	0.0	630.4	47.6	51.4	11.33	11.59	103.9	206
A-S-80-10	80.4	76.1	8.1	5.7	89.9	36.7	38.6	8.74	8.71	104.0	151
A-S-80-20	82.0	111.2	6.0	5.4	122.6	35.4	37.6	8.42	8.48	103.9	151
A-S-80-22.3	111.6	105.5	22.4	5.8	133.7	36.8	37.2	8.78	8.41	104.2	154
A-S-80-30	92.7	170.3	6.9	4.7	181.9	35.5	38.2	8.45	8.61	103.9	151
A-S-80-50	101.3	248.7	4.3	4.9	257.9	35.6	38.1	8.47	8.59	103.9	151
A-S-80-75	101.2	317.3	2.7	4.2	324.2	36.4	36.9	8.68	8.33	104.1	151
A-S-80-80	99.0	392.0	2.2	3.5	397.7	35.7	37.5	8.50	8.51	104.0	151
A-S-80	123.1	532.0	0.0	0.0	532.0	38.1	40.1	9.08	9.06	104.1	161
A-S-60-10	65.7	567.0	4.7	4.0	65.4	26.2	27.4	6.25	6.19	104.2	106
A-S-60-20	78.0	975.0	5.9	4.0	107.4	25.8	26.7	6.16	6.04	104.2	106
A-S-60-30	84.0	130.3	5.9	3.8	140.0	25.7	26.3	6.14	5.96	104.4	106
A-S-60-50	107.6	178.0	9.7	3.6	191.3	25.6	26.6	6.11	6.01	104.2	106
A-S-60-75	117.9	242.5	7.9	2.9	253.3	26.1	27.0	6.22	6.10	104.1	106
A-S-60-80	91.0	311.5	2.0	2.6	316.1	25.2	27.5	6.00	6.21	104.0	106
A-S-60	98.7	397.5	0.0	0.0	397.5	24.6	27.0	5.86	6.10	104.1	106

TABLE 12
OBSERVED VALUES AND COMPUTED RESULTS FOR ENGINE TESTS WITH AIR-STEAM MIXTURES

Designation	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	Designation
	Total Air			Total Steam, Terms of Dry Steam (Pres. in Item 1) lb. per hr.	Total Mixture lb. per hr. (Item 3 + Item 4)	Temperature (deg. F.)			Meeh. Efficiency	cu. ft. Free Air per min. (62° F., 14.7 lb. per sq. in. Abs.)	Air lb. per hr. per i. h. p.	Steam lb. per hr. per i. h. p.	Mixture lb. per hr. per i. h. p.	cu. ft. Free Air per min. (62° F., 14.7 lb. per sq. in. Abs.)	Air lb. per hr. per b. h. p.	Steam lb. per hr. per b. h. p.	Mixture lb. per hr. per b. h. p.	Barometer (in. Mercury)	Room Temperature (deg. F.)	B. t. u. Added per hr. by Steam = Heat Content of Steam in Item 4—Heat of Liquid at Exhaust Temp.	lb. Air Req. per i. h. p. hr. at 100° F., and Pressure in Item 1	i. b. p. hr. Obtainable from Air (Item 3) 100° F. Pressure in Item 1 (Item 3 ÷ Item 24)	i. h. p. hr. Added by Steam (Item 10 Less Item 25)	B. t. u. Req. per i. h. p. hr. Added by Steam (Item 23 ÷ Item 26)	lb. Dry Steam Used per i. h. p. hr. Added (Item 4 ÷ Item 26)	Engine Water-Rate Pressure in Item 1 Cut-off	i. b. p. hr. Obtainable from Steam (Item 4 ÷ Item 29)	i. h. p. hr. Obtainable Using Air and Steam Separately (Item 25 + Item 30)	i. h. p. Gained by Mixing (Item 10 Less Item 31)	Per Cent Gain (Item 32 ÷ Item 31)	lb. Air Req. per b. h. p. hr. at 100° F. and Pressure in Item 1	b. h. p. hr. Obtainable from Air (Item 3) 100° F. Pressure in Item 1 (Item 3 ÷ Item 24)	b. h. p. Added by Steam (Item 11 Less Item 35)	B. t. u. Req. per b. h. p. hr. Added by Steam (Item 23 ÷ Item 36)	lb. Dry Steam Used per b. h. p. hr. Added (Item 4 ÷ Item 36)	Engine Water-Rate Pressure in Item 1 25 % Cut-off	b. h. p. hr. Obtainable from Steam (Item 4 ÷ Item 39)	b. h. p. hr. Obtainable Using Air and Steam Separately (Item 35 + Item 40)	b. h. p. Gained by Mixing (Item 11 Less Item 41)	Per Cent Gain (Item 42 ÷ Item 41)				
	Pressure at Throttle lb. per sq. in. (Gage)	cu. ft. Free Air per min. (60° F., 14.7 lb. per sq. in. Abs.)	lb. per hr.			Mixture at Throttle	In Steam Chest	At Exhaust																																	i. h. p.	b. h. p.		
A-S-100-10	100	144	654	112	706	14.0	234	231	128	21.48	19.90	92.6	6.65	30.3	5.2	35.5	7.16	32.7	5.6	38.3	29.3	81	122.010	45.3	14.36	7.12	17.120	15.7	27.1	4.14	18.50	2.98	16.1	54.4	11.97	7.93	15.390	14.1	29.0	3.87	15.84	4.06	25.6	A-S-100-10
A-S-100-20	100	114	522	177	701	25.2	268	266	158	21.42	20.00	93.2	5.32	24.3	8.3	32.6	5.72	26.1	8.9	35.0	29.3	83	187.930	45.3	11.50	9.92	18.940	17.8	27.1	6.55	18.05	3.37	18.7	54.4	9.60	10.40	18.050	17.0	29.0	6.11	15.71	4.29	27.2	A-S-100-20
A-S-100-30	100	98	444	220	664	33.1	284	282	171	21.56	20.00	92.9	4.51	20.6	10.2	30.8	4.87	22.2	11.0	33.2	29.3	86	230.850	45.3	9.81	11.75	19.660	18.7	27.1	8.13	17.94	3.62	20.2	54.4	8.18	11.82	19.520	18.6	29.0	7.60	15.78	4.22	26.7	A-S-100-30
A-S-100-50	100	73	334	296	630	47.0	302	300	185	21.66	20.44	94.5	3.37	15.5	13.6	29.1	3.57	16.3	14.5	30.8	29.5	78	306.500	45.3	7.38	14.28	21.410	20.7	27.1	10.92	18.30	3.36	18.3	54.4	6.15	14.20	21.430	20.7	29.0	10.20	16.35	4.09	25.0	A-S-100-50
A-S-100-75	100	43	107	423	620	68.2	321	319	201	22.44	20.70	92.2	1.92	8.8	18.8	27.6	2.08	9.5	20.4	29.9	29.5	78	431.500	45.3	4.35	18.09	23.810	23.4	27.1	15.62	19.97	2.47	12.4	54.4	3.62	17.08	25.250	24.8	29.0	14.60	18.22	2.48	13.6	A-S-100-75
A-S-100-80	100	28	105	494	599	82.5	330	329	207	22.39	20.74	92.8	1.03	4.7	22.0	26.7	1.11	5.1	23.8	28.9	29.5	80	501.190	45.3	2.32	20.07	24.980	24.6	27.1	18.23	20.55	1.84	9.0	54.4	1.93	18.81	26.620	26.2	29.0	17.05	18.98	1.76	9.3	A-S-100-80
S-100	100	0	0	622	100.0	338	338	338	212	22.92	21.40	93.4	0.00	0.0	27.1		0.00	0.0	29.0	...	29.5	81	627.500			22.92	27.230	27.1	27.1							21.40	29.300	29.0	29.0					S-100
A-S-80-10	80	120	586	87	673	12.9	222	218	125	17.45	15.70	90.1	7.37	33.6	5.0	38.6	8.18	37.3	5.5	42.8	29.4	70	95.350	48.0	12.19	5.26	18.100	16.5	28.9	3.01	15.20	2.25	14.8	60.8	9.64	6.06	15.700	14.3	31.3	2.78	12.42	3.28	26.4	A-S-80-10
A-S-80-20	80	107	491	119	611	19.5	244	241	147	16.90	15.68	92.9	6.34	29.0	7.1	36.1	6.85	31.3	7.6	38.9	29.4	69	127.240	48.0	10.19	6.71	19.020	17.7	28.9	4.12	14.31	2.59	18.1	60.8	8.09	7.59	16.780	15.7	31.3	3.81	11.90	3.78	31.8	A-S-80-20
A-S-80-22.3	80	103	472	130	602	21.6	249	245	151	17.19	16.02	93.2	6.00	27.4	7.6	35.0	6.41	29.5	8.1	37.6	29.4	82	138.440	48.0	9.83	7.36	18.780	17.7	28.0	4.50	14.33	2.86	19.9	60.8	7.78	8.24	16.800	15.8	31.3	4.16	11.94	4.08	34.2	A-S-80-22.3
A-S-80-30	80	88	400	178	578	30.8	267	265	169	17.06	15.68	92.0	5.13	23.4	10.4	33.8	5.57	25.5	11.3	36.8	29.2	76	186.350	48.0	8.32	8.74	21.310	20.4	28.9	6.16	14.48	2.58	17.8	60.8	6.59	9.09	20.500	19.6	31.3	5.70	12.29	3.39	27.6	A-S-80-30
A-S-80-50	80	64	290	254	544	46.7	288	286	186	17.06	15.68	92.0	3.73	17.0	14.9	31.0	4.05	18.5	16.2	34.7	29.2	77	261.440	48.0	6.04	11.02	23.740	23.0	28.9	8.80	14.84	2.22	14.9	60.8	4.79	10.89	24.000	23.3	31.3	8.12	12.91	2.77	21.4	A-S-80-50
A-S-80-75	80	39	180	318	498	63.0	303	302	197	17.01	15.71	92.5	2.33	10.6	18.7	29.3	2.50	11.4	20.2	31.6	29.2	80	324.840	48.0	3.76	13.25	24.430	24.0	28.0	11.00	14.76	2.25	15.2	60.8	2.97	12.74	25.470	24.9	31.3	10.15	13.12	2.59	19.7	A-S-80-75
A-S-80-80	80	24	107	391	498	78.5	313	312	205	17.01	15.70	92.5	1.38	6.3	22.9	29.2	1.48	6.8	24.9	31.7	29.4	85	395.690	48.0	2.24	14.77	26.750	26.5	28.9	13.53	15.77	1.24	7.9	60.8	1.76	13.94	28.370	28.0	31.3	12.50	14.26	1.44	10.1	A-S-80-80
S-80	80	0	0	525		100.0	326	325	212	18.14	16.75	92.5	0.00	0.0	28.9		0.00	0.0	31.3	...	20.4	70	527.800			18.14	28.950	28.9	28.9							16.75	31.500	31.3	31.3					S-80
A-S-60-10	60	192	448	63	511	12.3	201	198	121	12.44	11.05	89.0	7.90	36.1	5.0	41.1	8.88	40.5	5.7	46.2	29.2	75	69.050	51.1	8.77	3.67	18.790	17.1	32.6	1.93	10.70	1.74	16.3	73.1	6.14	4.91	14.030	12.8	35.3	1.78	7.92	3.13	39.6	A-S-60-10
A-S-60-20	60	80	367	103	472	22.2	235	233	151	12.20	11.05	90.7	6.58	30.1	8.6	38.7	7.28	33.2	9.5	42.7	20.2	73	111.150	51.1	7.17	5.03	22.080	20.9	32.6	3.22	10.39	1.81	17.4	73.1	5.02	6.03	18.460	17.4	35.3	2.98	8.00	3.05	38.1	A-S-60-20
A-S-60-30	60	67	307	138	445	31.0	251	248	164	12.10	11.08	91.6	5.57	25.5	11.4	36.9	6.10	27.8	12.4	30.2	29.2	75	144.230	51.1	6.02	6.08	23.710	22.7	32.6	4.24	10.26	1.84	17.9	73.1	4.21	6.87	21.000	20.1	35.3	3.91	8.12	2.96	36.4	A-S-60-30
A-S-60-50	60	52	236	189	425	44.5	271	269	182	12.12	11.05	91.3	4.26	19.5	15.6	35.1	4.69	21.3	17.1	38.4	29.1	79	194.400	51.1	4.61	7.51	25.770	25.2	32.6	5.81	10.42	1.70	16.3	73.1	3.23	7.82	24.850	24.2	35.3	5.35	8.58	2.47	28.8	A-S-60-50
A-S-60-75	60	32	146	250	396	63.1	287	285	196	12.32	11.03	89.6	2.61	11.8	20.3	32.1	2.91	13.3	22.6	35.9	29.1	83	253.810	51.1	2.86	9.46	26.790	26.4	32.6	7.68	10.54	1.78	16.9	73.1	2.00	9.03	28.100	27.7	35.3	7.08	9.08	1.95	21.5	A-S-60-75
A-S-60-80	60	19	82	311	393	79.1	298	297	205	12.21	11.02	90.4	1.46	6.7	25.5	32.2	1.65	7.5	28.2	35.7	29.5	76	313.720	51.1	1.61	10.60	29.580	29.4	32.6	9.56	11.17	1.04	9.3	73.1	1.13	9.89	31.720	31.4	35.3	8.81	9.04	1.08	10.9	A-S-60-80
S-60	60	0	0	390		100.0	309	307	212	11.96	11.03	92.4	0.00	0.0	32.6			0.0	35.3	...	29.5	79	390.600			11.96	32.540	32.6	32.6							11.03	35.400	35.3	35.3				...	S-60

Designation	
A-S-100-10	
A-S-100-20	
A-S-100-30	
A-S-100-50	
A-S-100-75	
A-S-100-80	
S-100	
A-S-80-10	
A-S-80-20	
A-S-80-22.3	
A-S-80-30	
A-S-80-50	
A-S-80-75	
A-S-80-80	
S-80	
A-S-60-10	
A-S-60-20	
A-S-60-30	
A-S-60-50	
A-S-60-75	
A-S-60-80	
S-60	

from the predetermined ratio as shown by the last number of the test designation. This deviation, however, caused no difficulty. Items 7 and 8 show that the temperature difference between the throttle and the steam chest is small compared with that found when air alone is used. Even a small quantity of steam, 10 to 15 per cent of the mixture, will make this difference 4 degrees Fahrenheit or less; in the air tests at high temperatures this difference was often more than 40 degrees.

Item 6, the per cent of steam in the mixture, plotted against Item 7, the temperature at the throttle, for each total pressure gives the curves shown in Fig. 17. These curves suggest a method for regulating the proportions of the mixture without weighing the condensate. With a given cut-off and brake load, and for a specified total pressure, the valves in the air and steam lines can be adjusted to give the temperature at the throttle which results from the desired mixture.

In these air-steam tests the exhaust temperature, Item 9, is reliable and probably in no case is it more than 1 or 2 degrees in error. In all these tests it was considerably above the temperature of the atmosphere and surrounding objects; when steam alone was used, exhausting at atmospheric pressure, the thermometer read 212 degrees Fahrenheit. These exhaust temperatures show that the steam contained in an air-steam mixture is used more efficiently than it would be in a non-condensing steam plant; and, in the case of mixtures having small percentages of steam, it is used with efficiencies comparable to those obtained with condensing engines. In test A-S-100-10 the exhaust temperature was 128 degrees Fahrenheit, which is the temperature that would result in a condensing plant maintaining a 25 inch vacuum. The exhaust temperature may be regarded as a measure of the condenser effect produced from the use of mixtures of air and steam, resulting from the fact that the total exhaust pressure is the sum of the partial pressures of the steam and air.

When even a small quantity of steam was admitted to the cylinder with the air the difficulties with lubrication and the excessive internal friction were overcome, as shown by the indicated and brake horse power, Items 10 and 11. The difference between these items gives a friction horse power of about 1.5 for 100 pounds, 1.4 for 80 pounds, and 1.2 for 60 pounds. In the air tests with serial numbers 22-31 and 48-51, in which the air was 400 degrees at the throttle, about the same indicated horse power was developed as in the air-steam tests at 100

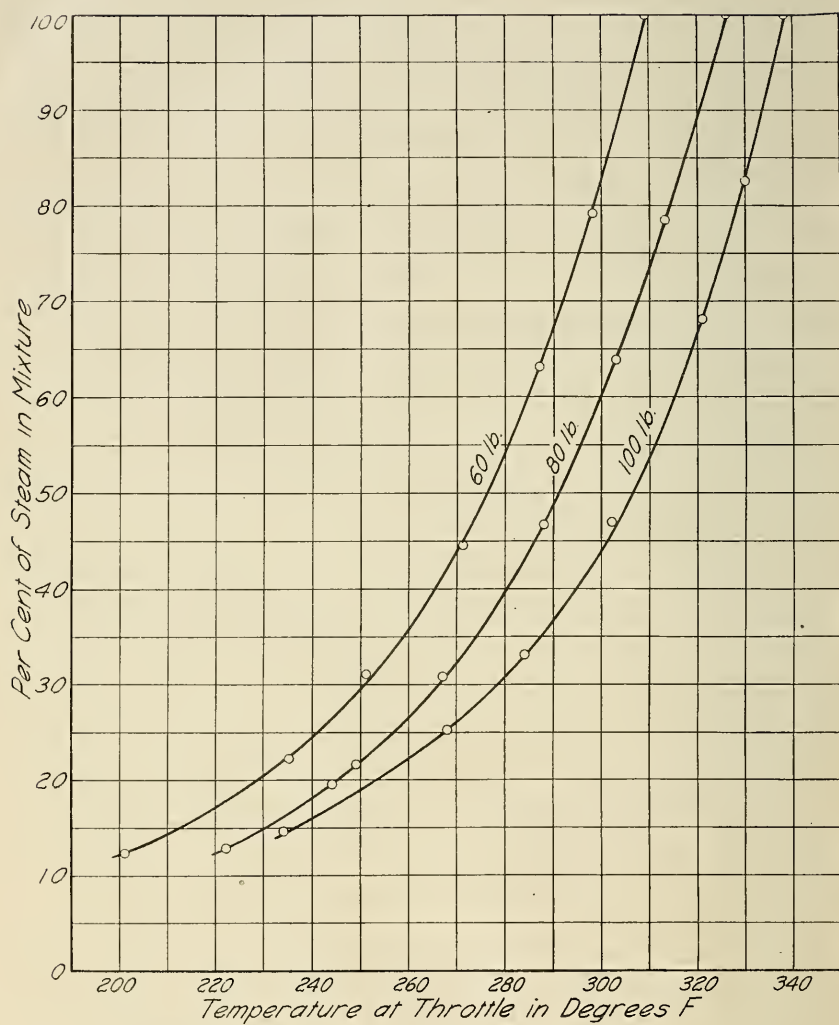


FIG. 17. RELATION BETWEEN TEMPERATURES AND PRESSURES AT THROTTLE AND PERCENTAGES OF STEAM IN AIR STEAM MIXTURE

pounds gage. In the air tests, however, the friction horse power was 1.7, showing that even with 400 degrees at the throttle the internal friction was not quite reduced to its minimum value, which, when the engine was developing 21 indicated horse power with 100 pounds pressure at the throttle, may be taken as 1.5 horse power.

Item 12 shows that when the load is nearly constant the mechanical efficiency does not vary with the per cent of steam in the mixture. When the initial pressure is decreased and the cut-off kept constant there is a slight decrease in the efficiency due to decrease of the load, as follows:

Pressure at Throttle (gage)	Average i.h.p.	Average Mechanical Efficiency
100	21.98	93.1
80	17.23	92.2
60	12.19	90.7

The air, steam, and mixture rate per indicated horse power given in Items 14, 15, and 16 are plotted to per cent of mixture in Figures 18, 19, 20. In each figure, curve (2) represents the air weight, curve (3) the steam weight, and curve (1) the weight of the mixture. Curve (1) is therefore the summation curve of curves (2) and (3). Figure 21 is the curve (1) of Figures 18, 19, and 20 drawn to a larger scale. It is unnecessary to plot the corresponding curves for brake horse power since the air and steam consumption per brake horse power per hour may be obtained from these curves by dividing particular values for a given pressure by the average mechanical efficiency for that pressure as shown above.

22. *Economic Results.*—The presentation of the economic effects resulting from mixing air and steam begins with Item 23, which gives the total heat above the exhaust temperature of the equivalent steam supplied during the test. This is the total heat chargeable to the engine, since the condensed exhaust steam is assumed to be used as boiler feed water. In Item 25 the indicated horse power per hour obtainable from the air in the mixture if used alone at 100 degrees Fahrenheit is found by dividing Item 3 by Item 24, which is the air-rate per indicated horse power at the given throttle pressure and 100 degrees Fahrenheit determined from air tests. Subtracting Item 25 from Item 10 gives the indicated horse power which may be assumed to

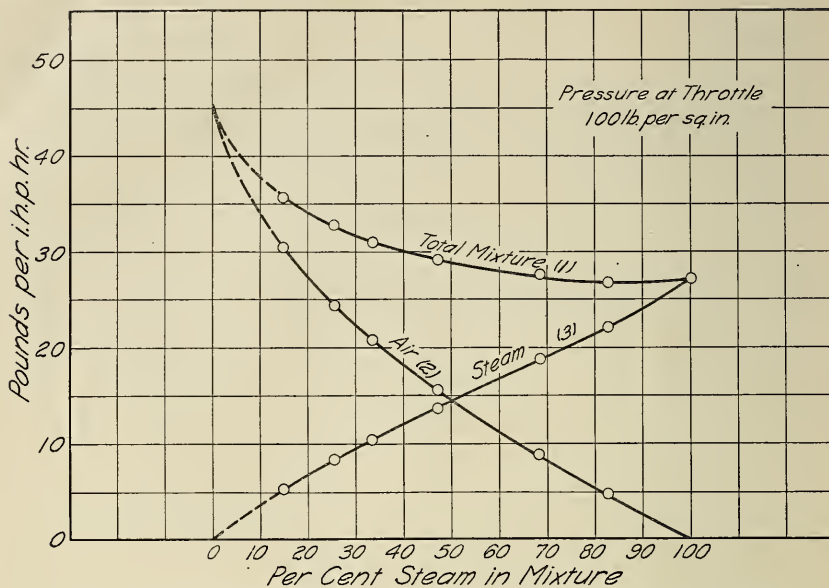


FIG. 18. CONSUMPTION OF AIR, STEAM, AND MIXTURE PER INDICATED HORSE POWER PER HOUR FOR DIFFERENT PERCENTAGES OF STEAM AND MIXTURE; THROTTLE PRESSURE 100 LB. PER SQ. IN.

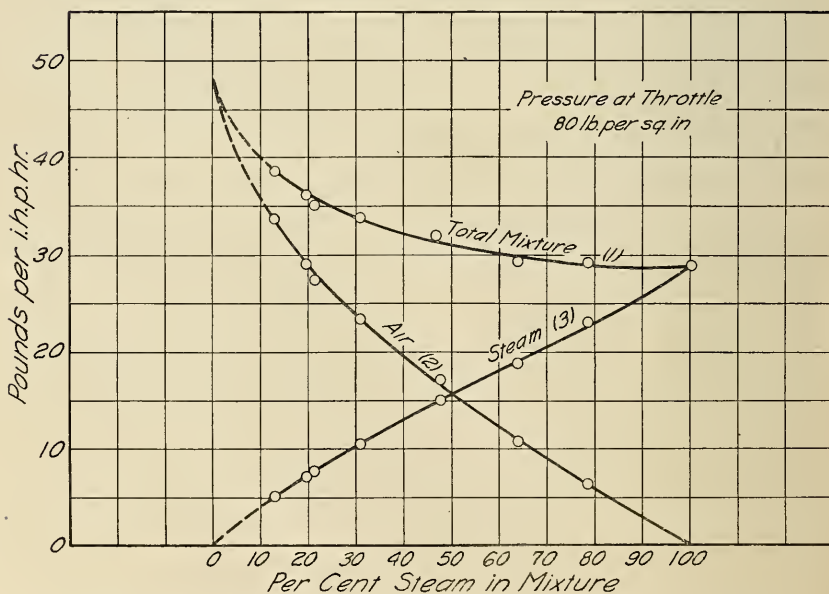


FIG. 19. CONSUMPTION OF AIR, STEAM, AND MIXTURE PER INDICATED HORSE POWER PER HOUR FOR DIFFERENT PERCENTAGES OF STEAM AND MIXTURE; THROTTLE PRESSURE 80 LB. PER SQ. IN.

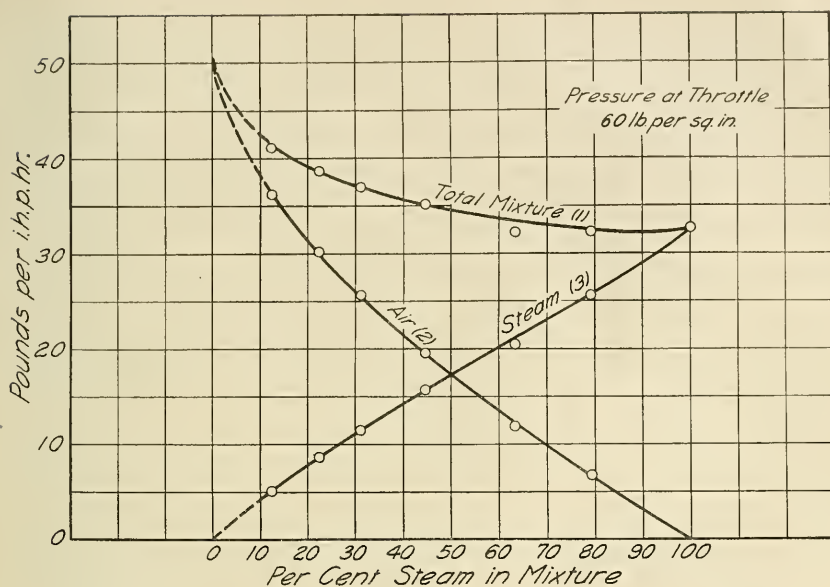


FIG. 20. CONSUMPTION OF AIR, STEAM, AND MIXTURE PER INDICATED HORSE POWER PER HOUR FOR DIFFERENT PERCENTAGES OF STEAM AND MIXTURE; THROTTLE PRESSURE 60 LB. PER SQ. IN.

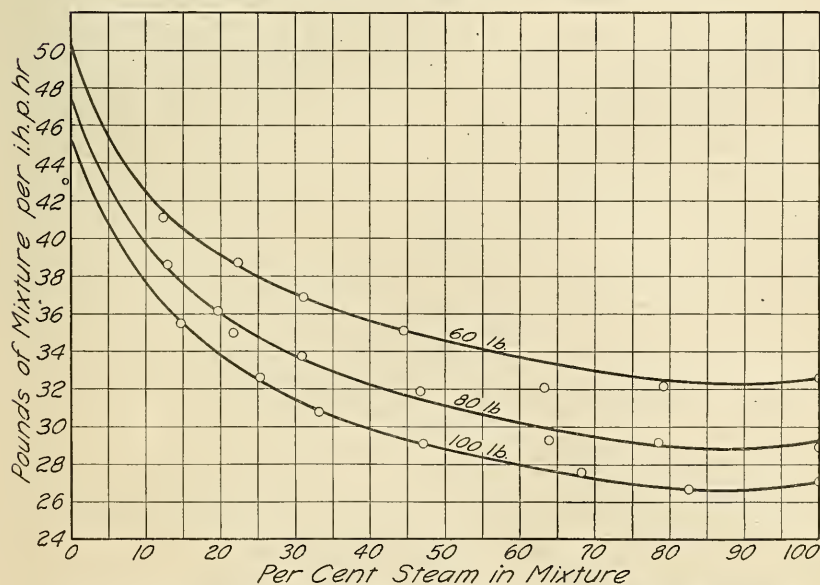


FIG. 21. CONSUMPTION OF AIR, STEAM, AND MIXTURE PER INDICATED HORSE POWER PER HOUR FOR DIFFERENT PERCENTAGES OF STEAM AND MIXTURE

be chargeable to the steam in the mixture. Items 27 and 28 give the cost of the added horse power in terms of British thermal units and pounds of dry steam. These two items increase steadily as the per cent of steam is increased. With 15 per cent of steam in the mixture the number of British thermal units required per indicated horse power per hour added is slightly more than one-half the number required per indicated horse power per hour when steam alone is used. Again, comparing the tests with 15 per cent steam with air tests at the same pressure, it is seen that the number of British thermal units required per indicated horse power per hour added in the case of air-steam mixtures is about twice as great as the number required per indicated horse power per hour gained by reheating air alone. Figure 22 shows the characteristics here discussed.

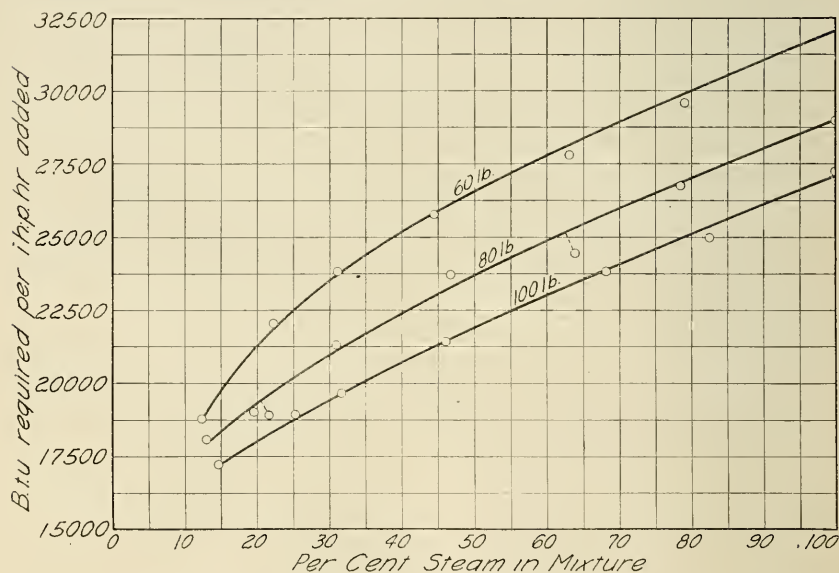


FIG. 22. HEAT REQUIRED PER ADDITIONAL INDICATED HORSE POWER PER HOUR FOR DIFFERENT THROTTLE PRESSURES AND DIFFERENT PERCENTAGES OF STEAM IN MIXTURE

Item 29 shows the best obtainable steam consumption in pounds per indicated horse power per hour, with cut-off at 25 per cent of the stroke and an initial pressure equal to the total initial pressure of the mixture in a particular test, as determined in a series of tests

using steam alone. If the engine had been operated at full load under the conditions specified in Item 29 until the weight of steam given in Item 4 was used, the number of indicated horse power hours shown in Item 30 would have been obtained. By a similar process the number of indicated horse power hours which could have been secured with the actual weight of air used in a particular test is given in Item 25. Therefore the sum of Items 25 and 30, Item 31, represents the power which could have been realized in the engine tested if the air and steam in the mixture had been used separately. The actual power secured from the mixture of air and steam in a particular test is shown under Item 10. Hence the difference between Items 10 and 31, Item 32, is the gain in power, in indicated horse power hours, which results when the engine is operated with the given mixture of the air and steam. Through this method of reasoning it is evident that the mixture of the air and steam affords a thermodynamic advantage over the use of the separate ingredients amounting to as much as 20.2 per cent, based on indicated horse power, (Test A-S-100-30), or 39.6 per cent based on brake horse power, (Test A-S-60-10).

By a similar process of calculation the economic results of the tests, based on brake horse power, are calculated and presented in Table 12 under Items 34 to 43 inclusive.

The astonishing results of these tests of air-steam mixtures must not be construed as an argument for the use of such a medium in the generation of power. If the air is compressed by a steam driven compressor the total steam consumption per horse power produced by an engine using a mixture of air and steam would probably be greater than through the use of steam alone, when the steam used in the compressor is included. When, however, a supply of compressed air is available and is used for generating power, and steam is also available, the mixture of the two affords a distinct advantage over the use of air alone or of steam alone if the cost of compressing the air is disregarded.

VI. GENERAL SUMMARY AND CONCLUSIONS

23. *Reheaters and Reheating.*—

(a) Commercial types of small external combustion reheaters will under favorable conditions and when operated at their best capacity supply 60 per cent of the heat in the fuel to the compressed air.

(b) The efficiency is dependent upon the design of the heating surface and its relation to the furnace or fire-pot. The maximum efficiency in improperly designed apparatus may be very low. Heating surface in direct contact with the fire-pot is three times as effective as such surface not in direct contact with the fire or very close to it.

(c) Internal combustion reheaters may attain efficiencies of 80 per cent or more when operating at their best capacity with reheated air temperatures of approximately 400 degrees Fahrenheit.

(d) The efficiencies of all types of reheaters decrease rapidly when the reheater is operated at low capacities. The size of the reheater should therefore be carefully selected with reference to the average capacity demanded by the service.

(e) The results of this investigation indicate that a chimney draft of from 0.15 to 0.25 inches of water is essential for the best results.

(f) One thousand pounds of air per hour or 219 cubic feet of free air per minute can be heated through a range of 100 degrees Fahrenheit with $3\frac{1}{4}$ to $3\frac{1}{2}$ pounds of fuel whose calorific value is 13 000 British thermal units per pound.

(g) To avoid excessive losses of heat the reheater must be installed as close as possible to the motor it serves.

24. *Expansive Use of Reheated Air.*—

(a) With the particular engine employed in this investigation operating with a cut-off at one-quarter stroke the air consumption per indicated horse power per hour decreases as follows, as the air temperature rises from 100 degrees to 400 degrees Fahrenheit:

With an initial gage pressure of 100 pounds per square inch the air consumption varies from approximately 45 pounds to 33 pounds, or from 10 cubic feet to 7.3 cubic feet of free air per minute.

With an initial gage pressure of 80 pounds per square inch the air consumption varies from approximately 48 pounds to 35.23 pounds or 10.5 cubic feet to 7.73 cubic feet of free air per minute.

With an initial gage pressure of 60 pounds per square inch the air consumption varies from approximately 51.1 pounds to 38.3 pounds, or 11.1 to 8.45 cubic feet of free air per minute.

(b) Similarly the air consumption per brake horse power per hour with other conditions as given above varies as follows:

at 100 pounds gage pressure, from 58.3 pounds to 36.2 pounds per hour, or from 12.77 cubic feet to 7.93 cubic feet of free air per minute;

at 80 pounds gage pressure, from 63.35 pounds to 39.45 pounds per hour, or from 13.9 cubic feet to 8.65 cubic feet of free air per minute;

at 60 pounds gage pressure, from 76.43 pounds to 45.84 pounds per hour, or 16.7 cubic feet to 10 cubic feet of free air per minute.

(c) Air at 400 degrees Fahrenheit will yield the same brake power with two-thirds the compressor capacity and steam consumption required when the air is used at 100 degrees Fahrenheit.

(d) When air is reheated from 100 degrees to 400 degrees Fahrenheit an indicated horse power per hour is gained by the expenditure of approximately 8700 to 9300 British thermal units, or 1.15 pounds of fuel, whose heating value is 13 000 British thermal units, burned in the reheater with 60 per cent efficiency. The heat required for the first 100 degree rise in temperature is particularly effective. In this range an indicated horse power was gained by the addition of about 7700 British thermal units or less than one pound of fuel.

(e) Reheating the air from 100 degrees to 400 degrees Fahrenheit permits the gain of a brake horse power per hour with an expenditure of heat as low as 8500 British thermal units. In Test 3-100-200-15 a brake horse power was gained with an ex-

penditure of only 7035 British thermal units or about 0.9 pounds of fuel. Test 44-60-200-25 shows an even more astonishing result for a rise of temperature of 100 degrees only.

(f) For the initial pressures employed in these tests the highest efficiencies were obtained when the cut-off was at about one-quarter of the stroke. Complete expansion does not show equally good results.

(g) The use of reheated air leads to an improvement in lubrication, an astonishing increase in mechanical efficiency, and the prevention of ice formation in motors using air expansively. In practice the chief reason for the use of reheaters is the prevention of interference with the operation of the motor through the accumulation of ice in the exhaust pipe.

25. *Use of Steam for Reheating Air.*—

(a) Steam is a convenient medium for reheating air, and a mixture of air and steam containing as little as 10 per cent of steam will entirely prevent the operating difficulties experienced when cold air is used in expansion motors through defective lubrication and the formation of ice in the exhaust pipe.

(b) A greater amount of work is obtained through the use of a mixture of air and steam than could be realized by the separate use of the same weights of each of the two substances. When the steam constitutes 15 to 20 per cent of the mixture there is a gain of from 16 to 20 per cent in the indicated horse power, and of from 25 to 35 per cent in the brake horse power of the engine. An increase in the weight of steam in the mixture reduces the gain.

(c) A much greater expenditure of heat is required per horse power gained when steam is used than is necessary with other methods of reheating.

LIST OF
PUBLICATIONS OF THE ENGINEERING EXPERIMENT STATION

Bulletin No. 1. Tests of Reinforced Concrete Beams, by Arthur N. Talbot. 1904. *None available.*

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Bulletin No. 9. An Extension of the Dewey Decimal System of Classification Applied to the Engineering Industries, by L. P. Breckenridge and G. A. Goodenough. 1906. Revised Edition, 1912. *Fifty cents.*

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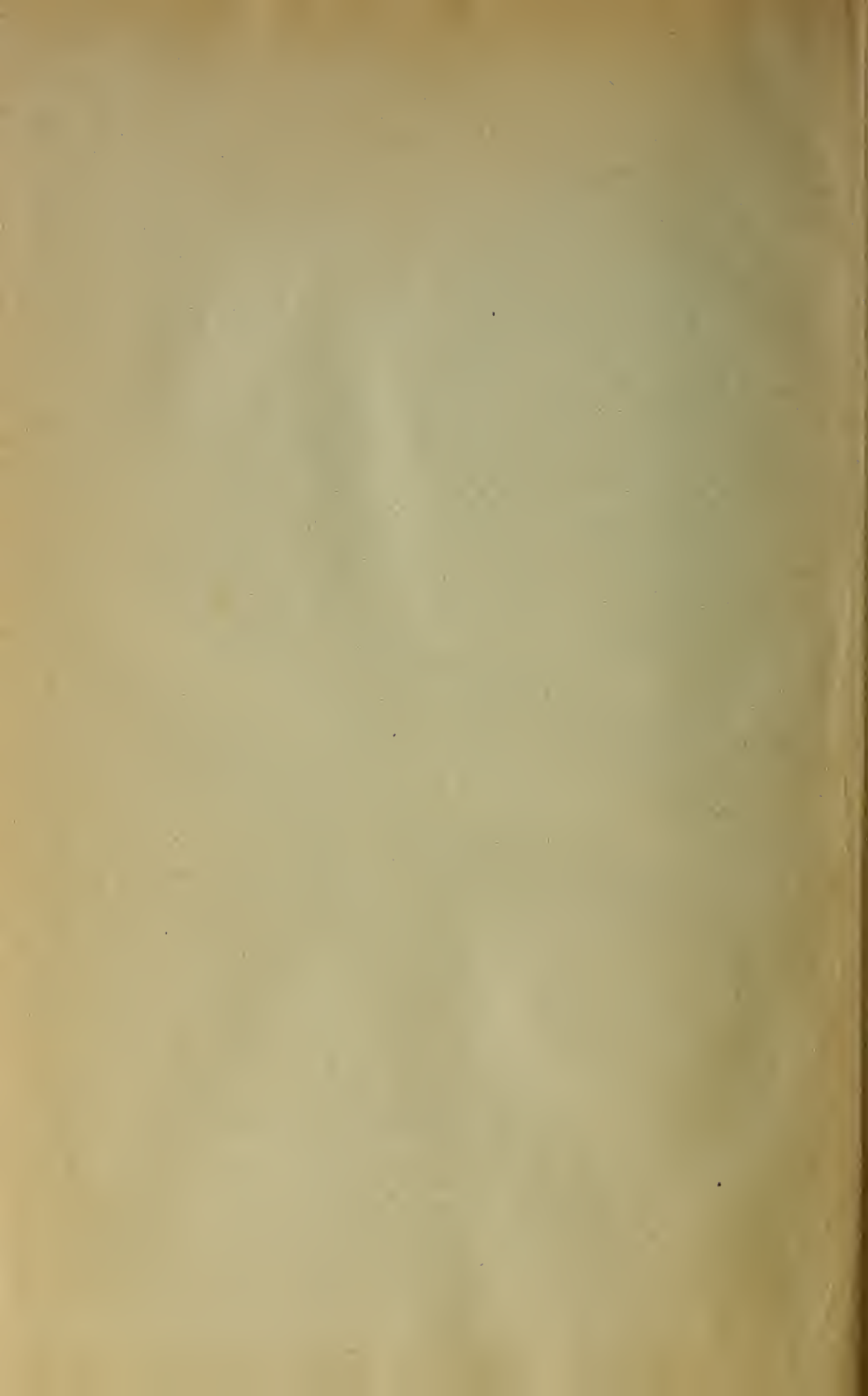
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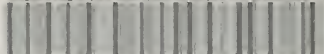
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